ORNL/TM-10764
(CESAR-88/19)

ornl

OAK RIDGE
NATIONAL
LABORATORY

MARTIN MARIETTA

Adaptive Optimal Control of Uncertain Nonlinear Systems: On-Line Microprocessor-Based Algorithm to Control Mechanical Manipulators

Carlos March-Leuba
R. B. Perez

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
CIRCULATION SECTION
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this report, send in name with report and the library will arrange a loan.

OPERATED BY
MARTIN MARIETTA ENERGY SYSTEMS, INC.
FOR THE UNITED STATES
DEPARTMENT OF ENERGY

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A03; Microfiche A01

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

ORNL/TM-10764
CESAR-88/19

Engineering Physics and Mathematics Division

ADAPTIVE OPTIMAL CONTROL OF UNCERTAIN NONLINEAR SYSTEMS: ON-LINE MICROPROCESSOR-BASED ALGORITHM TO CONTROL MECHANICAL MANIPULATORS

Carlos March-Leuba and R. B. Perez

Date Published: May 1988

Research sponsored by the Engineering Research Program
of the Office of Basic Energy Sciences,
U.S. Department of Energy

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831
operated by
MARTIN MARIETTA ENERGY SYSTEMS, INC.
for the
U.S. DEPARTMENT OF ENERGY
under contract DE-AC05-84OR21400

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0276327 6

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This paper presents an adaptive optimal control algorithm for uncertain nonlinear systems. A variational technique based on Pontryagin's Maximum Principle is used to track the system's unknown terms and to calculate the optimal control. The reformulation of the variational technique as an Initial Value Problem allows this microprocessor-based algorithm to perform on-line model-updating and control. To validate the algorithm a system representing a two-link mechanical manipulator is simulated. In the control model, the coupling and friction terms are unknown. The robot's task is to follow a prescribed trajectory and to pick up an unknown mass.

I. INTRODUCTION

"Models of realistic systems are seldom completely known and if known, they are seldom linear." This statement, issued at the Santa Clara Workshop[1] on "Challenges to Control: A Collective View," summarizes the motivation for the efforts being made lately on the development of control algorithms for uncertain nonlinear systems.

The newly available, fast, efficient and reliable microprocessors have opened the possibility of performing on-line numerical calculations to process the information given by the plant sensors. The analysis of these signals allows one to update the incomplete model of the plant and to calculate the control that should be input to the plant.

Although optimal control theory provides[2] the needed tools to perform this analysis, the actual application of Pontriagin's Maximum Principle (PMP) to control algorithms is hindered by the fact that drastic simplifications must be made to obtain on-line solutions. For instance, the Linear-Quadratic (LQ) algorithm, is based on linearized plant models and assumes that the functional relation between the state and adjoint vectors is always in steady-state. The LQ algorithm is able to give a fast feedback response, but the controls obtained are suboptimal, and LQ is not able to use the information coming from the detectors to improve the model assumed for the plant.

The ideal demand-following adaptive control algorithm should handle directly the nonlinear model, and provide an on-line solution for the unknown part of the model and for the optimal control. These conditions can be achieved by the implementation of the PMP to an approximated model of the plant. Two simultaneous optimizations must be performed: (a) the uncertain terms are determined by the optimal matching of the system signals to the control model, and (b) the optimal controls are obtained by matching a set of prescribed demands to the updated model. However, a major problem appears in regard with the numerical solution of the Two Point Boundary Value (TPBV) problem which arises in the implementation of the PMP methodology.

In this paper, a new approach to the PMP formulation is used,[3,4] which allows to solve the TPBV as an initial value problem for the case in which the plant is initially in equilibrium and both prescribed demands and uncertain terms are well behaved functions of time. The solution of the plant model equations coupled with the algorithm equations in a microprocessor, allows to update the model and control the plant on-line.

Sections II and III develop the basic control theory used. In Section IV, an example of how this control algorithm can be easily implemented is shown, by applying the control algorithm to a two-link mechanical manipulator. Section V presents the results and conclusions. Finally, some analytical details are given in Appendix A and B.

II. UNCERTAINTY TRACKING AND CONTROL

Given a dynamical system, described by the state vector, $\vec{X}(t)$, which satisfies the nonlinear set of coupled equations

$$\dot{\vec{X}} = \vec{F}\,(\vec{X},\ \vec{U},\ \vec{a}) \tag{1}$$

where $\vec{F}$ is a nonlinear vector-valued function of the state vector, we consider two classes of uncertain systems: (a) systems where there is a subset $\vec{a}_1$, of parameters which vary with time in a manner unknown to the system's model, and (b) systems where a part, $F_1(\vec{X},\vec{U},\vec{a})$, of the system dynamics is unavailable to the model. The first class of uncertain nonlinear systems was studied in detail in Ref. 3. In this work we shall elaborate on the second class. We consider the following control scenario:

(a) The system is initially at equilibrium ($\dot{\vec{X}} = 0$) and $\vec{X}(0)$ and $\vec{U}(0)$ are known.

(b) The system must follow a prescribed trajectory, $\vec{D}(t)$, up to a given position.

(c) The plant model is incomplete in the sense that only a subset, $\vec{F}_o(\vec{X},\vec{U},\vec{a})$, of the actual vector-valued function $\vec{F}(\vec{X},\vec{U},\vec{a})$ is accounted for by the model.

(d) A set of plant signals, $\vec{S}(t)$, is available to the model.

Within the framework of the above scenario the control problem can be solved by performing two simultaneous optimizations: In the first one, the uncertain term is obtained by matching the signals from the plant to the model; in the second, the optimal controls are calculated by matching the updated model to the prescribed demands.

Under the assumption that the system is at equilibrium at t=0, and it matches the prescribed demand at an undetermined time, T_F, both optimizations can be performed as a Free Terminal Time (FTT) problem.[2]

The adaptive optimal control can be formulated as follows:

Let

$$\dot{\vec{M}} = \vec{G}\ (\vec{M},\vec{U},\vec{a}) \tag{2}$$

define the incomplete model of the plant, where the vector, $\vec{M}$, is the model prediction for the actual state vector, $\vec{X}$, and the vector valued function, $\vec{G}(\vec{M},\vec{U},\vec{a})$, is given by

$$\vec{G}(\vec{M},\vec{U},\vec{a}) = \vec{F}_o(\vec{M},\vec{U},\vec{a}) + \vec{P}\ (t) \tag{3}$$

where $\vec{P}(t)$, the "uncertain" term, is the vector-valued function which should be manipulated to mimick the unmodeled part of the actual plant.

