ORNL/TM-10857

ornl

OAK RIDGE NATIONAL LABORATORY

MARTIN MARIETTA

The Aggregate Production Profile for U.S. Crude Oil

Daniel S. Christiansen
David B. Reister

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
CIRCULATION SECTION
4500N ROOM [illegible]
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this report, send in name with report and the library will arrange a loan.

Fossil Energy Program

OPERATED BY
MARTIN MARIETTA ENERGY SYSTEMS, INC.
FOR THE UNITED STATES
DEPARTMENT OF ENERGY

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A04; Microfiche A01

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

ORNL/TM-10857

THE AGGREGATE PRODUCTION PROFILE FOR U.S. CRUDE OIL*

Daniel S. Christiansen
Department of Economics and Management
Albion College
Albion, MI 49224

David B. Reister
Engineering Physics and Mathematics Division
Oak Ridge National Laboratory
Oak Ridge, TN 37831-6364

Date Published: October 1988

Prepared for the
Office of Planning and Environment
Office of Fossil Energy
U.S. Department of Energy

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennesssee 37831
operated by
Martin Marietta Energy Systems, Inc.
for the
U.S. Department of Energy
under contract No. DE-AC05-84OR21400

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0282901 8

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (cont'd)

LIST OF TABLES

ABSTRACT

In this paper we estimate the aggregate production profile for U.S. crude oil for the period 1961 to 1985. Using data on reserves, we find that there is a significant lag between the time that reserves are added and the time production peaks. We attribute this long lag to a system of production controls that existed until about 1970. When we re-estimate over the period from 1970 to 1985, we find that there is a much shorter period between reserve additions and peak production. We consider an alternative approach based on drilling data that gives similar results for the entire sample period but different results for the subperiod 1970-1985. We discuss the factors that are responsible for the general shapes of the profiles we estimate.

1. INTRODUCTION

The economics and engineering literature on crude oil production is filled with discussions of the production profile of a reservoir - the relationship between output and the time since production was begun. Such a profile is useful in predicting the future production from a given piece of property. Although several different mathematical forms are commonly recognized, there is little disagreement about the general shape of the relationship.

When one has an interest in explaining and forecasting petroleum supply on a regional or national level, the concept of an <u>aggregate</u> production profile is quite appealing. The idea is to relate the economy-wide additions to reserves, secured through discovery and development, to the production that will subsequently take place from these increments.

The purpose of this paper is to estimate the aggregate production profile. It is useful to do this if only because the concept is used by many oil industry analysts. More fundamentally, knowledge of the aggregate production profile contributes toward understanding the relationship between price and production. Price can have a very indirect effect on production beginning with its influence on drilling, continuing through the effect of drilling on reserve additions, and ending with the impact of reserve additions on production. It is the last link in the chain that we focus upon here.

If the production profile is stable over time, it follows that current production is a weighted sum of reserve additions in the current and previous years with the weights expressing the profile. Such a distributed lag can be estimated with standard econometric tools using readily available data on production and reserve additions. Alternatively the lag can be estimated using drilling data (such as feet drilled) instead of reserve additions. It is also possible to include other relevant economic and technological variables in the estimation procedure.

We find that the shape of the production relationship is sensitive to the time period over which it is estimated. This sensitivity is largely due to the presence of binding regulatory constraints on production up until about 1970. We provide estimates over periods that span the earlier years and over those that do not in order to show how severely these regulations affected the nature of the profile. We find that there is at least some difference between the aggregate profile and that of an individual reservoir. This difference is especially pronounced when our estimation is based on drilling data. We discuss in turn the factors that we believe are responsible for this difference.

The organization of this paper is as follows: In Section 2 we discuss the nature of the production profile at the individual reservoir level. In Section 3 we discuss the procedure we use to estimate the aggregate profile. In Section 4 we present several different estimates of the profile based on reserve data. In Section 5 we present an alternative set of estimates based on drilling data. In Section 6 we offer a discussion and a critique of the results we have generated. Section 7 is the conclusion.

2. THE PRODUCTION PROFILE OF AN INDIVIDUAL RESERVOIR

Because of the reduction in natural drive pressure as oil is removed from a reservoir, output typically peaks in the early years of production and then declines gradually over time. Of all the production profiles having this property, the one most commonly used by researchers and petroleum engineers is represented by the exponential function

$$Q(t) = Q(0)\ e^{-at} \tag{1}$$

where Q is production, a is the constant rate of decline, and Q(0) is production at time 0, the point at which output begins. It is easy to show (Bradley [1967, pp.46-7]) that under simple assumptions relating production to pressure the "decline curve" or production profile <u>must</u> be exponential.

Another functional form often seen in the literature is the hyperbolic function

$$Q(t) = Q(0)\ (1 + ct)^{-b} \tag{2}$$

where b and c are constants. For a hyperbolic decline curve the rate of decline is no longer constant but declines itself over time. Arps [1962] and Adelman, <u>et al</u>. [1983, p.404] argue that in practice most decline curves are hyperbolic.

Relying on an analysis of oil field production records, the engineering research firm Lewin and Associates (Kuuskraa [1985, pp.39-41]) produced generic profiles which might be used to estimate the pattern of recovery from a reservoir when both primary and secondary recovery techniques are employed. Reservoirs in small fields (class 6 or below) are assigned a 15-year life and those in large fields (class 7 and above) a 20-year life. The percentages of reserves that are typically recovered in each year are shown in Table 1.

Four sample profiles are illustrated in Fig. 1: an exponential profile (with a = .1), a hyperbolic profile (with a = .1, c = .5), Lewin's 15-year profile, and Lewin's 20-year profile. The horizontal axis shows the number of years since production began; the vertical axis shows the percentage of total production which occurs in the given year. Observe that the difference between the exponential and hyperbolic profiles is not particularly pronounced. Upon careful examination one can see that under the hyperbolic scheme production declines faster in the early years and slower in later years. While Lewin's generic profiles obviously cannot be called exponential or hyperbolic, they bear some similarity to these common forms. They suggest, in addition, that there may be significant plateaus in the early years of production.

One must take care to distinguish production profiles for a field, a reservoir, and a well. Such profiles are not necessarily independent. As additional development wells are drilled in a reservoir, for example, the rate of output for the reservoir as a whole will increase. At the same time, however, production from older wells will fall faster than if the newer wells had not been drilled.

Table 1. Lewin Generic Profiles

Year	15-Year Profile	20-Year Profile
0	9.0	9.0
1	9.0	9.0
2	9.0	9.0
3	8.0	9.0
4	8.0	8.0
5	8.0	7.0
6	8.0	6.0
7	8.0	5.5
8	8.0	5.0
9	7.0	5.0
10	6.0	5.0
11	5.0	4.5
12	4.0	4.0
13	2.0	3.5
14	1.0	3.0
15		2.5
16		2.0
17		1.5
18		1.0
19		0.5

To a very large extent production is determined by technological, geological, and physical conditions. But it is also true that producers have some latitude in determining the shape of the production profile. The petroleum engineering literature (see, for example, Craft and Hawkins [1959]) emphasizes the importance of attaining a maximum efficient rate (MER), an upper limit for the optimal depletion rate of a reservoir calculated on the basis of technical considerations alone. When oil production takes place at an excessive rate in a water-drive reservoir, for example, oil may be bypassed and permanently lost. In other cases high rates of production may lead to a rapid decline in reservoir pressure, premature release of dissolved gas, dissipation of gas and water, or other conditions which reduce the ultimate amount of oil that can be recovered.

Economic factors such as prices, interest rates, and taxes also play some role in determining the shape of the production relationship. Decisions as to the appropriate number of development wells to drill, for example, depend upon properly balancing the streams of costs and revenues. Similarly, decisions to employ secondary or tertiary recovery methods are primarily economic ones.

3. METHODOLOGY

Annual aggregate data on domestic reserve additions and production are readily available. For years prior to 1980, the data were reported by the American Petroleum Institute (API) [1980]; for 1977 and beyond, the data have been reported by the Energy Information Administration (EIA) (Wood, et al. [1986]), the statistical and analytic agency within the Department of Energy. We chose to estimate the production profile over a 25-year period from 1961 to 1985 or over a suitable subperiod. This required our using data on production from the sample time period and data on reserve additions from a much longer period, extending some 30 years further back. Since we suspect that the profile for the Arctic frontier region is significantly different from that of the lower 48 states, we have sought to subtract out the Alaska data whenever possible. Other than this, we have worked with the API and EIA data sets in their raw forms, thus choosing to ignore possible discrepancies within each data set and between the two. The historical data for production and reserve additions are plotted in Fig. 2.

For all years t in our sample period, we have data for aggregate production in that year, Y_t, and aggregate reserve additions, D_{t-j}, for $j = 0,1,2,\ldots 29$. We seek to estimate the relationship

$$Y_t = w_0 D_t + w_1 D_{t-1} + w_2 D_{t-2} + \ . \ . \ . + w_{29} D_{t-29} + u_t \qquad (3)$$

where the sequence w_0, w_1, $w_2,\ldots,w_{29}$ represents the production profile, i.e. each w_j represents the fraction of a given year's reserve additions that will show up as production j years later, and u_t is an error term. Our hope is that 30 years is a long enough time period to recover most of the reported reserve additions as production. We thus expect that the weights should add up to something close to one. We have not included a constant term in the regression because our supposition is that each barrel of oil produced in time period t can be identified with oil reported as discovered in one of the previous years. The data may, of course, be imperfectly recorded and the lag structure may not be truly stable over time, but there should be no factors contributing to production which cannot be identified with reserve additions. Looked at another way, given a stable production profile, if reserve additions are zero for a suitably long period of time, production must be zero as well.

Equation (3) represents a distributed lag model because the influence of reserve additions (D) on production (Y) is distributed over a number of lagged values of D. We clearly cannot estimate this equation without putting some additional restrictions on the coefficients w_j since we have more coefficients to estimate than we have data points. We will consider a variety of different ways of restricting the coefficients.

The purpose of this paper is to compare these models on the basis of some rather blunt statistical measures in order to help us understand the general nature of the shape of the aggregate production profile. We will argue that this kind of exercise in descriptive statistics is neces-sary given that the nature of the profile is not well understood. We are not seeking to pick out one model as best; we only want to say in a general way what kinds of models seem to perform very poorly and what kinds of models perform better.

Care needs to be taken in interpreting our comparison of models based on R^2 or adjusted R^2 statistics. R^2 is typically defined as

$$R^2 = \frac{\text{regression sum of squares}}{\text{total sum of squares}} \tag{4}$$

and the formula is often expressed as

$$R^2 = 1 - \frac{\text{error sum of squares}}{\text{total sum of squares.}} \tag{5}$$

The treatment of these two formulas as equivalent and the interpretation of R^2 as a measure of the proportion of variation of the dependent variable attributable to the sample regression are based on the presumption of linear least squares estimation applied to a regression equation with a constant term (Kmenta [1986, p.412]). In this case it follows that R^2 falls between zero and one. One way of seeing that R^2 is greater than zero is to note that the "total sum of squares," the error if only the constant term is used in the regression, is an upper bound for the "error sum of squares" since adding variables to the equation can only reduce the error.

Since we shall explore nonlinear as well as linear models and since our regressions are run without constant terms, the arguments made in the paragraph above break down. We shall follow a procedure of <u>defining</u> R^2 according to formula (5) above. R^2 will always be less than one but it may be less than zero as well! It is greater than or less than zero depending on whether the error is less than or greater than that of the best fit constant function. R^2 loses the interpretation given in the previous paragraph. It is used by us purely as a measure of goodness of fit. Caution is advised in focusing on it as an indicator of a correct specification of the model.

In addition to reporting the R^2 coefficient, we also report the adjusted R coefficient, one which makes a correction for the number of dependent variables relative to the number of sample points. This is defined by

$$\bar{R}^2 = 1 - \frac{n - 1}{n - k} (1 - R^2) \tag{6}$$

where n is the number of observations in the sample and k is the number of dependent variables.

In order to provide a frame of reference, the average value of production over the entire 25-year period was 2726.5 million barrels per year. Viewed as an estimate of production independent of all reserve additions, this fit has an R^2 value of zero.

An alternative approach to estimating the production profile involves using data on the number of feet drilled in place of data on additions to reserves. The advantage to focusing on drilling comes in tightening the connection between activities of producers that generate new reserves and the production that follows. This tightening occurs because production cannot begin until wells are drilled. We will have more to say about the usefulness of this approach later.

Data on drilling footage have been collected over the years by the American Petroleum Institute (API [1985]), the American Association of Petroleum Geologists (Johnson [1986]), and the Oil and Gas Journal. The data are subdivided by wells and footage, by oil production, gas production and dry holes, and by developmental and exploratory activity. To get a measure of the drilling activity for oil, we have included a fraction of the footage assigned to dry holes - the fraction representing the ratio of footage assigned to successful drilling of oil to that assigned to successful drilling of oil and gas. The further back in time we go, the less reliable and uniform the data set becomes. Between 1947 and 1965 only total footage was available, and we had to apportion this between gas and oil on the basis of successful oil and gas wells. Before 1946 we used World Oil data [1984], extrapolating, in some cases, to the best of our ability. The historical data for drilling are plotted in Fig. 3.

Now, for each year t we have data for production in that year, Y_t, and feet drilled, F_{t-j}, for $j = 0,1,2,\ldots 29$. We estimate the relationship

$$Y_t = w_0 F_t + w_1 F_{t-1} + w_2 F_{t-2} + \ldots + w_{29} F_{t-29} + u_t \qquad (7)$$

where the sequence $w_0, w_1, w_2, \ldots w_{29}$ represents the production profile. Note that the w_{t-j} coefficients now have units attached to them - barrels per foot. We do not expect these weights to add up to one.

4. ESTIMATING THE PROFILE USING RESERVE DATA

In this section our focus is on using reserve data to estimate the aggregate production profile. Our first set of estimates involves the use of data from the entire sample period, 1961 to 1985. It turns out that over this period the best fits are given by profiles with highly unusual features. We find that these results can be largely explained by the system of production controls that was effective over a significant part of the sample period. Next we estimate over the subperiod for which production controls were not effective and obtain shapes closer to those one might expect.

