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Excitation of Transmission Lines by Distributed Sources

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EXCITATION OF TRANSMISSION LINES BY DISTRIBUTED SOURCES

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TABLE OF CONTENTS

ABSTRACT

A new method of solving the problem of transmission lines excited by distributed series voltage sources and shunt current sources is presented. This solution procedure is convenient for electrical engineers familiar with transmission line theory. This theory can be applied in evaluating electrical transients induced on cables by externally originated excitation sources such as incident electromagnetic fields [EMI (electromagnetic interference), EMP (electromagnetic pulse), and lightning] or ionizing radiation.

INTRODUCTION

The solution described in this paper is offered as an alternative to classical solutions of the problem of transmission lines excited by distributed sources. The perceived merits of the new solution are that it is concise, easy to understand, and simple to use for engineers familiar with basic transmission line theory.

The problem of transmission line excitation by distributed sources is familiar to electrical engineers because it is encountered in the coupling of electromagnetic waves on lines. Present solution methods of this problem (in terms of Green's functions[1] and others[2]) are refinements of classical solutions.[3,4] These solutions are typically presented as unique and independent of the conventional transmission line theory. The new solution procedure described in this paper uses circuit analysis to translate the effects of these distributed sources into equivalent voltages and currents that are compatible with the voltage and current equations describing the normal mode of operation of the transmission line.

TRANSMISSION LINE THEORY RESULTS

In its normal mode of operation, the line of length ℓ, characteristic impedance Z_O, and propagation constant γ is driven by voltage source V_1 with internal impedance Z_1 and is terminated with load of impedance Z_2. The voltage $V(x)$ and current $I(x)$, at the point x from the input of the line, are given[5] by

$$V(x) = V_i(0)\ e^{-\gamma x}(1 + \rho_L) \quad , \tag{1}$$

$$I(x) = \frac{V_i(0)}{Z_O}\ e^{-\gamma x}(1 - \rho_L) \quad , \tag{2}$$

$$V_1(0) = \frac{V_1(1 - \rho_1)}{2\left(1 - \rho_1\rho_2 e^{-2\gamma\ell}\right)} \quad , \tag{3}$$

$$\rho_L = \rho_2\, e^{-2\gamma(\ell - x)} \quad , \quad \rho_S = \rho_1\, e^{-2\gamma x} \quad , \tag{4}$$

where

$\rho_1 = (Z_1 - Z_O)/(Z_1 + Z_O)$ and $\rho_2 = (Z_2 - Z_O)/(Z_2 + Z_O)$.

Left of point x (input), the line looks like an impedance $Z_S(x)$ and right of point x like an impedance $Z_L(x)$ given by

$$Z_S(x) = Z_o \frac{1 + \rho_S}{1 - \rho_S} \quad , \quad Z_L(x) = Z_o \frac{1 + \rho_L}{1 - \rho_L} \quad . \tag{5}$$

DESCRIPTION OF THE NEW SOLUTION

Consider the circuit configuration of Fig. 1. The sources E(x)dx and J(x)dx launch two propagating waves, one to the right (+) and one to the left (-), on the line. The analysis of the transmission line response to the sources E(x)dx and J(x)dx will be subdivided into two parts, one each for the line sections to the right and to the left of x.

The response of the line to the right of x can be obtained by replacing the line to the left of x by its Thevenin equivalent circuit. The Thevenin equivalent voltage is

$$dV_t = \left[E(x) + J(x)Z_S\right] dx = \frac{E(x)(1 - \rho_S) + Z_o J(x)(1 + \rho_S)}{(1 - \rho_S)}\, dx \quad . \tag{6}$$

The response of this section is described by Eqs. (1) through (5) with V_1 replacing dVt and Z_1 replacing $Z_S(x)$.