Then, to construct a control algorithm capable of following a prescribed demand while adjusting itself to match the plant signals coming from the actual system, we set up the following cost functions.

$$J_c = \int_o^{T_F} dt\ V_c\ (\vec{M},\vec{U}), \tag{4}$$

$$J_p = \int_o^{T_F} dt\ V_p\ (\vec{M},\vec{P}) \tag{5}$$

with the constraint that the model equation (2) must be satisfied at all times. By calling D(M) the subset of the state vector $\vec{M}$ which must follow prescribed demands, and S(M), the subset of the state vector, $\vec{M}$, associated with signals from the plant, the quantities V_c and V_p are given by

$$V_c = \frac{1}{2} \{(\vec{D}-\vec{D}(\vec{M}))^T \underline{Q}_c (\vec{D}-\vec{D}(M)) + (\vec{U}-\vec{U}_E)^T \underline{R}_c (\vec{U}-\vec{U}_E)\} \tag{6}$$

$$V_p = \frac{1}{2} \{(\vec{S}-\vec{S}(\vec{M}))^T \underline{Q}_p (\vec{S}-\vec{S}(\vec{M})) + (\vec{P}-\vec{P}_E)^T \underline{R}_p (\vec{P}-\vec{P}_E)\} \tag{7}$$

where, $\underline{Q}_c(E)$, $\underline{R}_c(t)$, $\underline{Q}_p(t)$ and $\underline{R}_p(t)$ are weight matrices, and $\vec{U}_E$ and $\vec{P}_E$ are respectively the equilibrium values of the control vector $\vec{U}_E$ and the "uncertain" vector-valued function $P(_T)$. The minimization of $\vec{J}_p$ determines the uncertain terms, $\vec{P}(t)$, whereas that of J_c provides the optimal controls to match the demands, $\vec{D}(t)$. Following Pointryagin's approach[2,3] we construct the following Hamiltonian functions:

$$H_c(\vec{M},\vec{U},\vec{W}) = V_c(\vec{M},\vec{U})+\vec{W}^T\vec{G}(\vec{M},\vec{U},\vec{P}) \tag{8}$$

$$H_p(\vec{M},\vec{Z},\vec{P}) = V_p(\vec{M},\vec{P})+\vec{Z}^T\vec{G}(\vec{M},\vec{U},\vec{P}) \tag{9}$$

where, $\vec{W}(t)$ and $\vec{Z}(t)$, are the adjoint vectors (Lagrange multipliers) of the control system. Note: (a) that both Hamiltonian functions vanish at equilibrium, as is required in FTT problems,[3] and (b) that in the control Hamiltonian, H_c, the uncertain quantity, $\vec{P}(t)$, is to be kept constant, whereas in the uncertainty tracking Hamiltonian, H_p, the control, $\vec{U}$, is kept constant. The usual PMP algorithm[2,3] determines the control vector, $\vec{U}$, and the uncertain quantity $\vec{P}(t)$ by means of the relations

$$\frac{\partial H_c}{\partial \vec{U}} = 0; \quad \frac{\partial H_p}{\partial \vec{P}} = 0 \tag{10}$$

and the adjoint vectors by the Hamilton equations

$$- \dot{\vec{W}} = \frac{\partial H_c}{\partial \vec{M}} \; ; \; - \dot{\vec{Z}} = \frac{\partial H_p}{\partial \vec{M}} \tag{11}$$

while the model equations (II.2) are recovered from

$$\dot{\vec{M}} = \frac{\partial H_c}{\partial \vec{W}} = \frac{\partial H_p}{\partial \vec{Z}} \tag{12}$$

with the conditions

$$H_c(T_F) = H_p(T_F) = 0 \quad . \tag{13}$$

To solve the set of equations (11) and (12) for a N^{th}-order model one needs 2N conditions. Since the state of the system is known at t=0, and $t=T_F$, the 2N quantities, $\vec{M}(t=0)$ and $\vec{M}(t=T_F)$ are already available. We have therefore defined a two-point boundary value (TPBV) problem, whereby the model equations are to be integrated forward in time and the adjoint equation backwards, following an iterative procedure which will last until the conditions, at both ends of the transient have been met.

III. PROBLEM REFORMULATION

As stated before, our goal is to develop an on-line algorithm which must be able to update a model while calculating the optimal control. Clearly, the iterative solution of the equations presented in Section II will be time consuming. However, as shown in Ref. 3, whenever the system is initially in equilibrium, and the demand is a well behaved function of time, the TPBVP can be reformulated as an Initial Value Problem allowing fast and efficient on-line solutions.

Let us now assume that at some time T_F the system will follow the demand. The feedback control obtained in Section II is a state variable function; therefore, does not depend on T_F, but on the difference, $\Delta T = T_F - t_o$, where, t_o, is the initial time. It follows, then, that the minimization of the cost function can be done backwards, assuming, T_F, known, and, t_o, unknown. Based on these ideas the FTT problem can be reformulated (see Ref. 3) as:

$$\dot{\vec{M}} = \frac{\partial H_c}{\partial \vec{W}} = \frac{\partial H_p}{\partial \vec{Z}} \tag{14}$$

$$\dot{\vec{W}} = \frac{\partial H_c}{\partial \vec{M}} \; ; \; \dot{\vec{Z}} = \frac{\partial H_p}{\partial \vec{M}} \tag{15}$$

with the conditions:

$$H_c(t=0) = H_p(t=0) = 0 \tag{16}$$

and the initial conditions

$$\vec{M}(t=0) = \vec{M}_o ; \; \dot{\vec{W}} = \dot{\vec{Z}} = 0 \tag{17}$$

whereas as before, the control vector, $\vec{U}$, and the uncertain vector, $\vec{P}$, are extracted from the relations in Eq. 10.