Consider first the entire sample period. Likely candidates for the aggregate production profile are those having the same general form as individual reservoirs. Using the Lewin generic profile for small fields, we fit a 15-year profile allowing for a scale factor so that

$$w_j = ab_j \tag{8}$$

where a is the scale factor to be estimated and the b_j coefficients are as given in Table 1. The estimates of the parameters are

$$\underset{(.032)}{a = 1.14} \qquad R^2 = \bar{R}^2 = -.50$$

where the number in parenthesis is the standard error. Likewise, the estimates using the 20-year Lewin profile are

$$\underset{(.029)}{a = 1.12} \qquad R^2 = \bar{R}^2 = -.27$$

It is clear that these are very bad fits to the data. The negative values for R^2 indicate that a constant production path fits the data better than either Lewin profile.

Consider a hyperbolic profile with the weights w_j given by

$$w_j = a(1 + cj)^{-b} \tag{9}$$

for $j = 0,1,2,\ldots 29$. Using a nonlinear optimization routine we found the least squares estimates of the parameters to be

$$a = .0462 \qquad b = 4.41 \qquad c = .00428 \qquad R^2 = .67 \qquad \bar{R}^2 = .66.$$

Figure 4 shows the distribution of weights over the 30-year period. There are further indications here that the aggregate production profile looks different from that of a typical reservoir. Individual profiles have the properties that the weights add up to one and decline to zero over the lifetime of the reservoir. While the weights here add up to 1.08, the individual values remain substantially above zero after 30 years. The profile simply does not decline fast enough to be compatible with anyone's conception of how a "typical" reservoir is emptied.

An examination of an exponentially declining weight structure would look very much like the hyperbolic case. Even a linear lag structure (i.e. a polynomial distributed lag of degree one)

$$w_j = a - bj \tag{10}$$

for $j = 0,1,2,\ldots 29$ produces a production profile almost identical to the hyperbolic one (see Fig. 5). The estimates here are

$$\underset{(.00619)}{a = .0454} \qquad \underset{(.000414)}{b = .000653} \qquad R^2 = .68 \qquad \bar{R}^2 = .66.$$

The hyperbolic and linear lag structures give a better fit to the data than do the Lewin profiles because they do not force the weights to go to zero. In fact, in the linear case, it is difficult to argue that the coefficient b is significantly different from zero, i.e. that the profile is not constant over time.

We considered the possibility that the shape of the aggregate profile might be the same as that for individual reservoirs but that it might be shifted in time due to a lag in setting up production. Using each of Lewin's generic profiles, we estimated the profiles by delaying initial production from the time reserves were added by one year, two years, three years, etc. The 15-year profile performed very poorly in each case with the R^2 coefficient reaching <u>zero</u> only after a 7-year delay. The 20-year profile performed somewhat better, but R^2 was never better than .50 with the highest values corresponding to delays of about 10 years.

We next looked at the possibility that the weights in the production profile might increase in value up to some point and then decline. We experimented with various kinds of lag structures which allowed for this kind of pattern. Our results for the Pascal distributed lag (Kmenta [1986, pp.536-7]) are typical. Here

$$w_j = b \frac{(j + r - 1)!}{j!(r - 1)!} (1 - a)^r a^j \tag{11}$$

for r a positive integer and j = 0,1,2,...29. We used a nonlinear optimization routine to produce the following estimates:

$$r = 3273 \qquad a = .00515 \qquad b = 1.000 \qquad R^2 = .69 \qquad \bar{R}^2 = .66.$$

The estimated profile is shown in Fig. 6. What is surprising from Fig. 6 is that the weights peak more than 15 years from the time reserve additions are reported.

Similar results were obtained when we examined simple inverted-V distributions given by

$$w_j = \frac{b\,j}{n} \qquad 0 \leq j \leq n \tag{12a}$$

$$w_j = \frac{b\,(29 - j)}{(29 - n)} \qquad n < j \leq 29 \tag{12b}$$

where n is a positive integer and b is a real number. Our estimates of the parameters are

$$n = 19 \qquad b = \underset{(.000778)}{.0707} \qquad R^2 = .76 \qquad \bar{R}^2 = .75.$$

This profile is shown in Fig. 7. Once again, the highest weights are associated with years well beyond those in which reserves are found.

Finally, we simply grouped the data into 5-year periods and estimated the profile

$$\begin{aligned} w_j &= b_1 \qquad j = 0,1,\ldots 4 \\ w_j &= b_2 \qquad j = 5,6,\ldots 9 \\ &\vdots \\ w_j &= b_6 \qquad j = 25,26,\ldots 29. \end{aligned} \tag{13}$$

Our estimates are

$$b_1 = \underset{(.0121)}{.0106} \qquad b_2 = \underset{(.0103)}{.0553} \qquad b_3 = \underset{(.0123)}{-.00639} \qquad b_4 = \underset{(.0139)}{.103} \qquad b_5 = \underset{(.00676)}{.0622}$$

$$b_6 = \underset{(.00867)}{-.0182} \qquad R^2 = .95 \qquad \bar{R}^2 = .93 \qquad DW = 1.55.$$

This profile is shown in Fig. 8. Testing for positive autocorrelation of the residuals at the 5% level, the Durbin-Watson statistic (DW) falls in the "inconclusive region." If we make a correction for autocorrelation using the Prais-Winsten method (Kmenta [1986, pp. 318-20]), the estimates become

$b_1 = .0362$ (.0167)	$b_2 = .0304$ (.0169)	$b_3 = .0274$ (.0156)	$b_4 = .0677$ (.0179)
$b_5 = .0409$ (.0138)	$b_6 = .00495$ (.0117)	$R^2 = .82$	$\bar{R}^2 = .77$.

The new profile is shown in Fig. 9.

Caution is advised in interpreting the results of this last model as it involves estimating a large number of parameters relative to the sample size. Nevertheless, the results are interesting because they lead us once again to the conclusion that, over the entire sample period, there was a significant lag between the time that reserves were added and the time that the greatest impact on production was felt.

The distorted profiles that emerge present a puzzle. It is really inconceivable that ten or fifteen years goes by, on average, between the time when reserves are found and production reaches its peak. We will at a later point try to make a case for a much shorter lag based on delays resulting from offshore production, the presence of some nonproducing reserves, and possibly even some overeager reporting of reserve additions. It is clear, however, that these factors cannot adequately explain what we have observed so far. We need to look elsewhere for a solution to the puzzle.

The answer comes in focusing on regulations that limit production. In the late 1920s and early 1930s, Texas and other large oil-producing states enforced state laws which restricted the output of oil wells based on their potential production. Such a system became known as "market-demand prorationing" since production in total was limited to an amount equal to the estimated market demand at the prevailing price and the output of each well was reduced proportionately.

Whether the intent of such a system was to avoid economic inefficiency and waste or whether it was simply to keep oil prices at levels above what they would otherwise be, the result was significant excess capacity in the U.S. oil industry for several decades. According to one estimate (Epple [1975, pp.18-19]), excess capacity for the U.S. remained above 40% for the first half of the 1960s and hit 69% in Texas in 1965. These figures dropped to about 20% at the beginning of the 1970s as U.S. oil production peaked, oil imports picked up markedly, and world oil markets neared the OPEC era. In terms of the percentage of potential production allowed in Texas (the "market demand factor"), the 1970s were a much different decade than the 1960s as shown in the following Texas Railroad Commission data reported in Mead [1976, p.141]:

Table 2. Texas Market Demand Factors

Year	Factor
1961	28%
1962	27%
1963	28%
1964	28%
1965	29%
1966	34%
1967	41%
1968	45%
1969	52%
1970	72%
1971	73%
1972	94%
1973	100%

Since 1973 the production constraints set by state regulatory agencies have not been binding.

It is clear that a changing production environment in the U.S. over the years of our sample period could be responsible for the distorted profiles we found. Because the instability caused by production constraints has not been present since about 1970, we chose to re-estimate the profile beginning in that year. The difference in our results is dramatic.

Consider now the subperiod 1970-1985. We begin by considering aggregate profiles of the same form as those of individual reservoirs. These profiles give much better fits than those obtained using the entire data set. For example, using equation (8) to fit the Lewin profiles, our estimates are

$$\underset{(.010)}{a = 1.26} \qquad R^2 = \bar{R}^2 = .93$$

for the 15-year profile and

$$\underset{(.011)}{a = 1.24} \qquad R^2 = \bar{R}^2 = .92$$

for the 20-year profile. Compare these results with the previous ones which had negative values for R^2. Note also that our estimates of the scale factor are much larger. The hyperbolic and exponential profiles give similarly good fits. Our estimates for the hyperbolic form (9) are

$$a = .0949 \qquad b = 10,700 \qquad c = .00000702 \qquad R^2 = .93 \qquad \bar{R}^2 = .92$$

and for the exponential profile

$$w_j = ae^{-bj} \tag{14}$$

are

$$\underset{(.0133)}{a = .0949} \qquad \underset{(.0138)}{b = .0752} \qquad R^2 = .93 \qquad \bar{R}^2 = .93.$$

The hyperbolic and exponential functions produce virtually the same estimated profile as evidenced by Fig. 10. While the hyperbolic declines at a rate equal to .0750 at the beginning and .0749 at the end, the exponential declines at a rate equal to .0752 throughout. A linear lag structure (10) also gives a better and much more reasonable fit than before:

$$\underset{(.00428)}{a = .0842} \qquad \underset{(.000274)}{b = .00310} \qquad R^2 = .97 \qquad \bar{R}^2 = .97.$$

This profile is shown in Fig. 11. Note that while we have not constrained the weights to be positive, they are negative (and only slightly so) for only two years out of 30. Based on R^2, this gives the best fit so far.

Next consider fitting the profile by allowing for a delay between the time reserves are added and production begins. In most cases the fits obtained above can be slightly improved. For example, R^2 for the Lewin 20-year profile and the linear lag can be increased, respectively, to .96 and .98 by allowing for a one year delay. In both cases one year is the optimal delay and the estimates for the parameters are similar to those obtained above.

As an alternative to delaying production we allowed for distributed lags with values that first increased and then decreased. Consider the Pascal lag (11). Our estimates of the parameters are now

$$r = 2 \qquad a = .818 \qquad b = 1.21 \qquad R^2 = .98 \qquad \bar{R}^2 = .98$$

and the profile is shown in Fig. 12. Whereas the fit generated from the entire sample period (shown in Fig. 6) peaked 16 years after reserves were added, this new fit peaks after only four years. Consider also the "inverted V" distribution given by equation (12). Our estimates for the parameters are

$$n = 1 \qquad b = \underset{(.000404)}{.0794} \qquad R^2 = .97 \qquad \bar{R}^2 = .97.$$

This profile is shown in Fig. 13. Note that since the weight for the current year is zero, the profile peaks after only one year.

Finally, consider a profile constant on five-year segments (13). Our estimates are

$$b_1 = \underset{(.0148)}{.0844} \qquad b_2 = \underset{(.0095)}{.0791} \qquad b_3 = \underset{(.0142)}{.0659} \qquad b_4 = \underset{(.0231)}{.0105}$$

$$b_5 = \underset{(.0176)}{.00652} \qquad b_6 = \underset{(.00737)}{.00236} \qquad R^2 = .99 \qquad \bar{R}^2 = .98$$

This profile is shown in Fig. 14. Consistent with our other results using data only since 1970, it no longer peaks many years after reserves are added. Note also the relatively small weights assigned to years 15 and beyond and the large standard errors associated with these estimates.

5. ESTIMATING THE PROFILE USING DRILLING DATA

When dealing with the entire period from 1961 to 1985, the profiles we obtained using drilling data exhibit a great deal of similarity to those obtained using reserve data. In contrast, when we restrict our attention to the 1970-85 subperiod, the two sets of profiles differ substantially.

Using the entire sample period, we found once again that profiles that characterize production from individual reservoirs do not explain production in the aggregate. The 15-and 20-year Lewin generic profiles, for example, produced large negative R^2 values (-2.91 and -1.95, respectively). The hyperbolic and linear lags fared little better, yielding R^2 values of -.36 and -.25. When we allowed for a delay between the time drilling occurred and production started, the individual profiles once again gave better fits. For example, the optimal delay for each of the Lewin scenarios was eight years. For the 15-year and 20-year profiles this produced R^2 values of .68 and .80, respectively.

As in the previous section, we get our best fits when we allow the weights to first increase and then decrease. Our estimates for the Pascal distributed lag (11) are

$$r = 8 \qquad a = .640 \qquad b = 22.0 \qquad R^2 = .91 \qquad \bar{R}^2 = .89$$

for the inverted-V distribution (12) are

$$n = 12 \qquad b = 1.49 \qquad R^2 = .90 \qquad R^2 = .90$$

and for the five-year grouped periods (13) are

$$\underset{(.182)}{b_1 = .650} \qquad \underset{(.202)}{b_2 = .980} \qquad \underset{(.192)}{b_3 = .871} \qquad \underset{(.176)}{b_4 = 1.63}$$

$$\underset{(.199)}{b_5 = .743} \qquad \underset{(.221)}{b_6 = -.644} \qquad R^2 = .98 \qquad \bar{R}^2 = .97.$$

These profiles are illustrated in Figs. 15-17. Recall that the vertical axis now has units of barrels per foot.

For most of the functional forms we estimated for the period from 1961 to 1985, the fit of the production profile, as measured by R^2, was somewhat better when we used drilling data than when we used reserve data. Furthermore, the profiles with the best fits using drilling data generally peaked a few years earlier than the corresponding profiles estimated with reserve data. Even so, none of the profiles we found came close to looking like the profile of a typical reservoir. Once again the production relationship was distorted by the presence of production controls over much of the sample period. As we will explain in the next section, the use of drilling data introduces additional distortions.