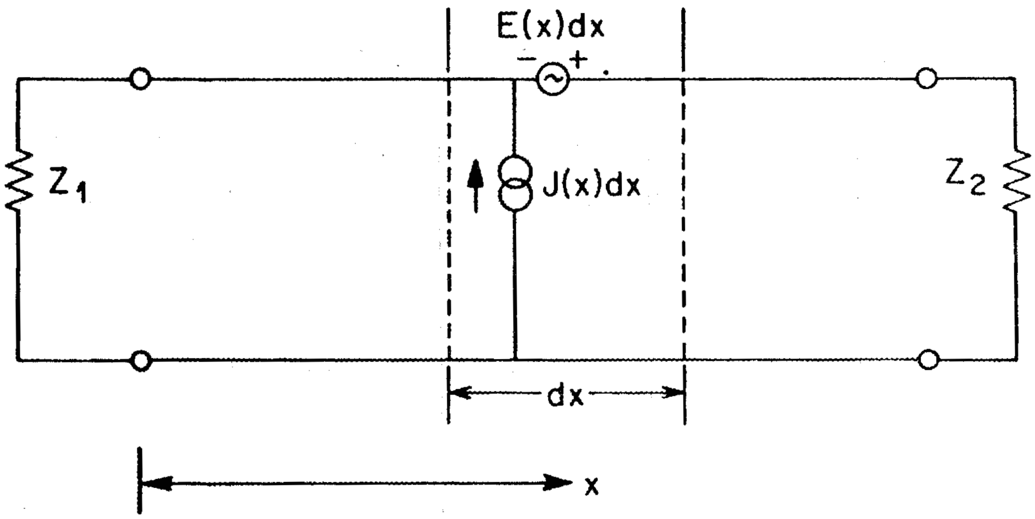

Fig. 1. Transmission line excited by distributed sources.

The incident voltage wave to the right of x is

$$dV_+(x) \equiv V_+(x)dx = \frac{\left[Z_oJ(x) + E(x)\right] + \left[Z_oJ(x) - E(x)\right] \rho_S}{\Delta} dx \quad , \qquad (7)$$

where

$$\Delta = 2\left(1 - \rho_1\rho_2 e^{-2\gamma\ell}\right) . \qquad (8)$$

The incremental voltage and current responses, at point z on the line, to the sources E(x)dx and J(x)dx are:

$$dV_+(z) = V_+(x)\left[1 + \rho_2 e^{-2\gamma(\ell - z)}\right] e^{-\gamma(z-x)} dx \quad , \quad z > x \quad , \qquad (9)$$

$$dI_+(z) = \frac{V_+(x)}{Z_o}\left[1 - \rho_2 e^{-2\gamma(\ell-z)}\right] e^{-\gamma(z-x)}\, dx \quad , \quad z > x \quad , \tag{10}$$

The subscript plus (+) denotes the wave traveling in the positive x direction.

Similarly, the response of the line section to the left of x can be obtained by representing the line section to the right of x by its Thevenin equivalent circuit. The Thevenin equivalent voltage is

$$dV_t = \left[-E(x) + Z_L J(x)\right] dx = \frac{-E(x)(1 - \rho_L) + Z_o J(x)(1 + \rho_L)}{1 - \rho_L}\, dx \quad . \tag{11}$$

Consequently, for the line section we find that the incident voltage wave to the left of x is

$$dV_-(x) = V_-(x)dx = \frac{\left[Z_o J(x) - E(x)\right] + \left[Z_o J(x) + E(x)\right]\rho_L}{\Delta}\, dx \quad . \tag{12}$$

The incremented voltage and current at a point $z < x$ are

$$dV_-(z) = V_-(x)\left(1 + \rho_1 e^{-2\gamma z}\right) e^{-\gamma(x-z)} dx \quad , \quad z < x \quad , \tag{13}$$

$$dI_-(z) = -\frac{V_-(x)}{Z_o}\left(1 - \rho_1 e^{-2\gamma z}\right) e^{-\gamma(x-z)} dx \quad , \quad z < z \quad . \tag{14}$$