IV. APPLICATION TO THE CONTROL OF A MECHANICAL MANIPULATOR

The previously developed Initial Value formulation of the FTT problem will be illustrated with an application to a two-link mechanical manipulator. The control problem consists of the determination of the optimal set of torques, T_1 and T_2, to be supplied to the arm, so that its end-effector moves along a prescribed trajectory, while only an uncertain (e.g.: incomplete) model of the arm dynamics is available. In the present illustration the "real world" mechanical manipulator (the plant) is simulated by the numerical solution of the Euler-Lagrange equations for a two-link arm, as developed in Paul's textbook,[5] with the addition of friction forces and a time-varying end-effector mass (see Appendix A). The control algorithm, which is based on an uncertain model, supplies the torques to the actual arm and updates itself to match the tachometer signals, S_3 and S_4, which are supplied by the plant.

IV.a. DYNAMICS AND CONTROL

The uncertain model for the control algorithm is derived from the following simplified Lagrangian functions (see Fig. 1 for symbol descriptions)

$$L_1 = \frac{1}{2} m_1 b_1^2 \dot{\theta}_1^2 + m_1 g b_1 \cos(\theta_1) \tag{18}$$

$$L_2 = \frac{1}{2} m_2 b_2^2 \dot{\theta}_2^2 + m_2 g (b_1 + b_2) \cos(\theta_1 + \theta_2) \tag{19}$$

written for each individual link, and where only inertial and gravitational terms were accounted for in the energy balance.

The corresponding Euler-Lagrange equations are:

$$\ddot{\theta}_1 = C_{1g} \operatorname{Sin} \theta_1 + C_{1T} T_1 \tag{20}$$

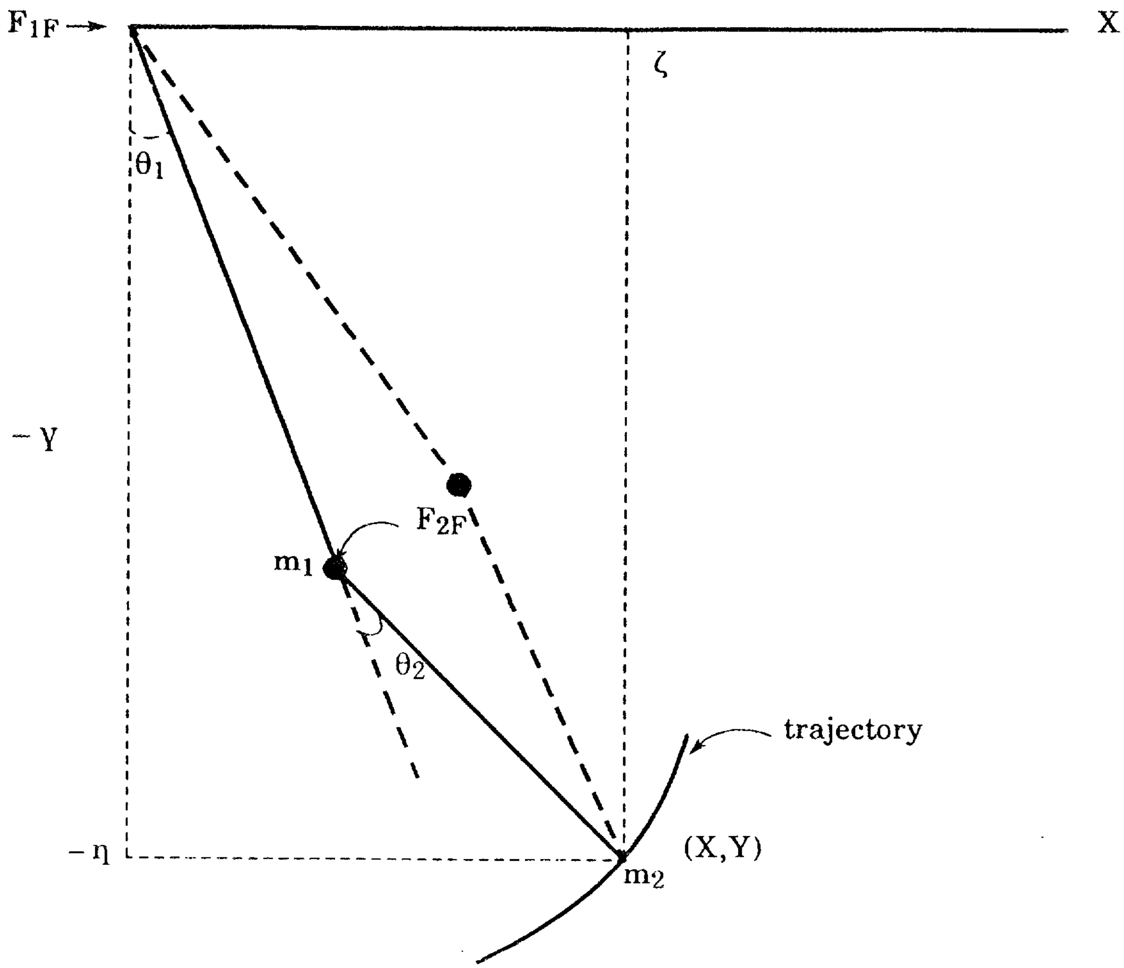

Fig. 1. Configuration of the two-link arm. The angular coordinates are θ_1 and θ_2. The trajectory coordinates are represented by (X,Y) and the cartesian coordinates by (ξ, η). The arm configuration in broken lines illustrates a singularity in the inverse coordinate transformation needed to locate the arm's end-effector on the prescribed trajectory. F_{1F} and F_{2F} are the friction forces added to the system.

$$\ddot{\theta}_2 = C_{2g} \text{ Sin } (\theta_1+\theta_2) + C_{2T}T_2 \tag{21}$$

with

$$C_{1g} = -\frac{g}{b_1}; \; C_{2g} = -\frac{1}{b_2{}^2}(b_1+b_2)g \tag{22}$$

$$C_{1T} = \frac{1}{m_1 b_1{}^2} \; ; \; C_{2T} = \frac{1}{m_2 b_2{}^2} \; . \tag{23}$$

By letting $M_1 = \theta_1$ and $M_2 = \theta_2$, the uncertain model is constructed via the conversion of Eqs. (20) and (21) into a set of first order differential equations, i.e.:

$$\dot{M}_1 = G_1; \; \dot{M}_2 = G_2; \; \dot{M}_3 = G_3; \; \dot{M}_4 = G_4 \tag{24}$$

with

$$G_1 = M_3; \; G_2 = M_4 \tag{25}$$

$$G_3 = C_{1g}\text{Sin}(M_1) + C_{1T} \; T_1 + P_1(t) \tag{26}$$

$$G_4 = C_{2g}\text{Sin}(M_1+M_2) + C_{2T} \; T_2 + P_2(t) \tag{27}$$

where $P_1(t)$ and $P_2(t)$ are the uncertain terms of the model.