Now consider estimating over the 1970-85 period only. Unlike the case using reserve data, we do not get a dramatic improvement in the fit of forms which characterize individual reservoirs. The Lewin profiles, files, for example, still have negative R^2 coefficients (-2.16 for the 15-year profile and -1.75 for the 20-year profile). The same is true for hyperbolic profiles (9)

$$a = .854 \qquad b = 115{,}000 \qquad c = .000000111 \qquad R^2 = -.48 \qquad \bar{R}^2 = -.70,$$

exponential profiles (14)

$$a = \underset{(.629)}{.855} \qquad b = \underset{(.0541)}{.0128} \qquad R^2 = -.48 \qquad \bar{R}^2 = -.58,$$

and linear lag profiles (10)

$$a = \underset{(.493)}{1.24} \qquad b = \underset{(.0326)}{.0350} \qquad R^2 = -.39 \qquad \bar{R}^2 = -.49.$$

The hyperbolic and exponential profiles are shown together in Fig. 18. The linear lag profile is shown in Fig. 19.

When we allow for long enough delays, we get much better fits. The optimal delay for the Lewin lags, for example, is five years in each case, and the R coefficients are .89 for the 15-year profile and .98 for the twenty-year profile.

Profiles which allow production to first increase and then decrease provide some of the best fits. Our results for the Pascal lag (11) are

$$r = 9 \qquad a = .570 \qquad b = 22.7 \qquad R^2 = .99 \qquad \bar{R}^2 = .99$$

and for the inverted V lag (12) are

$$n = 10 \qquad b = 1.52 \qquad R^2 = .93 \qquad \bar{R}^2 = .92.$$

These profiles are shown in Figs. 20 and 21. Note that, according to these profiles, production peaks about ten years after drilling takes place.

Finally, we present our results for profiles constant on five-year segments (13):

$$b_1 = \underset{(.895)}{.960} \qquad b_2 = \underset{(1.64)}{.177} \qquad b_3 = \underset{(.701)}{1.53} \qquad b_4 = \underset{(.401)}{1.65}$$

$$b_5 = \underset{(1.27)}{.563} \qquad b_6 = \underset{(.701)}{-.702} \qquad R^2 = .99 \qquad R^2 = .99.$$

This profile is shown in Fig. 22.

6. DISCUSSION

We have seen that the aggregate production profile, whether based on reserve data or drilling data, has not been stable over the last thirty years. We have attributed this instability largely to the presence of controls on production which were binding up until about 1970. In order to estimate the profile in the post-1970 period, we have simply dropped the earlier years from our sample. This yielded results, at least for the case using reserve data, that are much more intuitive than those obtained earlier. Even so, there are reasons to believe that we have not completely corrected for the effect of prorationing. Given that production was limited in the early years, some additions to reserves before 1970 were left idle or were utilized at a rate less than they would have been if production had not been constrained. This means that their contribution to production after 1970 should have been larger than would otherwise have been the case. As we move further away from 1970, however, this problem becomes less significant.

On the basis of our estimates using reserve data, we are led to the conclusion that, unlike the profile of an individual reservoir, the aggregate production profile does not decline from the beginning. Instead, there is a period of something between one and five years before the profile begins to fall. This can be represented by traditional profiles that have a delay associated with them or by profiles with values that first rise and then fall.

We maintain that there are good reasons for the existence of a short period between the time reserves are added and the time production reaches its peak. Total reserve additions are of two kinds: those that begin producing immediately and those that do not. If we can establish that a significant fraction of reserve additions belong to the second category, then the profile derived from the combination of the two categories will exhibit the kind of properties we have uncovered.

The definition of additions to proved reserves (Wood, et al. [1986, pp.91-2]) includes reserves that are not yet producing. This comes about because the definition is largely based on inference. Most reserve additions come not from discoveries of new oil but rather from revisions and extensions based on previous estimates. There is explicit acknowledgment that some reserve additions may come from adjoining portions that are as yet undrilled. The definition also emphasizes that actual production is not necessary to justify making an addition to proved reserves; indeed, other ways of supporting economic producibility are spelled out explicitly. The EIA even acknowledges "nonproducing crude oil reserves" and suggests that they have been about 1.8 billion barrels, on average, between 1977 and 1985 (Wood [1986, p.21]).

The notion that there might be a lag between the time reserves are found and the time that production begins makes sense especially when dealing with offshore production. Adelman and Jacoby [1979, p.25] have argued that this set-up lag could amount to something like two years for onshore production and four years for offshore production.

When we used drilling data to estimate the aggregate production profile, we established the existence of a long lag between the time that drilling took place and production peaked. This lag amounted to something like five to ten years even after the prorationing years had been removed from the sample. Since drilling coincides with real activity in the crude oil sector and is not based on inference, this lag results from a different set of factors than those discussed above. First, the approach taken in Section 5 relies on a stable relationship between drilling and reserve additions - i.e. it presumes a relatively constant finding rate. But over the period of this study there has been a general downward trend in the finding rate. The finding rate was 29.4 barrels per foot for the 1940s, 18.9 for the 1950s, 21.1 for the 1960s, 18.1 for the 1970s, and 11.9 for the 1980s. It is quite likely that this fact has distorted our estimates.

Second, there are conditions other than drilling under which additions to proved reserves can take place. These include the use of improved recovery methods, the occurrence of more favorable economic conditions, and an attainment of better knowledge of the reservoirs. We would expect that when these factors contribute substantially to reserve additions, the production profile based on drilling data should be significantly different from that based on reserve data.

We have not been able to discern meaningful relationships between the price of oil and the production profile. This is not meant to suggest that production does not depend on price; indeed, the indirect effect of price on production through drilling is hard to dispute. It seems likely to us, however, that changes in oil prices should have some direct impacts on production. Perhaps the presence of strong correlation between price and the relevant explanatory variables in our study prevented us from detecting this.

It is interesting to examine the sum of the weights in the profiles we have generated. When we use reserve data, the weights should add up to something close to one. For the 1970-85 period the sum is substantially above one as evidenced in Table 3.

Table 3: Sum of Profile Weights
Reserve Data 1970-85

Model	Sum
Hyperbolic	1.17
Exponential	1.18
Linear	1.22
Lewin 20 w/delay	1.21
Pascal	1.19
Inverted V	1.15
5 Year Groups	1.22

This can be explained partially by the lingering effects of prorationing (i.e. the presence after 1970 of previously underutilized reserves). It is unclear just how much of the difference actually results from obtaining more production than was ever reported as additions to reserves. (When we used the entire sample period, the sum of the weights was typically closer to one. For example, it was 1.08 for the hyperbolic and linear, 1.00 for the Pascal, and 1.03 for the inverted V and 5 year groups.)

When we use drilling data, the weights in the profiles should add up to an average finding rate measured in barrels per foot. The following table shows the sum of the weights for several formulations of our model:

Table 4: Sum of Profile Weights
Drilling Data 1970-85

Model	Sum
Lewin 15 w/delay	23.4
Lewin 20 w/delay	22.9
Pascal	21.6
Inverted V	22.0
5 Year Groups	20.9

These numbers are all higher than the 19.1 finding rate over the relevant period, 1941-1985. This once again suggests that more oil is being produced than is accounted for in reserve additions. (When we used the entire sample period, the sums were similar. For example, it was 21.5 for the Pascal, 21.6 for the inverted V, and 21.2 for the 5 year groups.)

7. CONCLUSION

In this paper we set out to estimate the aggregate production profile for the period from 1961 to 1985. Using reserve data, we found that there was a significant lag between the time reserves were added and production reached a peak. We attributed this lag largely to a system of production controls that existed up until about 1970. We re-estimated the profile for the years 1970-1985 and found that the best fits all had the feature that production did not decline from the time reserves were added but reached a peak after a short period --- something like from one to five years. We attributed this lag to a definition of reserve additions, based on inference, that includes reserves that are not presently producing oil.

We considered an alternative approach based on drilling data. When we used data from the entire sample period, our results looked much like those obtained using reserve data. But when we restricted our sample to the period from 1971 to 1985, we found that there was a lag of between five and ten years from the time drilling began to the time production peaked. We attributed this lag to a declining finding rate over the period of our study and to reserve additions that occur without drilling taking place.

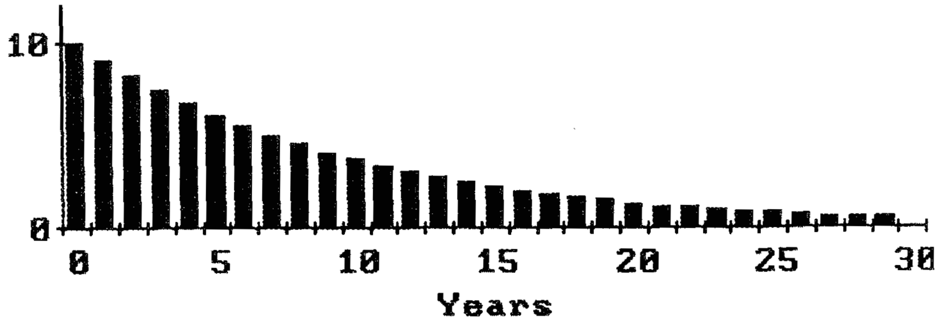

Fig. 1a. Exponential profile (percent).

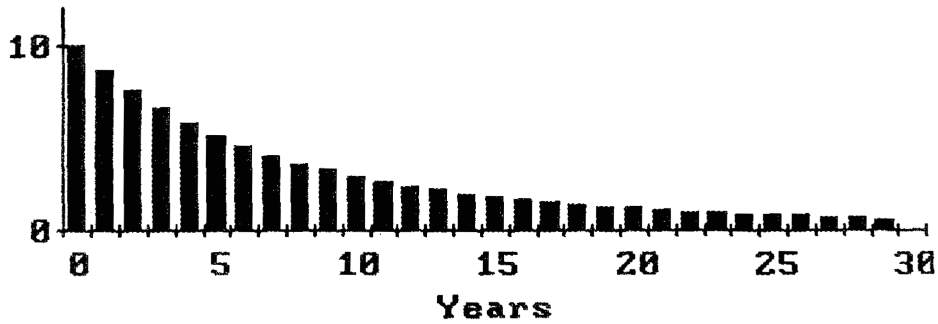

Fig. 1b. Hyperbolic profile (percent).

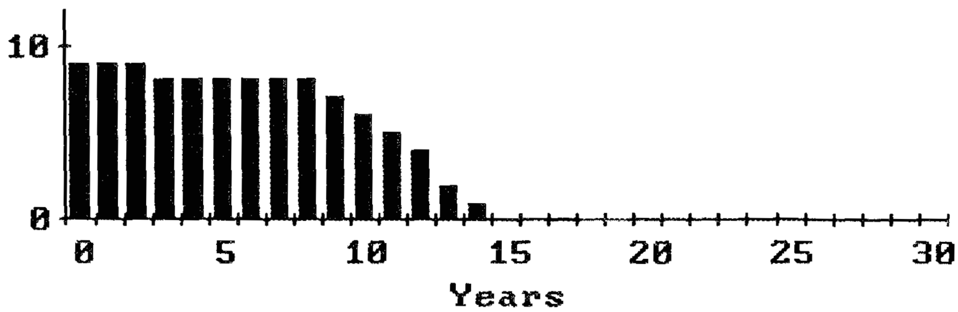

Fig. 1c. 15-year Lewin profile (percent).

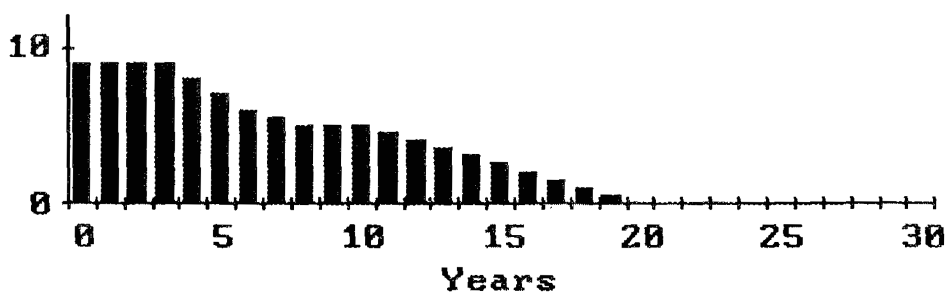

Fig. 1d. 20-year Lewin profile (percent).

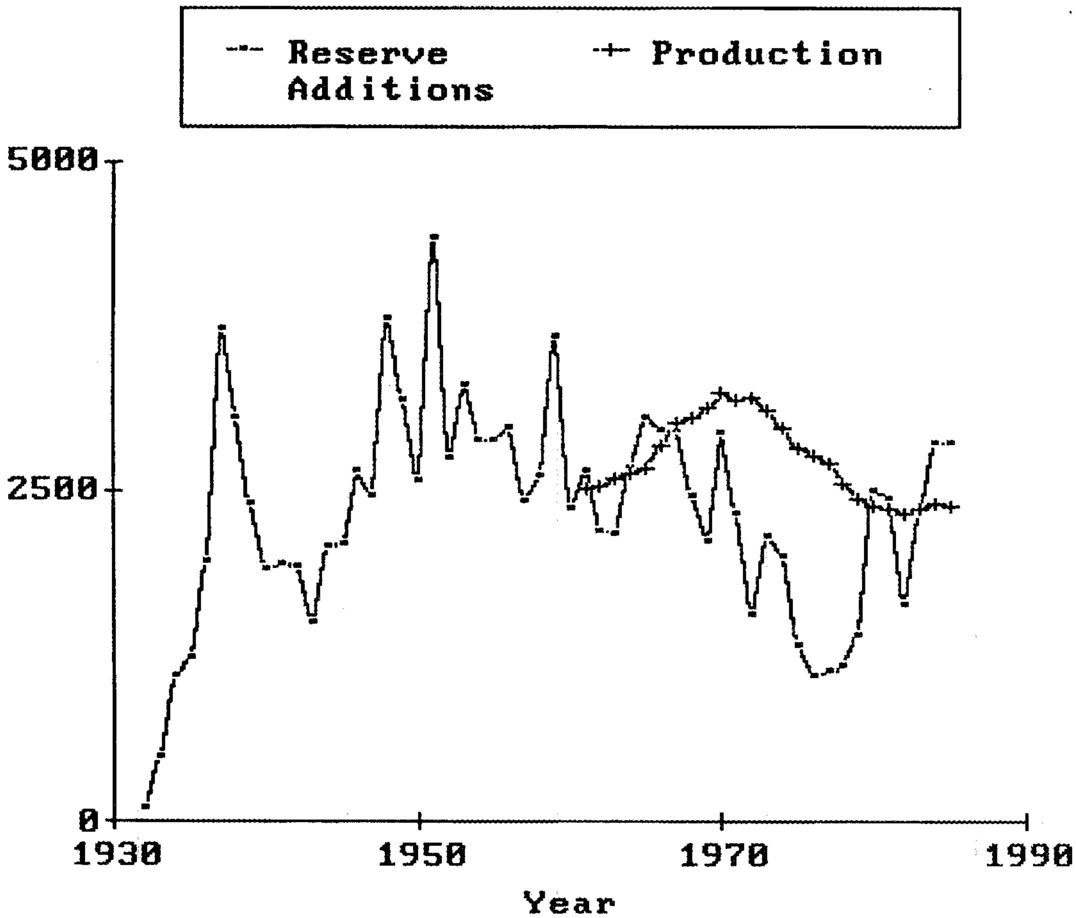

Fig. 2. Historical data on production and reserve additions (millions of barrels).