The total voltage and current responses to E(x)dx and J(x)dx, at any point z of the line are:

$$V(z) = \int_o^z dV_+(z) + \int_z^\ell dV_-(x)$$

$$= \left[1 + \rho_2 e^{-2\gamma(\ell-z)}\right] e^{-\gamma z}\, V_+(z) + \left(1 + \rho_1 e^{-2\gamma z}\right) e^{\gamma z}\, V_-(z) \quad , \tag{15}$$

$$I(z) = \int_0^z dI_+(z) + \int_z^\ell dI_-(z)$$

$$= \left[1 - \rho_2 e^{-2\gamma(\ell - z)}\right] e^{-\gamma z} \frac{V_+(z)}{Z_o} - \left(1 - \rho_1 e^{-2\gamma z}\right) e^{\gamma z} \frac{V_-(z)}{Z_o} \quad , \qquad (16)$$

where

$$V_+(z) = \int_0^z V_+(x)\, e^{\gamma x} dx \quad , \qquad (17)$$

$$V_-(z) = \int_z^\ell V_-(x)\, e^{-\gamma x} dx \quad . \qquad (18)$$

The response at the termination Z_2 can be found by the substitution $z = \ell$ in the above equations; they are:

$$V(\ell) = (1 + \rho_2)\, e^{-\gamma \ell} \int_0^\ell V_+(x)\, e^{\gamma x} dx \quad , \qquad (19)$$

$$I(\ell) = \frac{(1 - \rho_2)\, e^{-\gamma \ell}}{Z_o} \int_0^\ell V_+(x)\, e^{\gamma x} dx \quad . \qquad (20)$$

Similarly, the voltage and current at the termination Z_1, are

$$V(0) = (1 + \rho_1) \int_0^\ell V_-(x)\, e^{-\gamma x} dx \quad , \qquad (21)$$

$$I(0) = -\frac{(1 - \rho_1)}{Z_o} \int_0^\ell V_-(x)\, e^{-\gamma x} dx \quad . \qquad (22)$$

The above analysis and results apply also for lumped (point) sources. Point sources E and J, at point x_o of the line, can be expressed as in terms of the delta function as

$$E(x) = E\,\delta(x - x_o) \text{ and } J(x) = J\,\delta(x - x_o) \quad . \tag{23}$$

Substitution of these point sources in Eqs. (15) and (16) yield:

for $z > x_o$

$$V(z) = \left[1 + \rho_2\, e^{-2\gamma(\ell - z)}\right] V_+(x_o)\, e^{-\gamma(z - x_o)} \quad , \quad z > x_o \quad , \tag{24}$$

$$I(z) = \left[1 - \rho_2\, e^{-2\gamma(\ell - z)}\right] \frac{V_+(x_o)}{Z_o}\, e^{-\gamma(z - x_o)} \quad , \quad z < x_o \quad , \tag{25}$$

for $z < x_o$

$$V(z) = \left(1 + \rho_1\, e^{-2\gamma z}\right) V_-(x_o)\, e^{-\gamma(x_o - z)} \quad , \quad z < x_o \quad , \tag{26}$$

$$I(z) = \left(1 - \rho_1\, e^{-2\gamma z}\right) \frac{V_-(x_o)}{Z_o}\, e^{-\gamma(x_o - z)} \quad , \quad z < x_o \quad . \tag{27}$$

As a demonstration, consider the line driven by the source $E = V_1$ at the input ($x_o = 0$). Substitution in Eqs. (24) and (25) yields

$$V(z) = \frac{V_1}{\Delta}\,(1 - \rho_1)\, e^{-\gamma z} \left[1 + \rho_2\, e^{-2\gamma(\ell - z)}\right] \quad , \tag{28}$$

and

$$I(z) = \frac{V_1(1 - \rho_1)}{\Delta Z_o}\, e^{-\gamma z} \left[1 - \rho_2\, e^{-2\gamma(\ell - z)}\right] \quad , \tag{29}$$

as expected from conventional transmission line theory, Eqs. (1) and (2).

This procedure was applied to the solution of EMP-induced transients in transmission lines. By comparison with previous solution procedures, this method is more convenient for use by engineers familiar with transmission line theory.

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