The control and signal tracking Hamiltonian functions, Eqs.(8) and (9) respectively are now written as:

$$H_c = \frac{1}{2}(D_1-M_3)^2 Q_{c1} + \frac{1}{2}(D_2-M_4)^2 Q_{c2} + \frac{1}{2}(T_1-T_{1E})^2 R_{c1} + \frac{1}{2}(T_2-T_{2E})^2 R_{c2}$$

$$+ \sum_{i=1}^{4} W_i G_i \tag{28}$$

and

$$H_p = \frac{1}{2}(S_3-M_3)^2 Q_{p3} + \frac{1}{2}(S_4-M_4)^2 Q_{p4} + \frac{1}{2} P_1{}^2 R_{p1} + \frac{1}{2} P_2{}^2 R_{p2}$$

$$+ \sum_{i=1}^{4} Z_i \; G_i \tag{29}$$

where in writing Eq. (29), we did set to zero the equilibrium values for the uncertain terms P_{1E} and P_{2E}.

Note that in the above equations, the quantities, M_3 and M_4 are the angular velocities predicted by the uncertain model; the demands D_1 and D_2 are the angular velocities that the arm should have to follow the prescribed trajectory, whereas the signals S_3 and S_4 are the actual arm angular velocities. Clearly the sets of angular velocities, $(\dot{M}_3, \dot{M}_4)$, (S_1, S_2) and (D_1, D_2) should attain the same numerical value under perfect control conditions. In writing the control Hamiltonian function above, we have found convenient to define the equilibrium torques T_{1E} and T_{2E} as those functions which at any given time would make $\dot{M}_3$ and $\dot{M}_4$ to vanish. Consequently, one obtains by setting to zero G_3 and G_4 in Eqs. (26) and (27).

$$T_{1E}(t) = -\frac{1}{C_{1T}}\left(P_1(t) + C_{1g}\right) \text{Sin}\,\left(M_1(t)\right). \tag{30}$$

$$T_{2E}(t) = -\frac{1}{C_{2T}}\left(P_2(t) + C_{2g}\ \text{Sin}\,(M_1+M_2)\right). \tag{31}$$

Insertion of the above equations in Eq. (28) for the control Hamiltonian, H_c yields

$$H_c = \frac{1}{2}\left(D_1-M_3\right)^2 Q_{c1} + \frac{1}{2}\left(D_2-M_4\right)^2 Q_{c2} + \frac{1}{2}\left(T_1 + \frac{1}{C_{1t}}[P_1 + C_{1G}\text{Sin}(M_1)]\right)^2 R_{c1}$$

$$+ \frac{1}{2}\left(T_2 + \frac{1}{C_{2T}}[P_2 + C_{2g}\text{Sin}(M_1+M_2)]\right)^2 R_{c2} + \sum_{i=1}^{4} W_i G_i \tag{32}$$

Optimal estimates of torques and uncertain terms are obtained from the conditions $\partial H_c/\partial T_i = \partial H_p/\partial P_i = 0$, $(i=1,4)$, i.e.:

$$T_1 = -\frac{1}{C_{1T}}\left(C_{1g}\mathrm{Sin}(M_1) + P_1\right) - \frac{C_{1T}}{R_{c1}} W_3 \tag{33}$$

$$T_2 = -\frac{1}{C_{2T}}\left(C_{2g}\mathrm{Sin}\ (M_1 + M_2) + P_2\right) - \frac{C_{2T}}{R_{c2}} W_4 \tag{34}$$

$$P_1 = -\frac{Z_3}{R_{p1}}\ ;\ P_2 = -\frac{Z_4}{R_{p2}} \tag{35}$$

The equations for the adjoint vectors, $\vec{W}$ and $\vec{Z}$ are obtained from the relations (15) in Section III, using the Hamiltonians (29) and (32). Insertion of Eqs. (33) up to (35) for the optimal torques and uncertain terms into the model Eq. (24) and into the adjoint equations obtained in the previous step, yields:

$$\dot{M}_1 = M_3;\ M_2 = \dot{M}_4 \tag{36}$$

$$\dot{M}_3 = -\frac{C_{1T}{}^2}{R_{c1}} W_3 \tag{37}$$

$$\dot{M}_4 = -\frac{C_{2T}{}^2}{R_{c2}} W_4 \tag{38}$$

$$\dot{W}_1 = \dot{W}_2 = 0 \tag{39}$$

$$\dot{W}_3 = W_1 - \left(D_1 - M_1\right)Q_{c1} \tag{40}$$

$$\dot{W}_4 = W_2 - \left(D_2 - M_2\right)Q_{c2} \tag{41}$$

$$\dot{Z}_1 = C_{1g}\ \cos\ \left(M_1\right)\ Z_3 + C_{2g}\ \cos\ \left(M_1 + M_2\right)\ Z_4 \tag{42}$$

$$\dot{Z}_2 = C_{2g} \cos \left(M_1 + M_2\right) Z_4 \tag{43}$$

$$\dot{Z}_3 = Z_1 - \left(S_3 - M_3\right) Q_{p3} \tag{44}$$

$$\dot{Z}_4 = Z_2 - \left(S_4 - M_4\right) Q_{p4} \tag{45}$$

The initial conditions for the state vector, $\vec{M}$, are (on account that the system is initially at equilibrium):

$$M_i(0) = 0 \ (i = 1,4). \tag{46}$$

The initial condition for the adjoints are obtained as previously stated in Section III, by the condition that the adjoint vectors are initially at equilibrium. Then setting the time derivatives equal to zero and taking in account that at t=0, $D_i=S_i=M_i$ (i=1,4) vanish, one obtains from Eqs. (40), (41), (44) and (45),

$$W_1(0) = W_2(0) = Z_1(0) = Z_2(0) = 0 \tag{47}$$

and from Eqs. (42) and (43)

$$Z_3(0) = Z_4(0) = 0. \tag{48}$$

Finally from Eqs. (37) and (38) at equilibrium one obtains

$$W_3(0) = W_4(0) = 0 \tag{49}$$

IV.b. TRAJECTORY PRESCRIPTION AND KINEMATICS

We must now address the problems of trajectory prescription and of the determination of the arm angular velocities, D_1 and D_2 which locate the end effector on the trajectory.