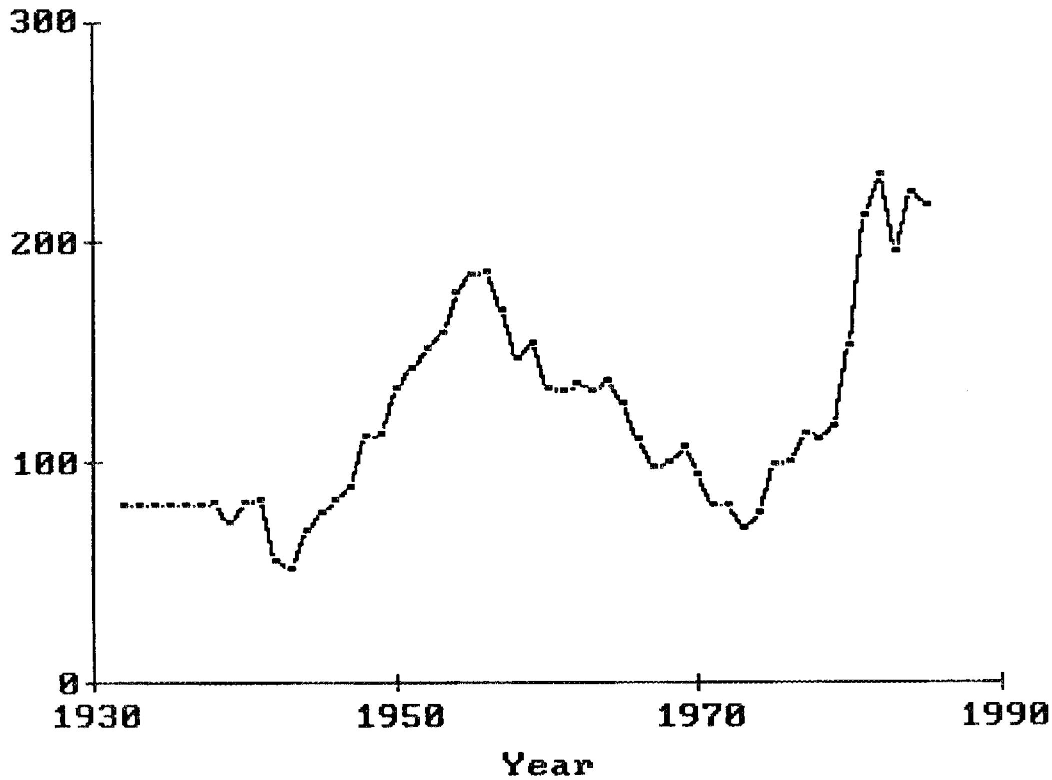

Fig. 3. Historical data on drilling (millions of feet).

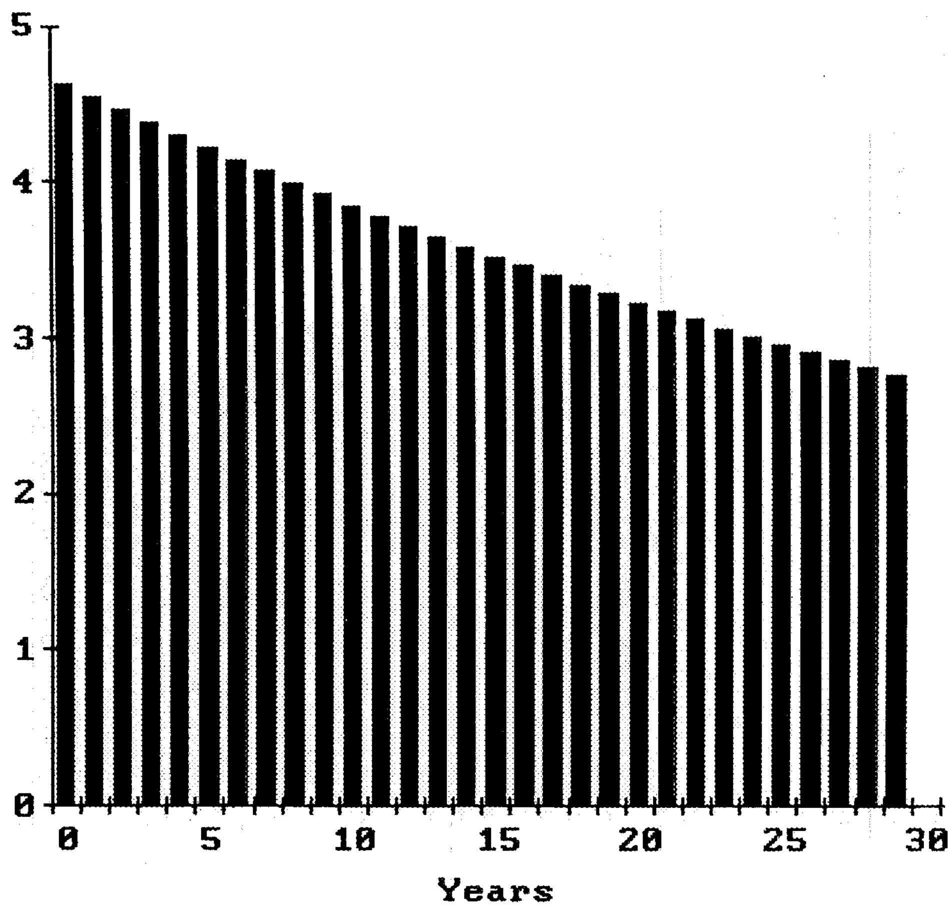

Fig. 4. Estimated hyperbolic profile 1961-85 (percent).

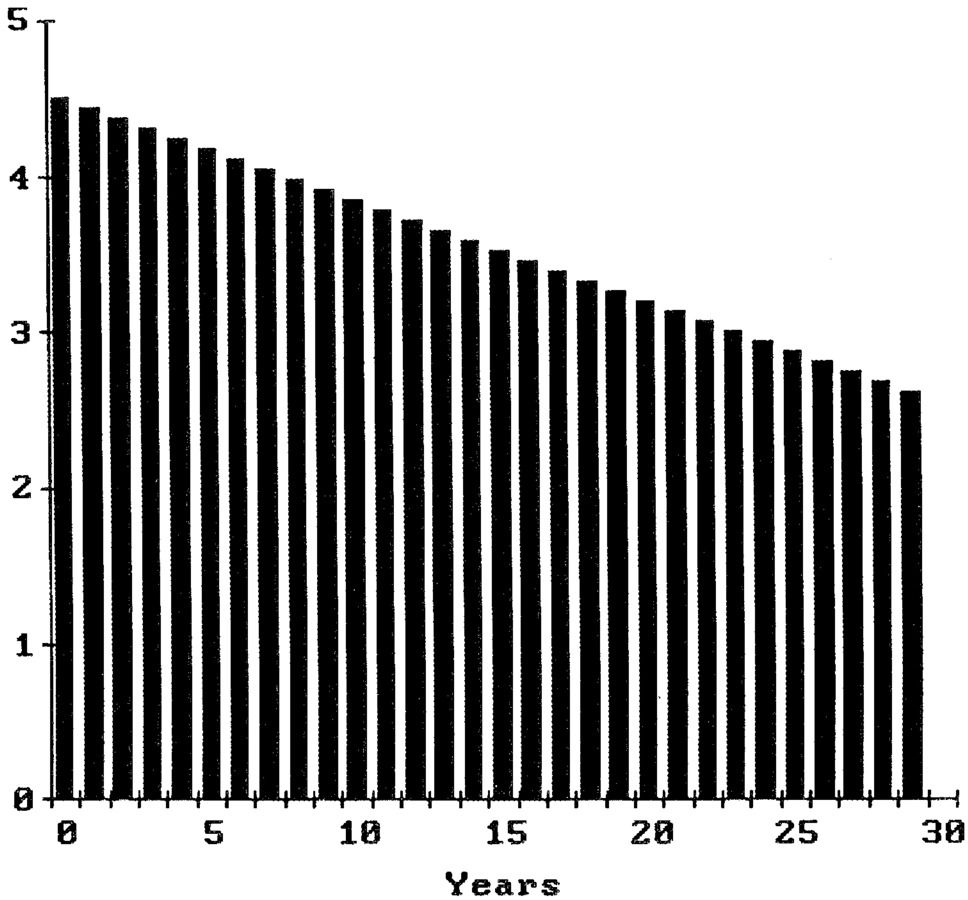

Fig. 5. Estimated linear profile 1961-85 (percent).

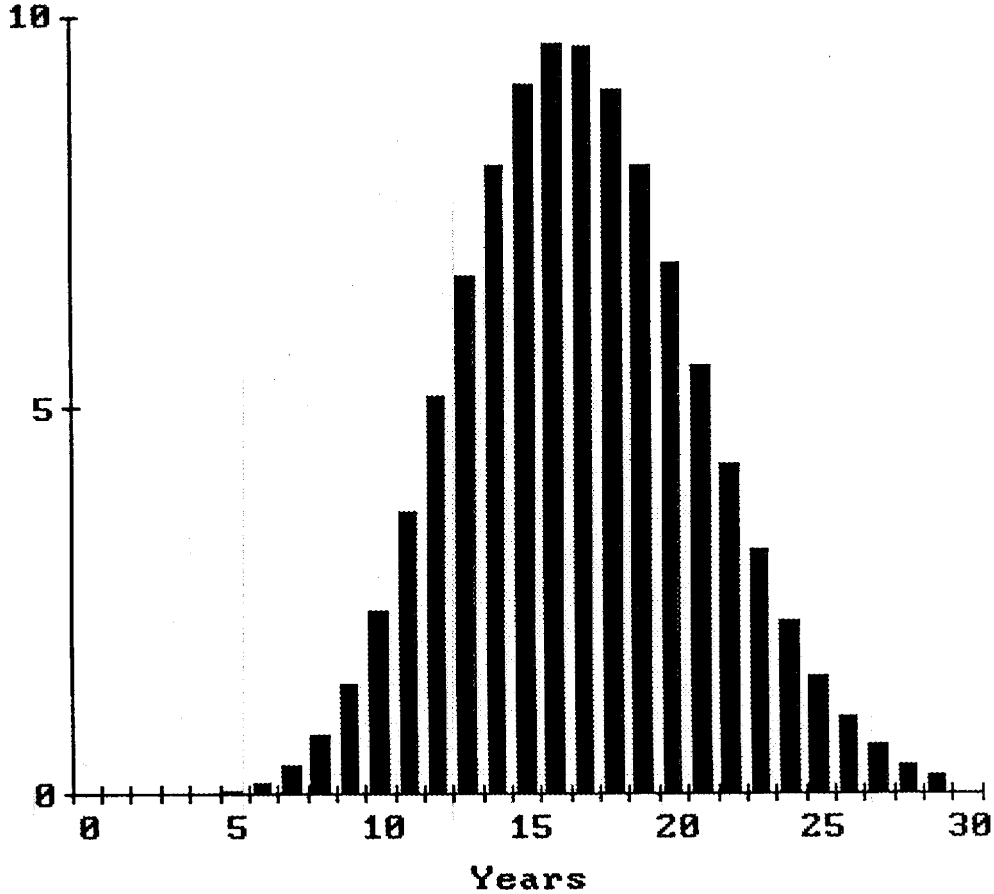

Fig. 6. Estimated Pascal profile 1961-85 (percent).