Let the Cartesian coordinates, X, Y, of the prescribed trajectory be parameterized in the form

$$X(\tau) = F_1(\tau);\ Y(\tau) = F_2(\tau) \tag{50}$$

where the parameter, τ, $(0\leq\tau\leq1)$, is given as a function of time by the relation

$$\tau = \frac{1}{2}\left(1 - \cos\left(\frac{\pi t}{T_F}\right)\right);\ 0\leq\tau\leq T_F \tag{51}$$

then the Cartesian components of the velocity along the trajectory are given by

$$\dot{X} = \frac{\partial F_1}{\partial \tau}\ \frac{d\tau}{dt}\ ;\ \dot{Y} = \frac{\partial F_2}{\partial \tau}\ \frac{d\tau}{dt} \tag{52}$$

which satisfy the required conditions of zero velocity at the beginning and end of the trajectory, since in view of (51) $\frac{d\tau}{dt}_{t=0} = 0$ and $\frac{d\tau}{dt}_{t=T_F} = 0$.

For the present illustration the prescribed trajectory has been defined by the following, $F_1(t)$ and $F_{2(t)}$, functions:

$$F_1 = \text{Sin}\ (2\pi t) + \text{Sin}\ (6\pi t);\ F_2 = -\cos\ (2\pi t) - \cos\ (6\pi t) \tag{53}$$

The kinematics problem of finding the angular velocities, D_1 and D_2 which will place the arm end-effector on the prescribed trajectory, is treated as an optimization problem with the cost function

$$J_T = \int_0^{T_F} dt\ V_T\left(\xi, \eta, D_1, D_2\right) \tag{54}$$

with

$$V_T\left(\xi,\eta,D_1,D_2\right) = \frac{1}{2}\left\{(X-\xi)^2 Q_{1T} + (Y-\eta)^2 Q_{2T} + R_{1T} D_1{}^2 + R_{2T} D_2{}^2\right\} \tag{55}$$

and the constraints:

$$\dot{\theta}_1 = D_1; \ \dot{\theta}_2 = D_2 \tag{56}$$

and where the dimensionless Cartesian coordinates of the trajectory, ξ and η, are given by the relations (see Fig. 1).

$$\xi = \text{Sin}(\theta_1) + \frac{b_2}{b_1} \text{Sin}\ (\theta_1 + \theta_2) \tag{57}$$

$$\eta = -\cos\ (\theta_1) - \frac{b_2}{b_1} \cos\ (\theta_2 + \theta_1) \tag{58}$$

The Hamiltonian function, H_T, is given in view of the cost function (54) and the constraints (56) by

$$H_T(\xi, \eta, D_1, D_2, \Lambda_1, \Lambda_2) = V_T(\xi, \eta, D_1, D_2) + \sum_{i=1}^{2} \Lambda_i D_i \tag{59}$$

where, Λ_1 and Λ_2 are the components of the adjoint vector for the arm kinematics. The optimal controls, D_1 and D_2 are obtained from the relations $\partial H_T/\partial D_1 = \partial H_T/\partial D_2 = 0$ i.e.

$$D_1 = -\frac{1}{R_{1T}} \Lambda_1; \ D_2 = -\frac{1}{R_{2T}} \Lambda_2 \tag{60}$$

Insertion of the result (60) into (56), and use of the relations, $\Lambda_1 = \partial H_T/\partial \theta_1$, $\Lambda_2 = \partial H_T/\partial \theta_2$, yields the following set of equations for the solution of the arm kinematics problem

$$\dot{\theta}_1 = -\frac{1}{R_{1T}} \Lambda_1; \ \dot{\theta}_2 = -\frac{1}{R_{2T}} \Lambda_2 \tag{61}$$

$$\dot{\Lambda}_1 = \eta\ (X-\xi)\ Q_{T1} - \xi\ (Y-\eta)\ Q_{T2} \tag{62}$$

$$\dot{\Lambda}_2 = -\ b_2 \cos\ (\theta_1 + \theta_2)\ (X-\xi)\ Q_{1T} - b_2\ \text{Sin}\ (\theta_1 + \theta_2)\ (Y-\eta)\ Q_{T2} \tag{63}$$

with the initial conditions

$$\theta_1(0) = \theta_2(0) = 0 \tag{64}$$

$$\Lambda_1(0) = \Lambda_2(0) = 0 \tag{65}$$

V. RESULTS AND DISCUSSION

To validate the present methodology for uncertain nonlinear models we have used the computer simulation shown in the block diagram of Fig. 2. The PLANT (full model of the two-link arm) was simulated by converting the Euler-Lagrange equations, (see Eqs. (A.2) and (A.3) in Appendix A) into four coupled first order differential equations, which are solved with zero initial conditions for angle, angular velocities and torques. The PLANT algorithm sends the tachometer signals S_3 and S_4 and receives the torques T_1 and T_2 from the CONTROL algorithm. The TASK algorithm constructs the trajectory (Eq. 53) and locates an object with a given mass on a predetermined point along the trajectory. The parametric representation, $X(\tau)$, $Y(\tau)$, of the trajectory is transferred to the KINEMATICS algorithm which solves Eqs. (B.9) and (B.10), of Appendix B and provides the demands D_1 and D_2.