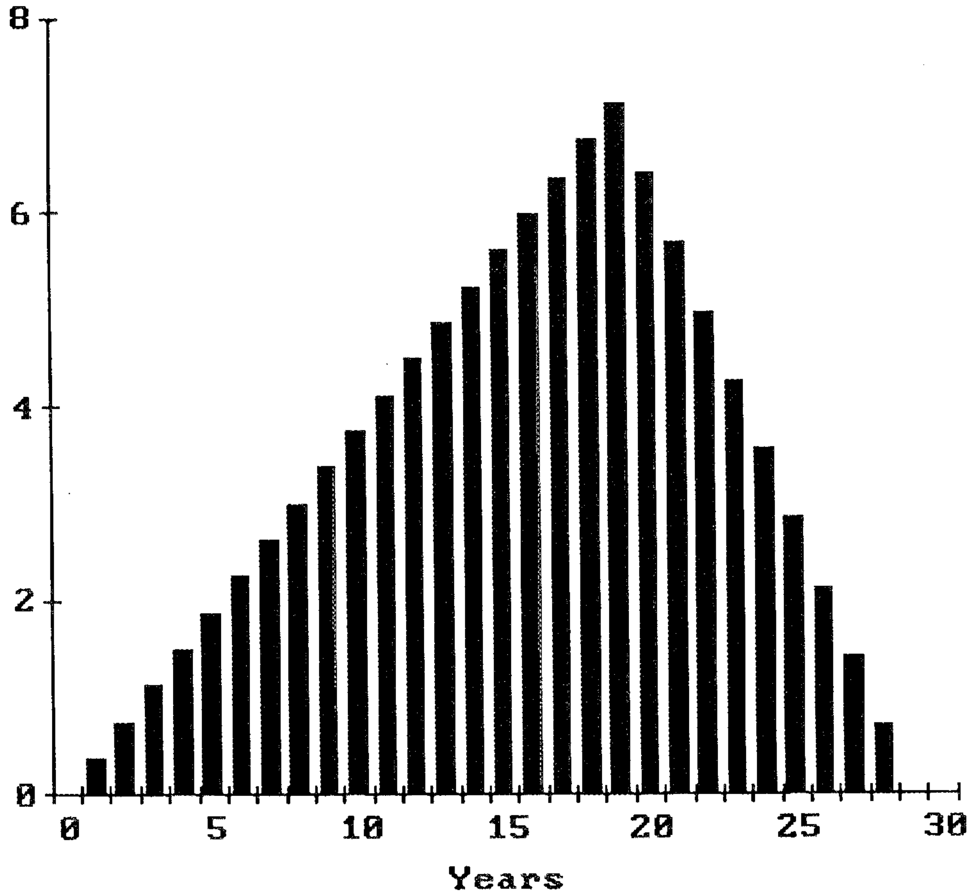

Fig. 7. Estimated inverted - V profile 1961-85 (percent).

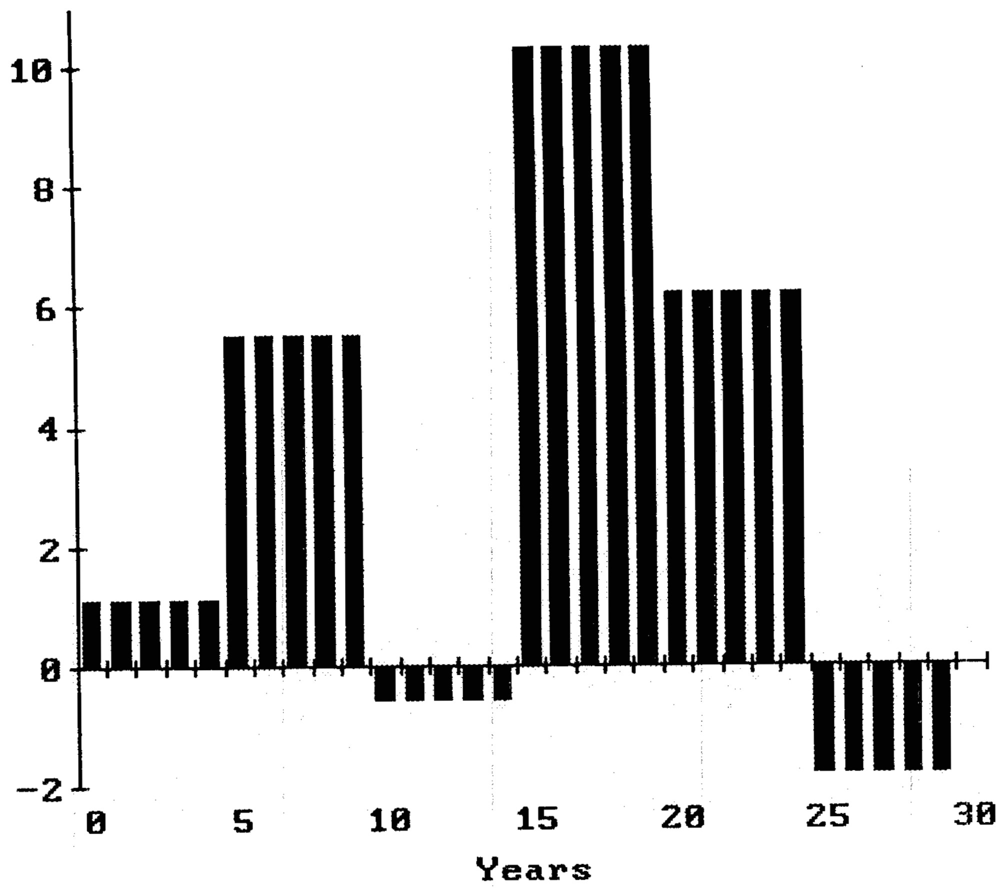

Fig. 8. Estimated grouped profile 1961-85 (percent).

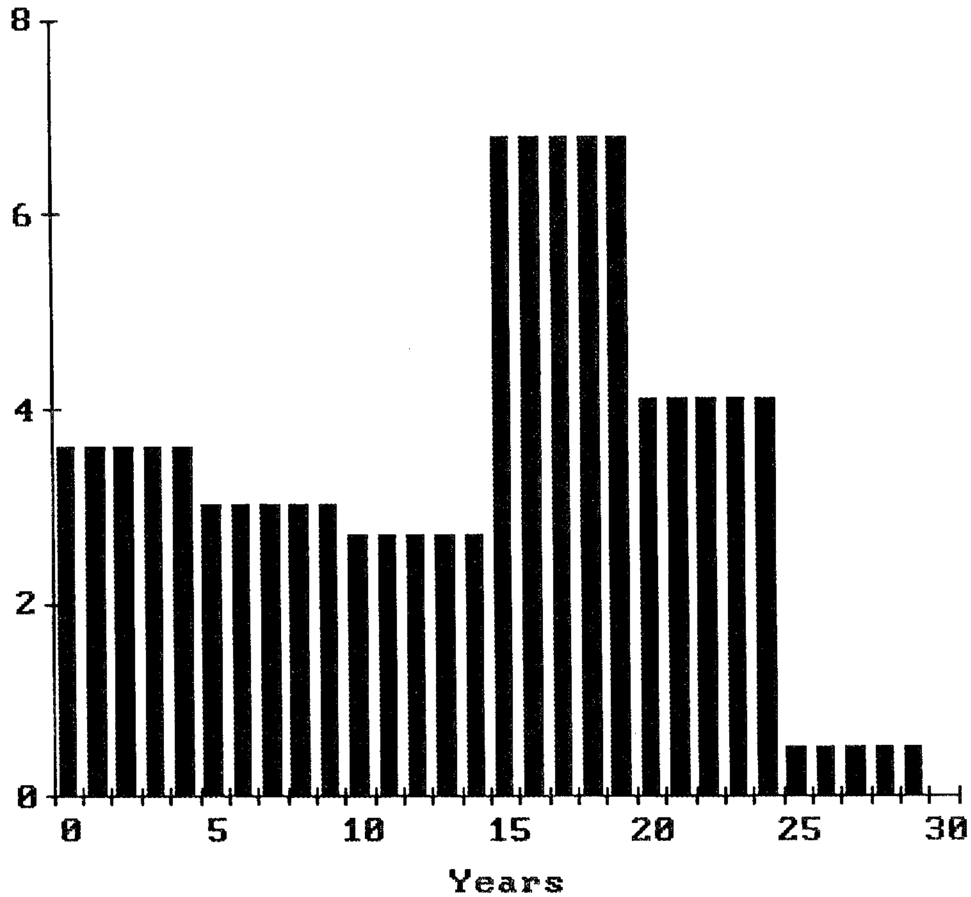

Fig. 9. Estimated grouped profile corrected for autocorrelation 1961-85 (percent).

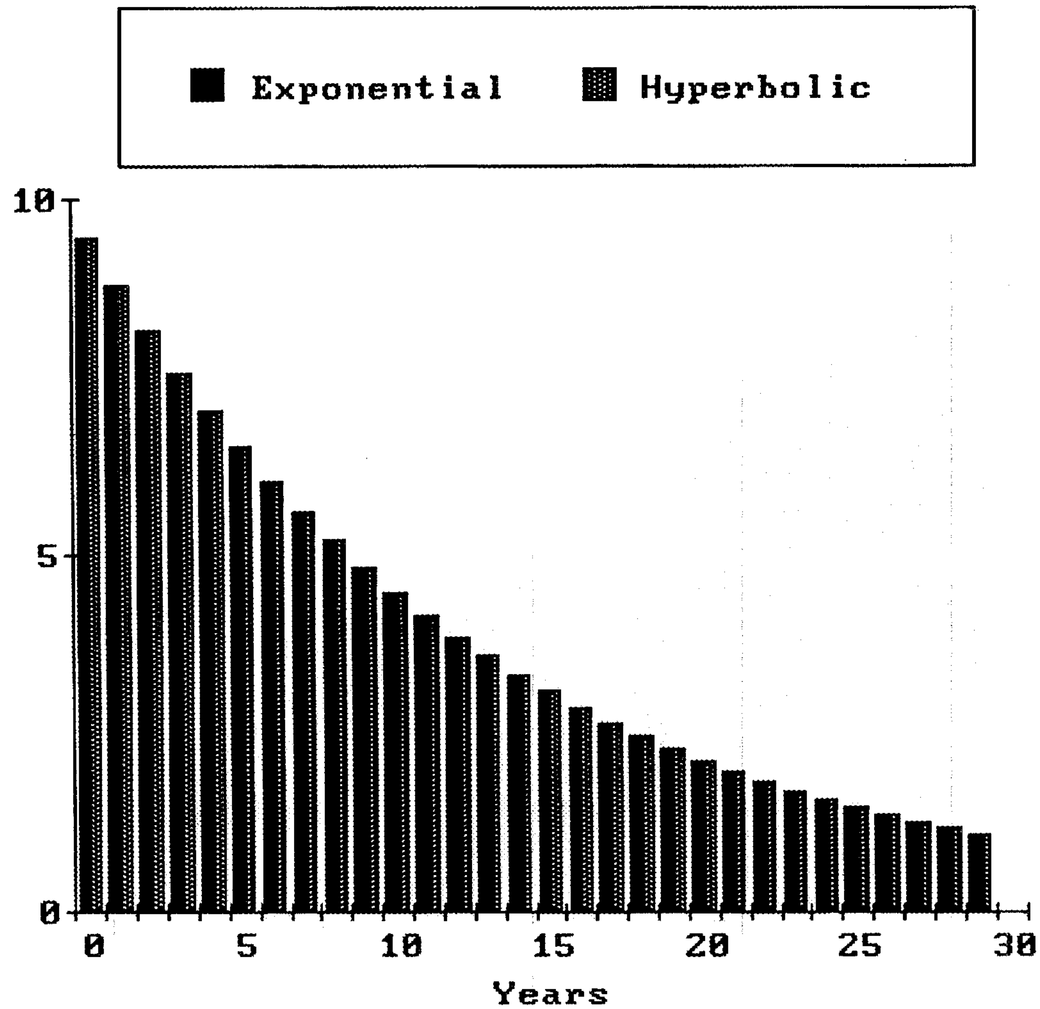

Fig. 10. Estimated exponential and hyperbolic profiles 1970-85 (percent).

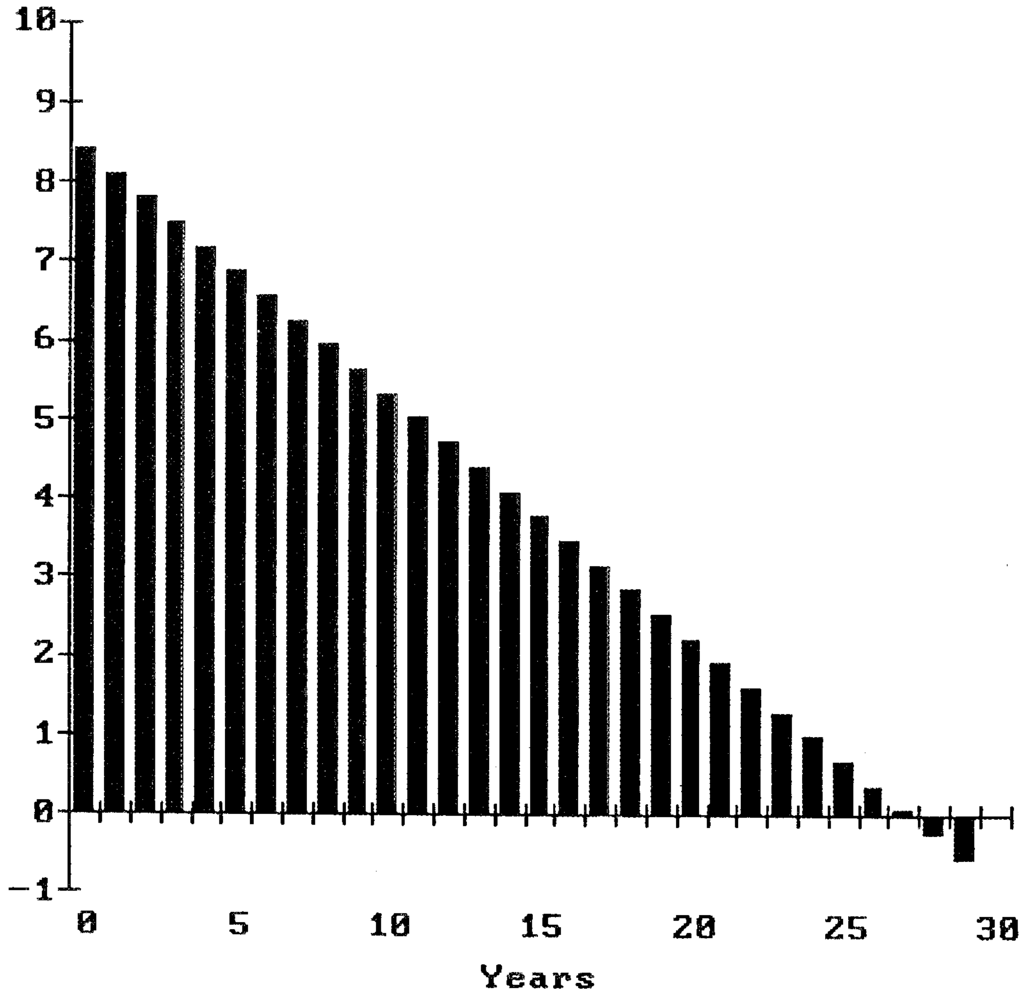

Fig. 11. Estimated linear profile 1970-85 (percent).

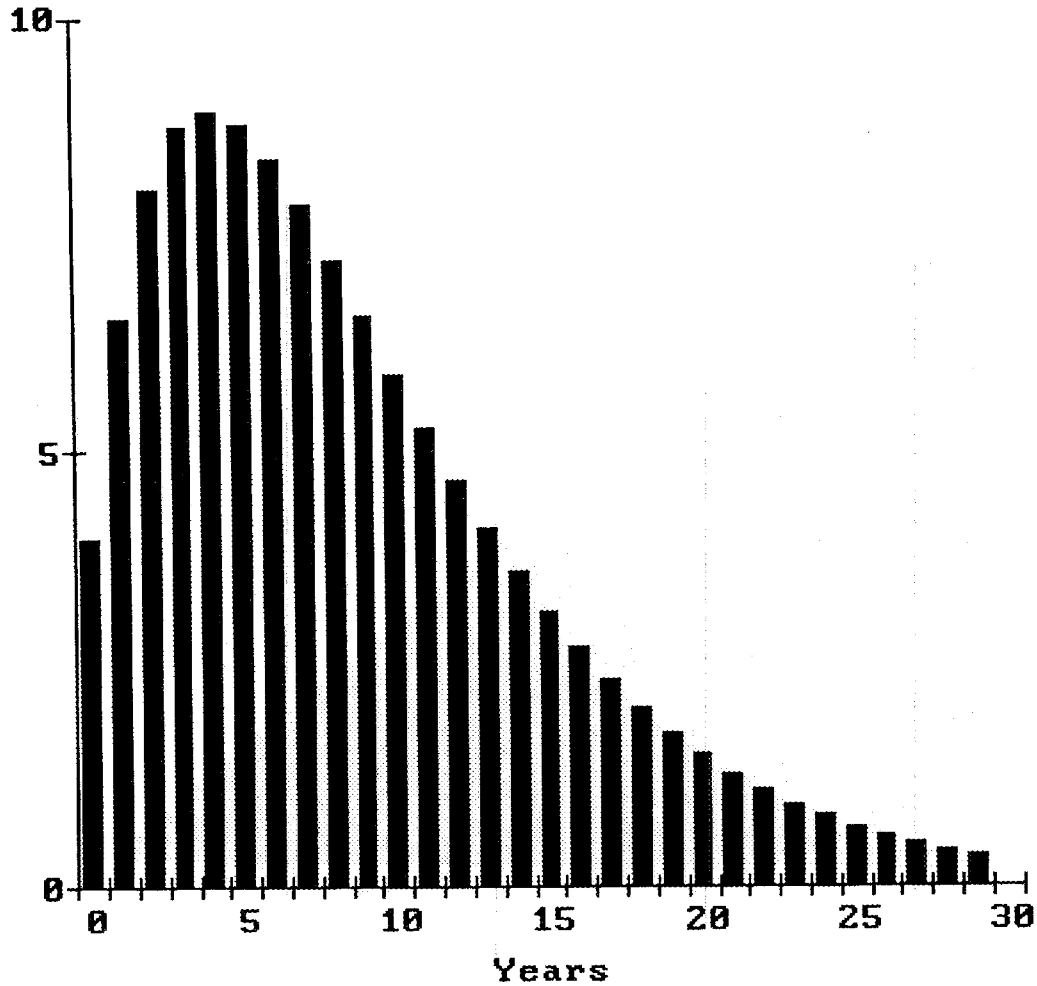

Fig. 12. Estimated Pascal profile 1970-85 (percent).

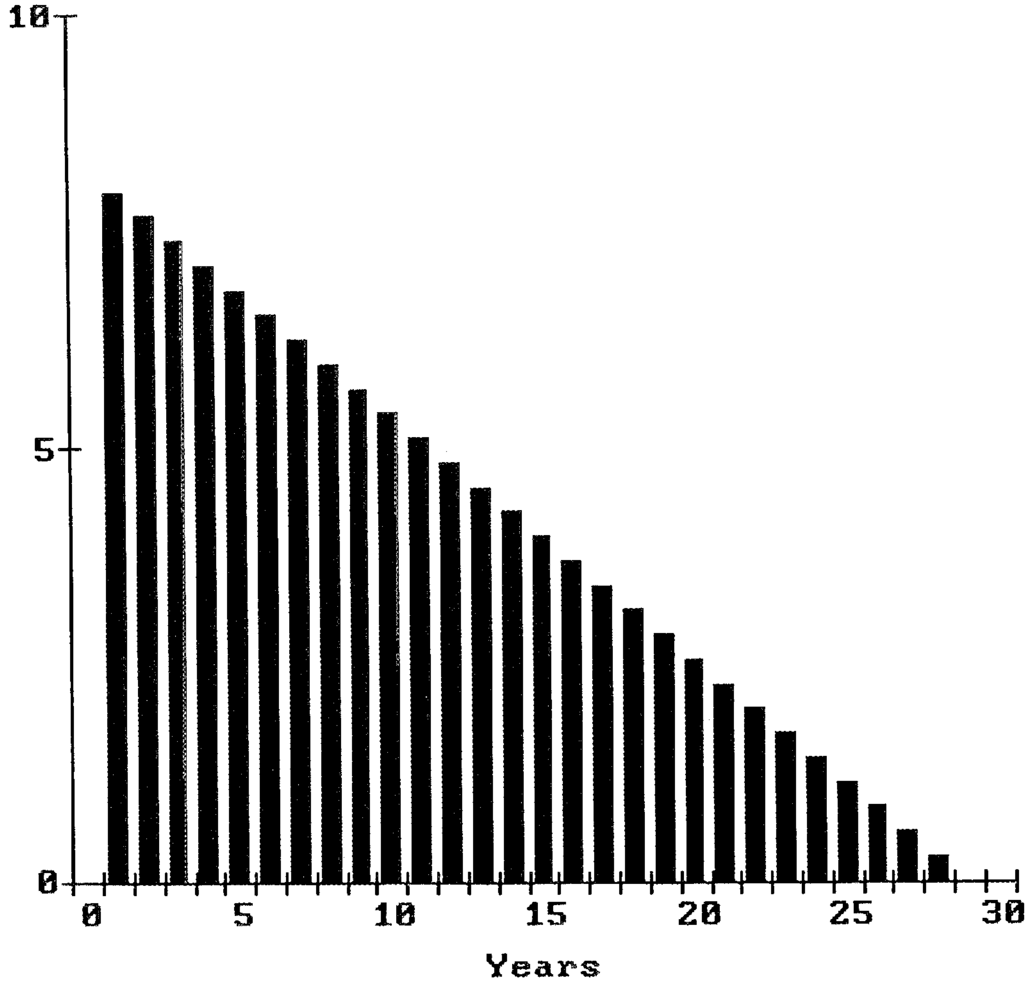

Fig. 13. Estimated inverted - V profile 1970-85 (percent).

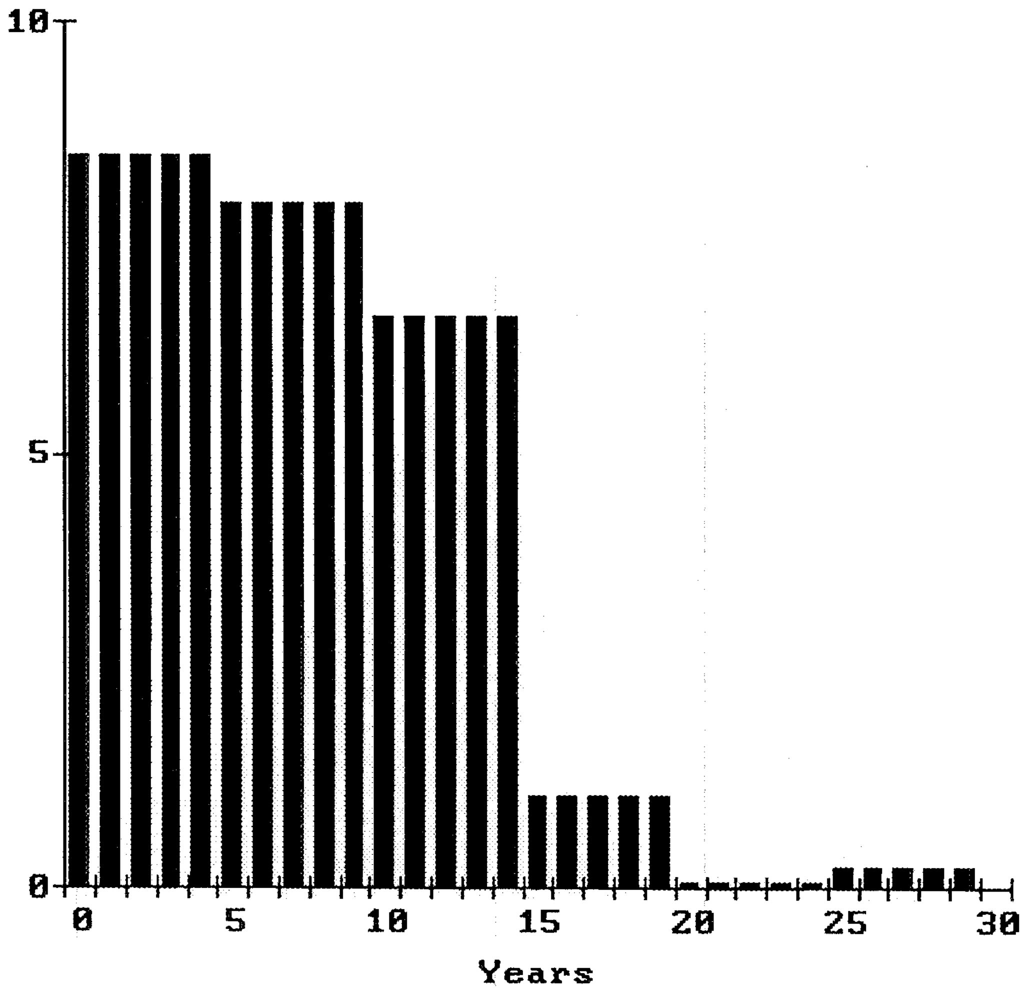

Fig. 14. Estimated grouped profile 1970-85 (percent).

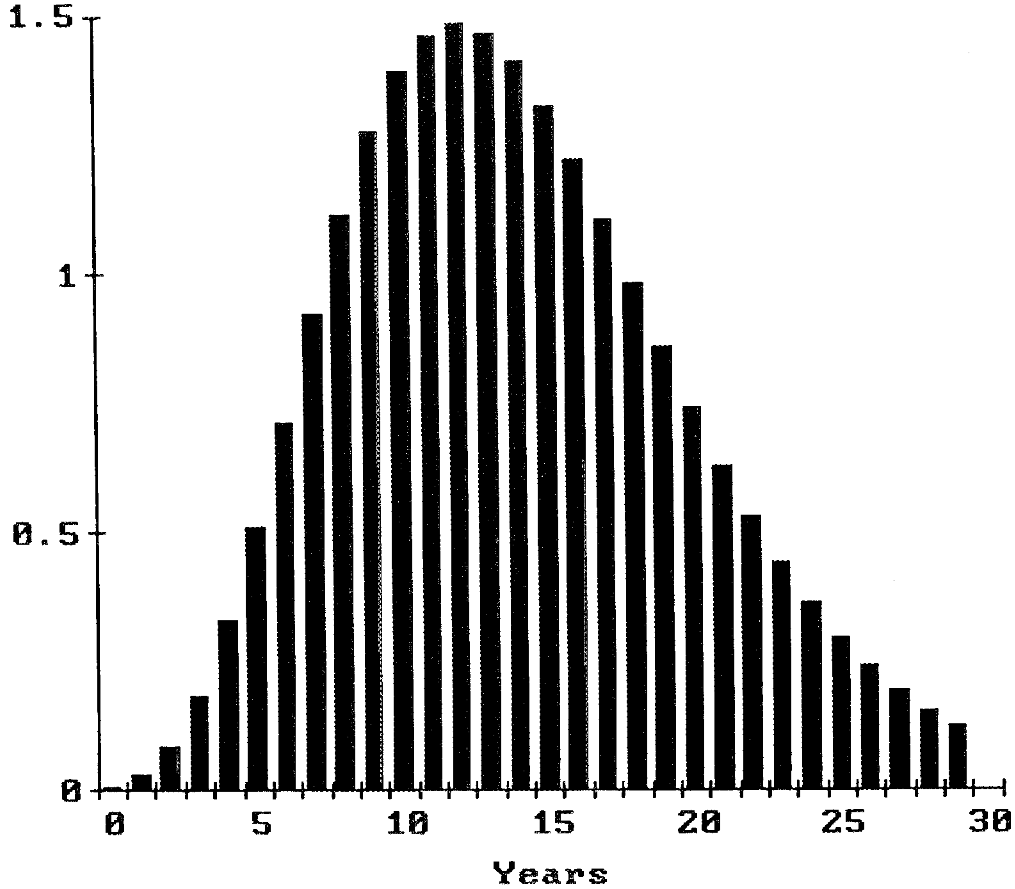

Fig. 15. Estimated Pascal profile 1961-85 (barrels per foot).