Signals and demands are fed into the MODEL UPDATE algorithm which solves the set of differential equations (B.1) up to (B.8) of Appendix B. The adjoint variables W_3, W_4, Z_1, Z_2, Z_3 and Z_4 are supplied to the CONTROL algorithm which calculates torques and uncertain terms, using equations (33) up to (35). The result of this simulation is shown in Fig. 3, which pictures the motion of the two-link arm along the prescribed trajectory. Note that the uncertain terms, $P_1(t)$ and $P_2(t)$ updated the highly simplified model used for the robot dynamics, hence compensating for friction forces and for the nonlinear terms which were not accounted for in the model. Note though that from a strict theoretical viewpoint, the uncertain terms should be made functions of the state variables. Neglection of this functional dependence infringes upon the optimality of the control motion, but in no way affects the

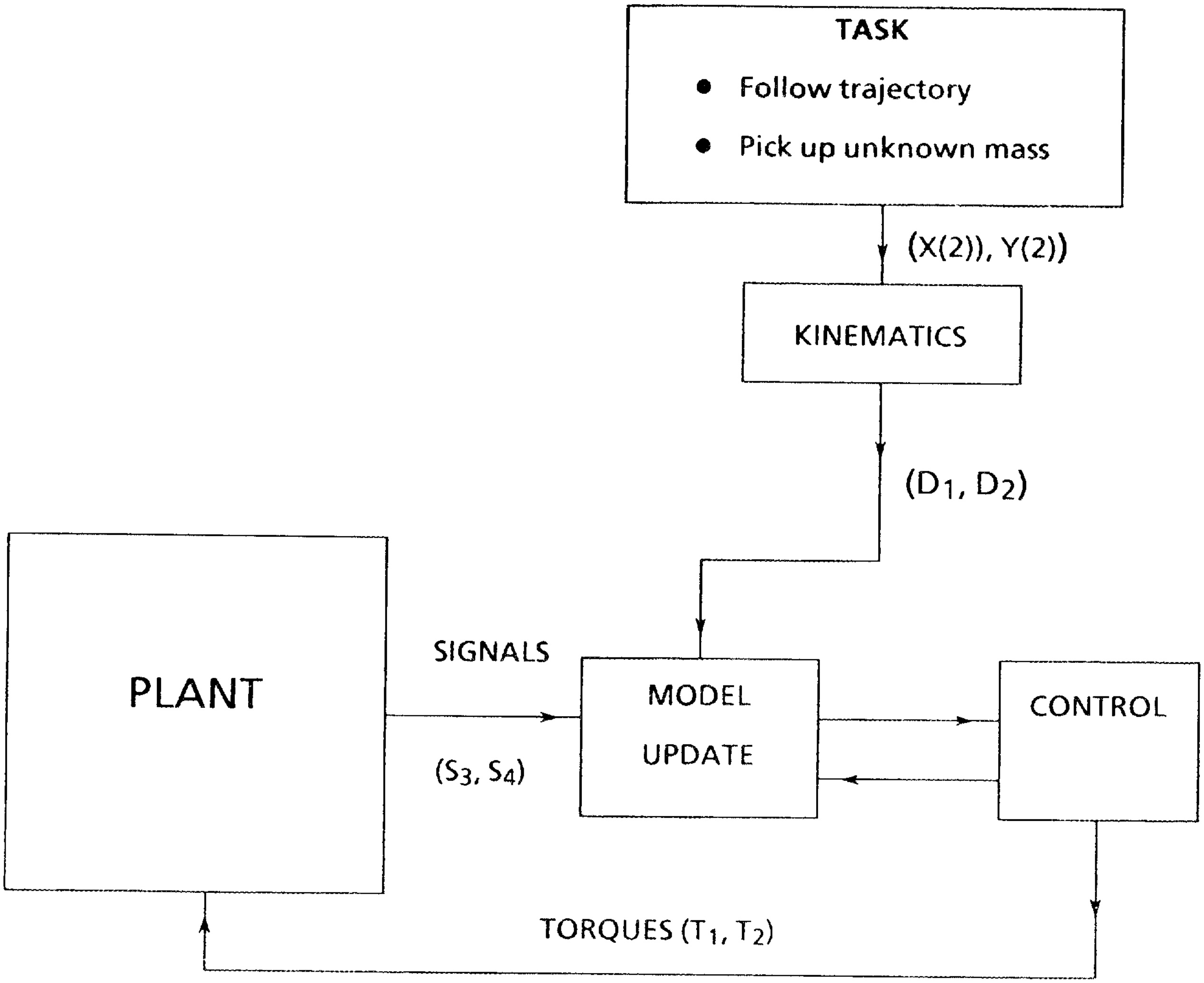

Fig. 2. Block diagram of the microprocessor-based simulation.

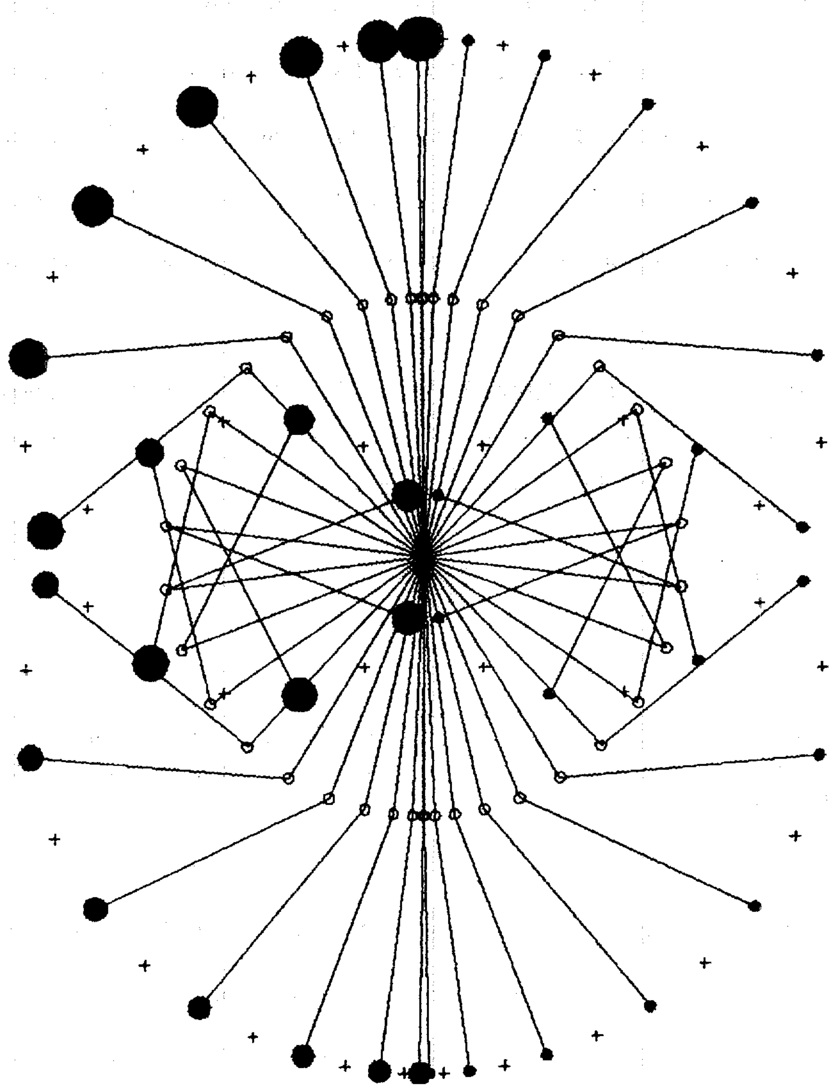

Fig. 3. Motion of the robotic arm along the prescribed trajectory. The arm task is to follow the trajectory, stop at the point labeled, M, in the figure, and then pick up a mass of 6 kg, which is time-varying up to a value of 2 kg at the end of the trajectory. The uncertain model does not have any information either on the time-varying mass, m_2, or on the added friction forces.

demand-following capabilities of the control algorithm. As is always the case in optimal control, one has to supply values for the various Q_i and R_i weight factors introduced in the cost functions. We have adopted the policy to set all the R-weights to unity and then "tune" the algorithm by assigning trial values for the Q-weights. The search for those weight factors, proceeds swiftly after a few trials. Table 1 gives the values of the arm parameters and weights, Q_i, used in the present simulation.

Table 1. Parameters and Weight Factors for the Computer Simulation

$b_1 = b_2 = 1_m$; $m_1 = 1(\text{kgm})$; $m_2{}^{(a)} = 2(\text{kgm})$

$Q_{c1} = Q_{c2} = 10^3$; $Q_{p3} = Q_{p4} = Q_{1T} = Q_{2T} = 10^8$

Transient Duration = 10 seconds

[a] In the plant, m_2 has an initial value of 2(kgm) up to the point labeled, M_1 in Fig. 3, where m_2 = 6 kgm. Thereafter, m_2, decreases up to 2 kgm at the trajectory end.

We have developed novel approaches for the analytical representation of prescribed trajectories and for the kinematics problem of transforming from trajectory coordinates to arm coordinates. In our approach the prescribed trajectory is represented by twice-differentiable parametric representations of the cartesian coordinates, in terms of a parameter, τ, which is related to the time variable by Eq. (51). This approach provides a simple method to construct trajectories satisfying both smoothness and the required conditions at the beginning and end of the robotic arm motion.

The treatment of the kinematics problem as an optimal control problem significantly reduces the number of singularities appearing in the usual methodology. This feature of the algorithm arises from the fact that to ensure the stationarity of the cost function, the algorithm must minimize the change of the arm coordinates. This property is confirmed by the results shown in Fig. 3, whose arm configurations involving larger changes of the arm coordinates (see Fig. 1) are not observed. There is however, one singularity which appears in the present approach when the system is prescribed to move along the Y-axis. In this instance there are two symmetric arm configurations which can be reached by the same change in the arm coordinates. Nevertheless, our new approach to robot kinematics has proven to be robust against the singularities which appear in the transformation from trajectory to arm coordinates. The microprocessor-based control algorithm operated faster than real time for task durations longer than a few seconds.

The results of the computer simulation validate the uncertain model formalism developed in this work. We consider this conclusion of great relevance for the control of "real-world" dynamic systems which complexity cannot be fully modeled.

REFERENCES

1. "Challenges to Control: A Collective View," IEEE Transactions on Automatic Control, Vol. AC-32, No. 4 (1987).

2. D. G. Luenberger, "Introduction to Dynamic Systems," John Wiley and Sons, Inc., New York (1979).

3. C. March-Leuba, R. B. Perez, "Optimal Control Theory of Load-Following and Parameter Tracking of Nonlinear Systems: An Application of Pontryagin Maximum Principle to Reactor Dynamics ORNL/TM-10662 (December 1987).

4. C. March-Leuba, R. B. Perez, "Optimal Parameter Tracking and Control of Nonlinear Systems with Time-Varying Parameters," IEEE International Conference on Acoustics, Speech, and Signal Processing, April 11-14, 1988, New York City, USA.

5. R. P. Paul, "Robot Manipulators: Mathematics, Programming and Control," The MIT Press, Cambridge, USA (1982).

APPENDIX A. SIMULATION OF THE TWO-LINKS ARM

The simulation of the mechanical manipulator is based on the Euler-Lagrange equations derived from the Lagrangian in Paul's textbook, with the additional features of including friction forces and a time-varying mass, m_2, in the second link.