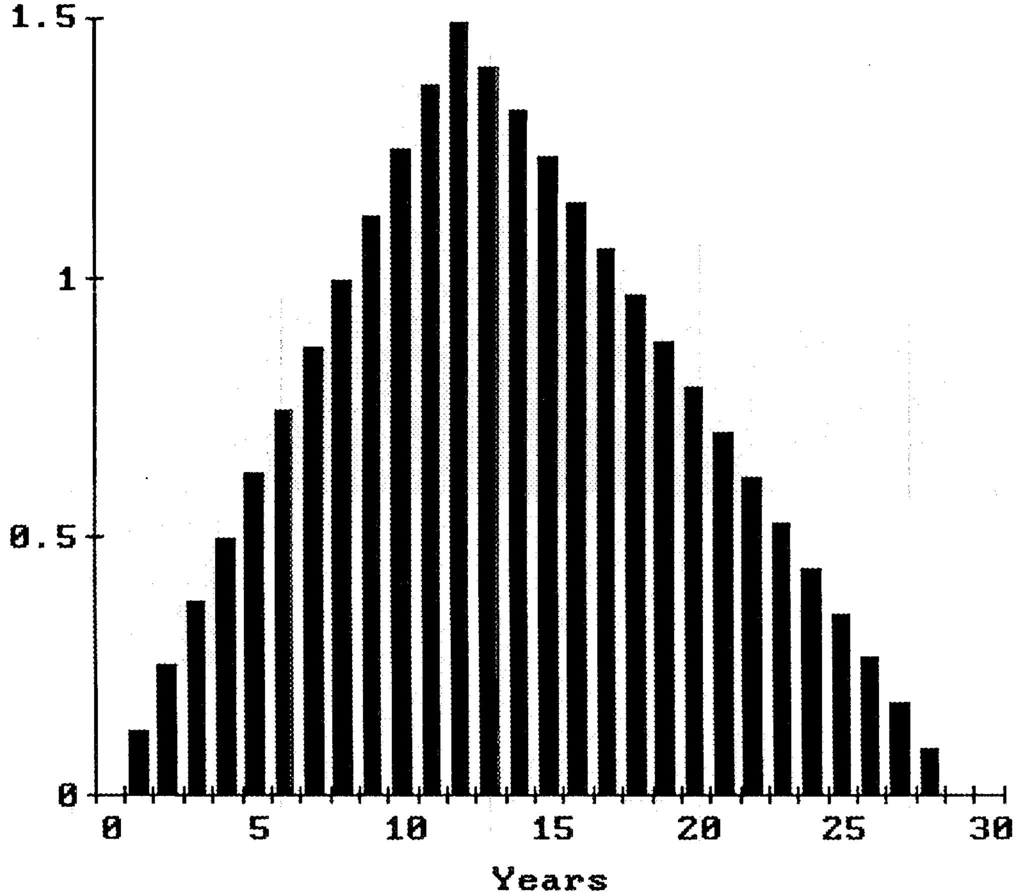

Fig. 16. Estimated inverted - V profile 1961-85 (barrels per foot).

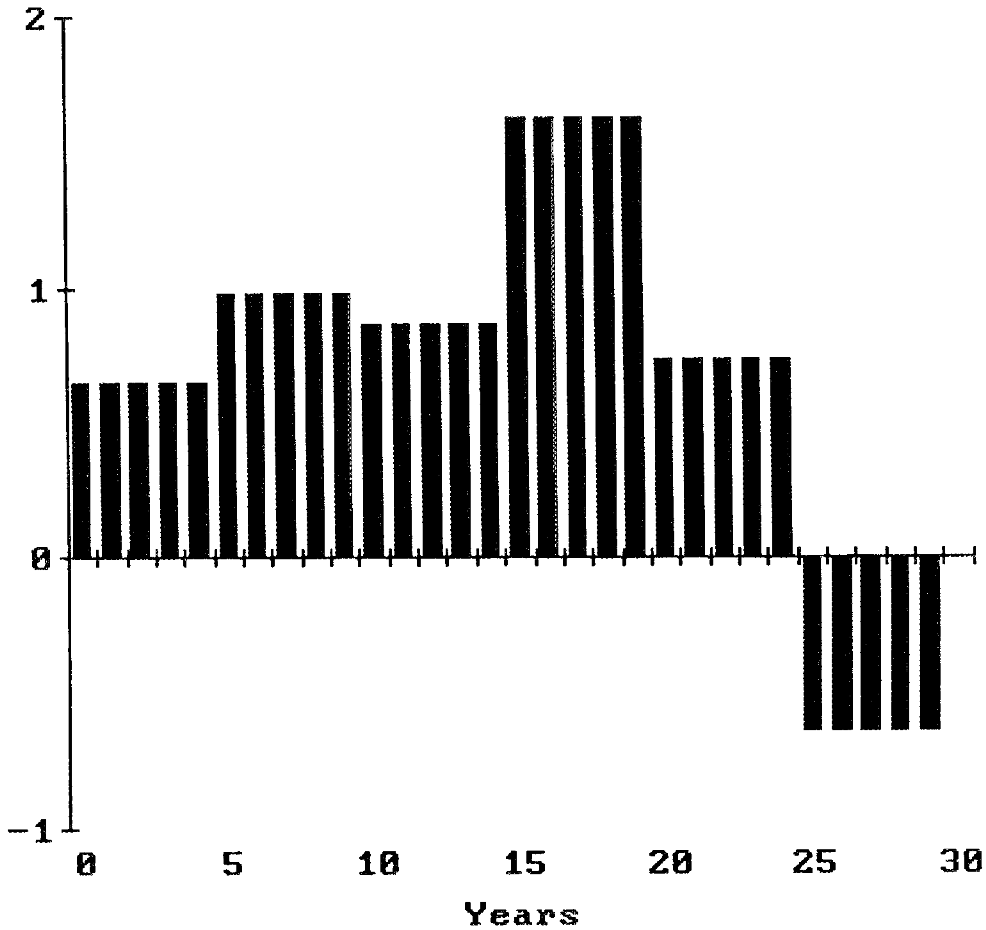

Fig. 17. Estimated grouped profile 1961-85 (barrels per foot).

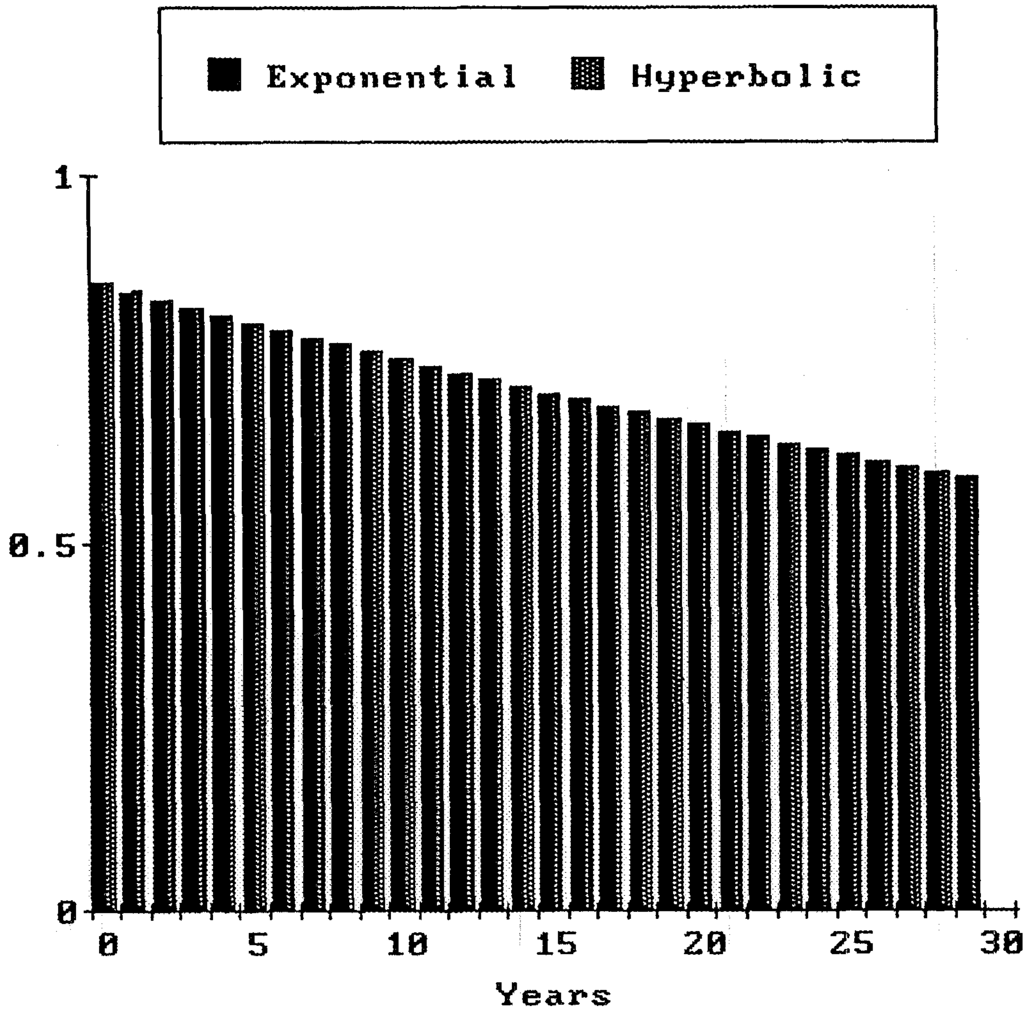

Fig. 18. Estimated exponential and hyperbolic profiles 1970-85 (barrels per foot).

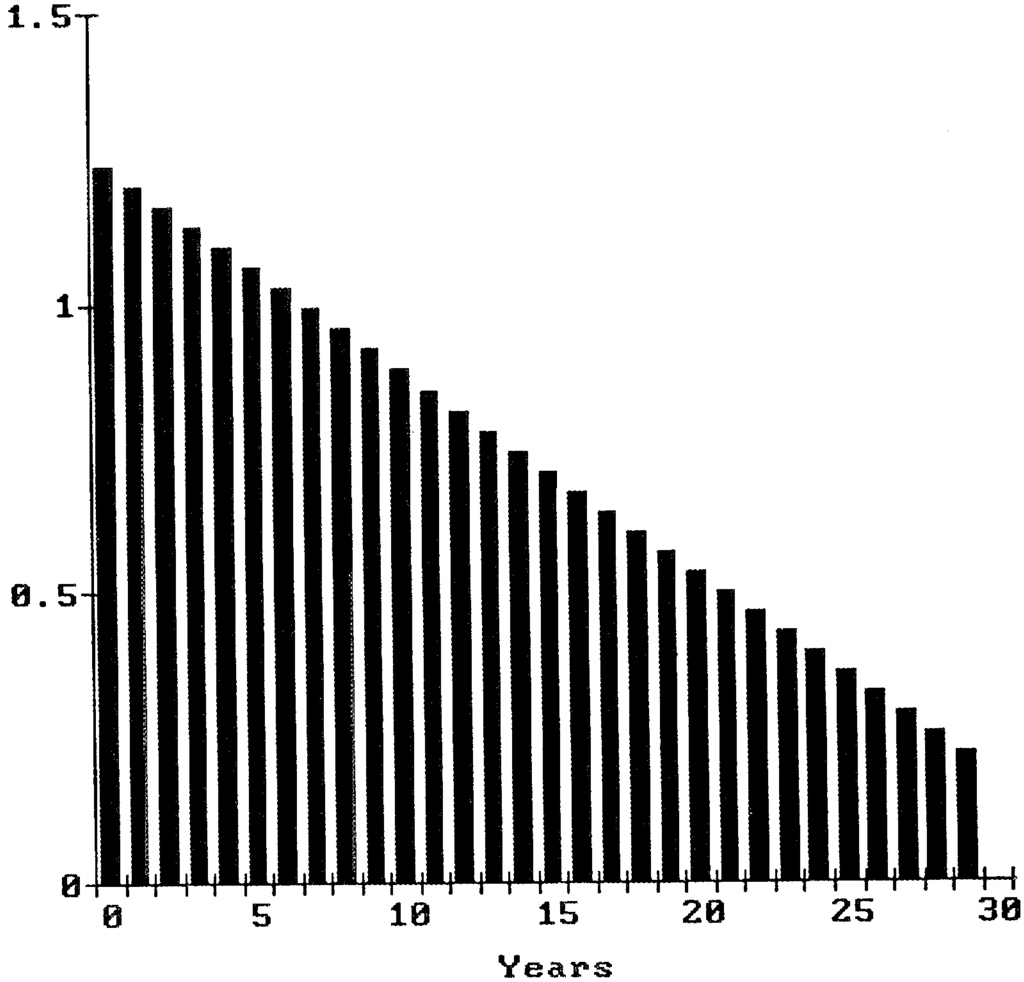

Fig. 19. Estimated linear profile 1970-85 (barrels per foot).