From the Lagrangian, L, below:

$$L = \frac{1}{2}(m_1 + m_2)b_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2b_2^2(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) + m_2b_1b_2\cos(\theta_2)(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)$$
$$+ (m_1 + m_2)gb_1\cos(\theta_1) + m_2gb_2\cos(\theta_1 + \theta_2) \tag{A.1}$$

we obtain

$$T_1 - F_{1F} = M_{12}\frac{dm_2}{dt} + D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + D_{112}\dot{\theta}_1\dot{\theta}_2 - D_{122}\dot{\theta}_2^2 + D_{1g} \tag{A.2}$$

$$T_2 - F_{2F} = M_{22}\frac{dm_2}{dt} + D_{22}\ddot{\theta}_2 + D_{12}\ddot{\theta}_1 + D_{211}\dot{\theta}_1^2 + D_{2g} \tag{A.3}$$

where the friction forces, F_{1F} and F_{2F} are given by

$$F_{1F} = k_1\dot{\theta}_1$$

$$F_{2F} = k_2\dot{\theta}_2$$

and

$$M_{12} = b_1^2\dot{\theta}_1 + b_2^2(\dot{\theta}_1 + \dot{\theta}_2) + b_1b_2(2\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2$$

$$M_{22} = b_2^2(\dot{\theta}_1 + \dot{\theta}_2) + b_1b_2\dot{\theta}_1\cos(\theta_2)$$

$$D_{11} = (m_1 + m_2)b_1^2 + m_2b_2^2 + 2m_2b_1b_2\cos(\theta_2)$$

$$D_{12} = m_2b_2^2 + m_2b_1b_2\cos(\theta_2)$$

$$D_{112} = -2m_2b_1b_2\sin(\theta_2)$$

$$D_{122} = - m_2 d_1 d_2 \sin(\theta_2)$$

$$D_{1g} = m b_1 g \sin(\theta_1)$$

$$D_{2g} = (m_1 + m_2) g b_1 \sin(\theta_1) + m_2 g b_2 \sin(\theta_1 + \theta_2)$$

$$D_{22} = m_2 b_2^2$$

$$D_{211} = m_2 b_1 b_2 \sin(\theta_2)$$

APPENDIX B. SUMMARY OF THE CONTROL ALGORITHM

We summarize here the set of coupled nonlinear equations which describe the control algorithm:

(a) Control and Uncertain Terms

$$\dot{M}_1 = M_3 \ ; \ \dot{M}_2 = M_4 \tag{B.1}$$

$$\dot{M}_3 = -\frac{C_{1T}{}^2}{R_{c1}} W_3 \ ; \ \dot{M}_4 = -\frac{C_{2T}{}^2}{R_{c2}} W_4 \tag{B.2}$$

$$\dot{W}_3 = -(D_1 - M_1)1Q_{c1} \tag{B.3}$$

$$\dot{W}_4 = -(D_2 - M_2)Q_{c2} \tag{B.4}$$

$$\dot{Z}_1 = C_{1g} \cos(M_1)Z_3 + C_{2g} \cos(M_1 + M_2) \tag{B.5}$$

$$\dot{Z}_2 = C_{2g} \cos(M_1 + M_2)Z_4 \tag{B.6}$$

$$\dot{Z}_3 = Z_1 - (S_3 - M_3)Q_{p3} \tag{B.7}$$

$$\dot{Z}_4 = Z_2 - (S_4 - M_4)Q_{p4} \tag{B.8}$$

with zero initial conditions for the state and adjoint vectors. Note that the differential equations for the adjoints, W_1 and W_2, are omitted in view that from Eqs. (39) and for initial conditions, $W_1(0) = W_2(0) = 0_2$ these two adjoints are always zero.

The torque T_1, T_2 and the uncertain terms, P_1 and P_2 are given by Eqs. (33) to (35)

(b) Kinematics

$$\dot{\theta}_1 = -\frac{1}{R_{1T}} \Lambda_1 \ ; \ \dot{\theta}_2 = -\frac{1}{R_{2T}} \Lambda_2 \tag{B.9}$$

$$\dot{\Lambda}_1 = \eta(X - \xi)Q_{T1} - \xi(Y - \eta)Q_{T2} \tag{B.10}$$

$$\dot{\Lambda}_2 = - b_2 \cos(\theta_1 \theta_2)(X - \xi)Q_{T1} - b_2 \sin(\theta_1 + \theta_2)(Y - \eta)Q_{T2} \qquad (B.11)$$

Zero initial conditions are for state and adjoint vectors. The demand, D_1 and D_2, is given by Eq. (60).

ORNL/TM-10764
CESAR-88/19

INTERNAL DISTRIBUTION

1. B. R. Appleton
2. S. M. Babcock
3. C. R. Brittain
4. G. de Saussure
5. D. N. Fry
6. W. R. Hamel
7. D. P. Kuban
8-12. F. M. Maienschein
13. R. C. Mann
14-18. C. March-Leuba
19. J. March-Leuba
20. S. A. Meacham
21. J. R. Merriman
22. E. M. Oblow
23. P. J. Otaduy
24-28. B. R. Perez
29. F. G. Pin
30. J. T. Robinson
31. C. R. Weisbin
32. J. T. White
33. B. A. Worley
34-35. Central Research Library
36. Y-12 Technical Library Document Reference Section
37-38. Laboratory Records Department
39. Laboratory Records ORNL, RC
40. ORNL Patent Office
41. EPMD Reports Office

EXTERNAL DISTRIBUTION

42. Office of the Assistant Manager, Energy Research and Development, DOE-ORO, P.O. Box E, Oak Ridge, TN 37830.
43. Dr. John J. Dorning, Department of Nuclear Engineering and Engineering Physics, Thornton Hall, University of Virginia, Charlottesville, VA 22901.
44. Professor Gene H. Golub, UNIACS — 2321, Computer Science Building, University of Maryland, College Park, MD 20742
45. Dr. Robert M. Haralick, Department of Electrical Engineering, University of Washington, Seattle, WA 98195.
46. Dr. Don Steiner, Institute Professor, Department of Nuclear Engineering, Rensselaer Polytechnic Institute, Troy, NY 12181.
47. Dr. Oscar P. Manley, Division of Engineering, Mathematical, and Geosciences, Office of Basic Energy Sciences, ER-15, U.S. Department of Energy - Germantown, Washington, DC 20545.
48. T. F. Kerlin, University of Tennessee, Knoxville, TN 37996.
49. P. F. Pasqua, University of Tennessee, Knoxville, TN 37996.
50. L. A. Rovere, University of Tennessee, Knoxville, TN 37996.
51. R. E. Uhrig, University of Tennessee, Knoxville, TN 39996.
52. B. R. Upadhyaya, University of Tennessee, Knoxville, TN 37996.
53. F. Senent, Departamento de Fisica Fundamental, University of Valencia, Burjasot, Valencia, Spain.
54. J. March Espana, Albacete 3 pta 7, 46007 Valencia, Spain.
55-64. Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37830.