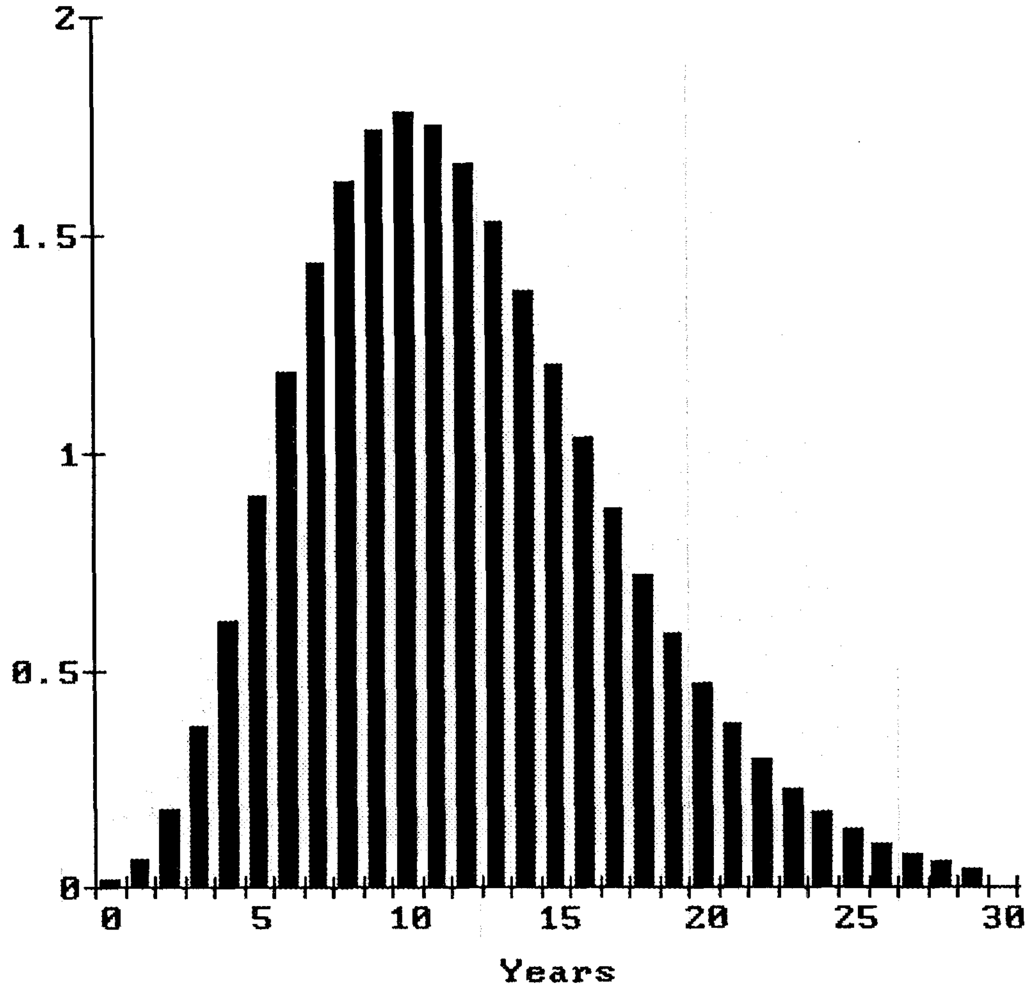

Fig. 20. Estimated Pascal profile 1970-85 (barrels per foot).

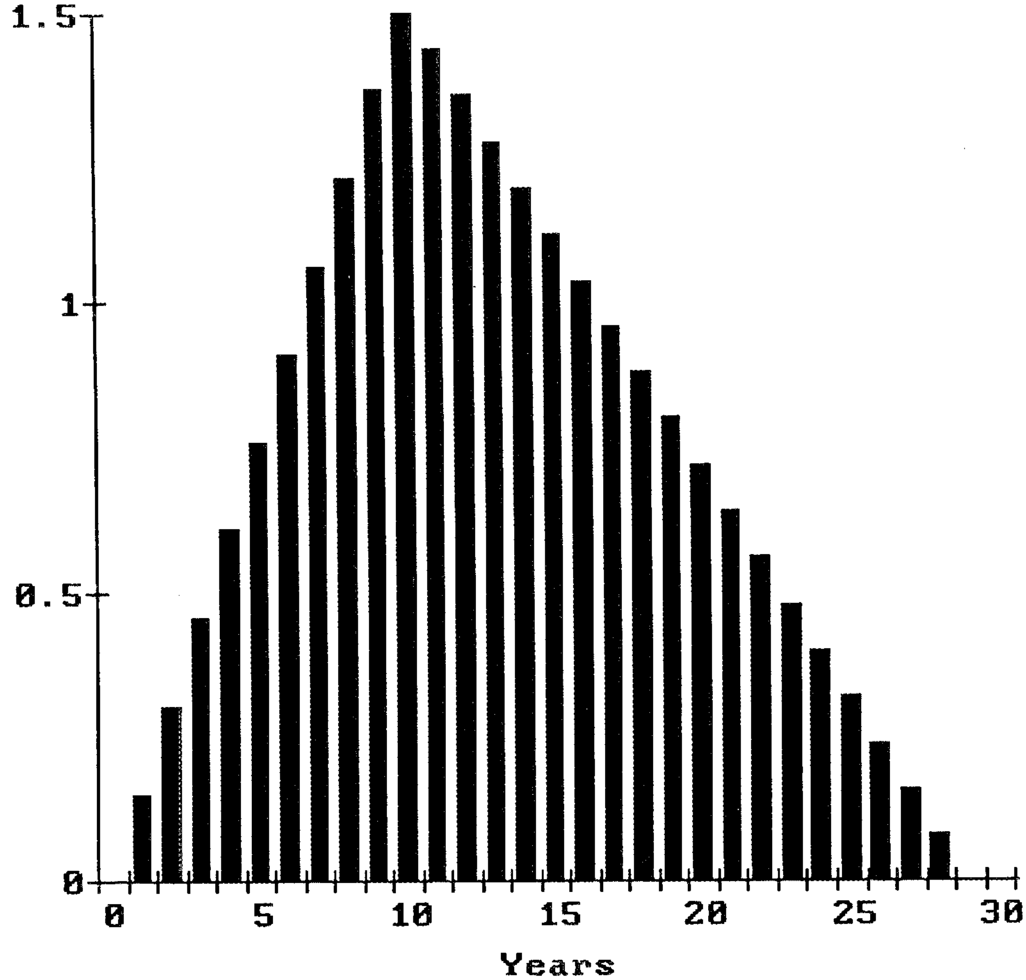

Fig. 21. Estimated inverted - V profile 1970-85 (barrels per foot).

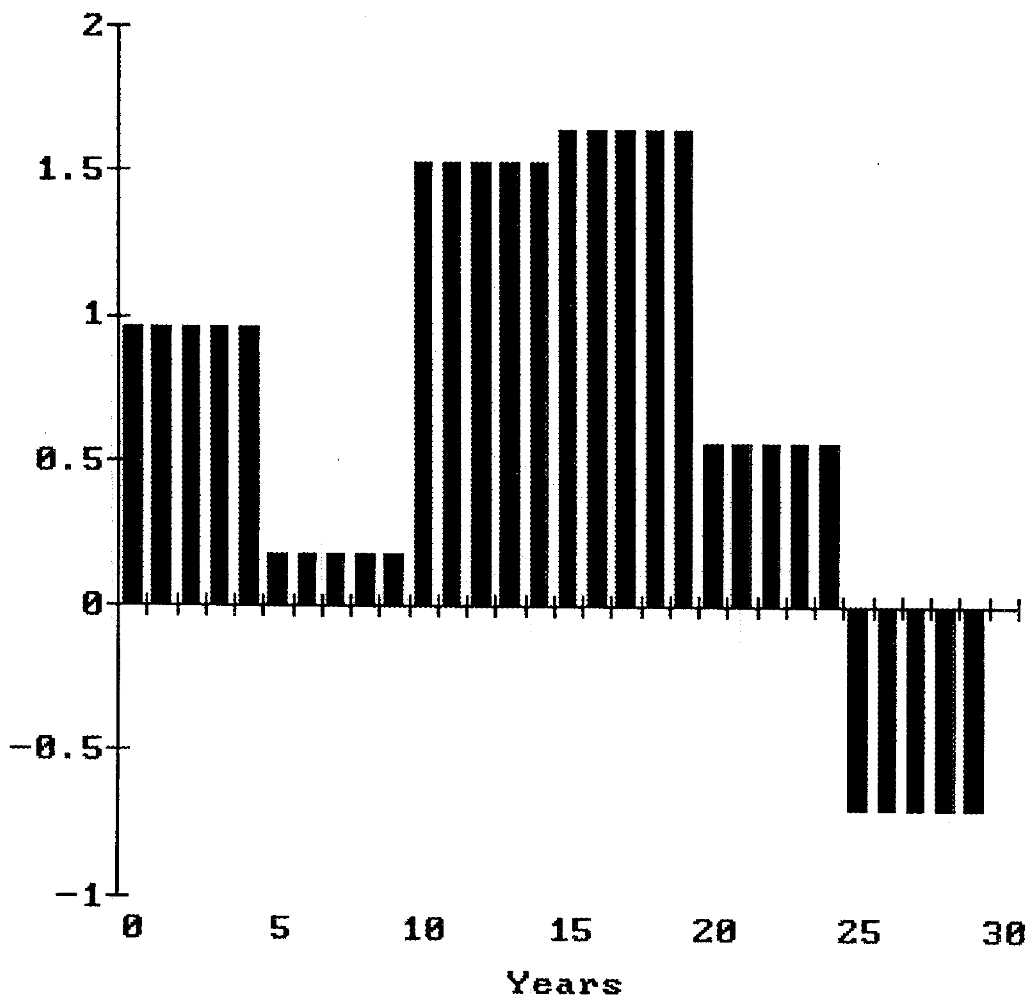

Fig. 22. Estimated grouped profile 1970-85 (barrels per foot).

ACKNOWLEDGMENTS

We are grateful to the following for helpful comments: Jerry Hadder, Paul Leiby, and Max Morris of Oak Ridge National Laboratory; Bill Finger and his colleagues at Exxon USA; and John Wood and his colleagues at the Dallas Field Office of the Energy Information Administration.

REFERENCES

1. Adelman, M.A., J. Houghton, G. Kaufman and M. Zimmerman, Energy Resources in an Uncertain Future, Cambridge: Ballinger, 1983.

2. Adelman, M.A. and H.D. Jacoby, "Alternative Methods of Oil Supply Forecasting," Advances in the Economics of Energy and Resources, 2(1979), 1-38.

3. API, Reserves of Crude Oil, Natural Gas Liquids, and Natural Gas in the United States and Canada as of December 31, 1979, Washington, D.C.: American Petroleum Institute, 34(1980).

4. API, Basic Petroleum Data Book: Petroleum Industry Statistics, Washington, D.C.: American Petroleum Institute, 5(1985)

5. Arps, J.J., "Production Decline Curves" in T.C. Frick and R. W. Taylor, eds., Petroleum Production Handbook, Volume II: Reservoir Engineering, New York: McGraw-Hill, 1962, 37-41 to 38-18.

6. Bradley, P.G., The Economics of Crude Oil Production, Amsterdam: North Holland, 1967.

7. Craft, B.C. and M.F. Hawkins, Jr., Applied Petroleum Reservoir Engineering, Englewood Cliffs: Prentice Hall, 1959.

8. Epple, D.N., Petroleum Discoveries and Government Policy, Cambridge: Ballinger, 1975.

9. Johnson, R.R., "North American Drilling Activity in 1985," The American Association of Petroleum Geologists Bulletin, 70(1986), 1195-1223.

10. Kmenta, J., Elements of Econometrics, New York: Macmillan, 1986.

11. Kuuskraa, V.A., et al, Replacement Costs of Domestic Crude Oil - Supply Analysis Methodology, DOE/FE/30012-1 (Vol. 2), Washington, D.C.: U.S. Department of Energy, 1985.

12. Mead, W.J., Petroleum: An Unregulated Industry?" in R.J. Kalter and W.A. Vogely, eds., Energy Supply and Government Policy, Ithaca, N.Y.: Cornell University Press, 1976, 130-160.

13. Wood, J.H., et al, U.S. Crude Oil, Natural Gas, and Natural Gas Liquids Reserves, DOE/EIA-0216(85), Washington, D.C.: Energy Information Administration, 1986.

14. World Oil, February 15,1984, p. 72.

INTERNAL DISTRIBUTION

1. T. D. Anderson
2. B. R. Appleton
3. T. R. Curlee
4. R. M. Davis
5. W. Fulkerson
6. G. R. Hadder
7. E. A. Hirst
8. R. R Judkins
9. R. M. Lee
10. P. N. Leiby
11-15. F. C Maienschein
16. M. D. Morris
17. A. M. Perry, Jr.
18-22. F. G. Pin
23-27. D. B. Reister
28. M. W. Rosenthal
29. R. B. Shelton
30. D. P. Vogt
31. J. J. Dorning (Consultant)
32. R. M. Haralick (Consultant)
33. Central Research Library
34. Y-12 Technical Library Document Reference Section
35-36. Laboratory Records - RC
37. ORNL Patent Office
38-42. EPMD Reports Office

EXTERNAL DISTRIBUTION

43. Office of Assistant Manager, Energy Research and Development, DOE-ORO, Oak Ridge, Tennessee 37831

Albion College, Department of Economics and Management, Albion, MI 49224
44. D. S. Christiansen

American Gas Association, 1515 Wilson Boulevard, Arlington, VA 22209
45. L. Tucker

Data Resources, Inc., 29 Hartwell Ave., Lexington, MA 94022
46. S. Smith

Decision Focus, Inc., 4984 El Camino Real, Los Altos, CA 94022
47. D. M. Nesbitt

Duke University, 4875 Duke Station, Durham, NC 27706
48. M. Gillis

Electric Power Research Institute, P.O. Box 10412, 3412 Hillview Ave., Palo Alto, CA 94303
49. F. R. Kalhammer
50. O. Yu
51. S. C. Peck

Emory University, Economics Department, Atlanta, GA 30330
52. R. F. Conrad

Energy Information Administration, Dallas Field Office, Room 804, 1114 Commerce, Dallas, Texas 75242
53. J. Wood

Exxon Company, P.O. Box 2180, Houston, TX 77252
54. W. R. Finger

Gas Research Institute, Suite 730 North, 1331 Pennsylvania Ave., NW, Washington, DC 20004-1703
55. R. H. Hilt
56. T. J. Woods
57. J. I. Rosenberg

John F. Kennedy School of Government, Harvard University, Cambridge, MA 02138
58. W. W. Hogan
59. J. P. Kalt

ICF/Lewin Energy, Room 702, 9300 Lee Highway, Fairfax, VA 22031
60. M. Godec
61. V. A. Kuuskraa

Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139
62. M. A. Adelman
63. D. O. Wood

Stanford University, Stanford, CA 94305
64. A. S. Manne
65. J. P. Weyant
66. J. L. Sweeney

U.S. Department of Energy, Office of Fossil Energy, Washington, DC 20545
67. R. J. Braitsch, Jr.
68. R. Folstein
69. H. W. Hochheiser
70. D. F. Swink

U.S. Department of Energy, Washington, DC 20585
71. J. D. Pearson
72. J. Stanley-Miller

U.S. 6th Circuit Court, 417 U.S. Courthouse, Cincinnati, OH 45202
73. D. J. Boggs

74-83. Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37830