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Estimating Finding Rates for U.S. Crude Oil

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Fossil Energy Program

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Engineering Physics and Mathematics Division

ESTIMATING FINDING RATES FOR U.S. CRUDE OIL

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ABSTRACT

In this paper we provide estimates of both exploratory and developmental finding rate equations for crude oil for six onshore regions in the lower 48 United States. We estimate the finding rate in two different ways - one based on the relationship between the finding rate and cumulative reserve additions (a "derivative approach") and one based on the relationship between cumulative reserve additions and cumulative drilling (an "integral approach"). We develop the integral approach according to two different methods. We investigate differences in finding rates between approaches and between regions. The regional data set is included as an appendix.

1. INTRODUCTION

The purpose of this paper is to estimate finding rate equations by region for U.S. crude oil. We do so for both exploratory and developmental drilling for each of six onshore regions making up the lower 48 states.

The finding rate is the ratio of the amount of oil found to the effort expended in the search for oil. This ratio may differ by region and by class of drilling (exploratory or developmental). In each case, however, finding rates are likely to decline over time as the resource becomes more difficult to locate. In order to estimate how rapidly this takes place, we construct a data base on drilling and reserve additions. Our data, summarized in the Appendix, covers both exploratory and developmental drilling in six onshore regions (West Coast, Rockies, Mid Continent, West Texas, Gulf Coast, and Appalachia) over the years 1970 to 1986.

We estimate the finding rate equation in two different ways. First, we estimate it directly by focusing on the relationship between the finding rate and cumulative reserve additions. Second, we estimate it indirectly by concentrating on the relationship between cumulative drilling and cumulative reserve additions. As we will explain later, the first approach might be called a "derivative" approach and the second an "integral" approach. We actually develop the integral approach according to two separate methods.

In general, we find that the two approaches lead to somewhat different predictions for the finding rate. Given a choice, we find the integral estimates more attractive. For most regions and classes of drilling, we can establish that the finding rate declines as more drilling takes place. There are some exceptions to this pattern, and we discuss likely explanations in each particular case. We also find that there are differences in the finding rate function among regions.

The organization of this paper is as follows: In Section 2, we present a conceptual framework for dealing with finding rates. In Section 3, we describe our two approaches and present estimates of the finding rate equations. In Section 4, we discuss our results. In Section 5, we offer our conclusions. The Appendix contains the regional data sets.

2. FINDING RATE METHODOLOGY

It is useful to first discuss the notion of a finding rate. Intuitively, the finding rate should indicate the amount of crude oil which is found per unit of activity which generates the finding. Thus the finding rate is a fraction with numerator expressing the barrels of oil found and denominator expressing the effort generated.

In defining the finding rate, there are many possible candidates for numerator and denominator. Considering the numerator first, additions to the stock of oil can be measured either in terms of reserves (an estimate of what can be extracted under a given set of assumptions) or "oil-in-place" (an estimate of what lies under the ground regardless of its recoverability). Since there are many different categories of additions to the stock of oil (adjustments, revisions, extensions, new field discoveries, and new reservoir discoveries in old fields), some or all of these may be included. In particular, these categories can be grouped into developmental additions or exploratory additions. Finally, if future additions to the stock of oil can be associated with the additions that take place in a given year, the numerator can reflect an estimate of all the additions that will ultimately take place.

The simplest choice for a denominator is time itself, but time does not capture the intensity of drilling. Drilling activity is usually measured in terms of feet drilled or wells completed. Again, interest may be restricted to exploratory or developmental activity or even to subsets of these groups. There are further complications that arise with respect to the denominator because some drilling results in the finding of gas and some results in dry holes. If one is interested in exploratory footage, for example, drilling activity could be measured in terms of total exploratory footage, successful oil exploratory footage, or a measure which allocates to oil some fraction of the exploratory drilling associated with dry holes.

It is helpful to have a conceptual framework for dealing with finding rates. The model of Arps and Roberts [1958] provides a place to begin. The Arps-Roberts model is a model of discovery. As such, it focuses on wildcat wells as the measure of drilling activity. The fundamental premise behind the model is that for each additional wildcat well drilled, the probability of finding a field of a certain size class is proportional to the number of remaining undiscovered fields of that class and to the size of fields in that class as measured by their areal extent. As is well known, these assumptions lead to the equation

$$F_i(w) = F_i^* \, [1 - \exp(-C \, A_i \, w)] \qquad (2.1)$$

where $F_i(w)$ is the cumulative number of fields of class i found after w wildcat wells have been drilled, F_i^* is the total number of fields of class i that originally existed, A_i is the average areal size of a field region in which drilling takes place. Arps and Roberts choose to use $C = 2/B$ to reflect the fact that drilling is generally based on geological and geophysical leads.) It is reasonable to suppose that the original distribution of field sizes F_i^* is highly skewed so that there are fewer large fields than small and medium sized ones.

The Arps-Roberts model tells a simple but satisfying story about the discovery of oil. Discoveries do not constitute a random sample drawn from a general population. Instead, since large fields have the largest areas, they are more likely to be found early in the exploration process. As the process continues, discoveries will consist almost entirely of medium and small size fields and finding rates will fall. Finding rates will not be driven to zero, however, because at some point the distribution of remaining field sizes (concentrated mostly on smaller size fields) will not provide the incentive to continue drilling. Thus all of the oil will not be found but only that which is expected to be economically recoverable. The Arps-Roberts model helps quantify the discovery process and provides a basis for a discussion of why exploratory finding rates fall.

Since we will measure effort in terms of feet instead of wells, we first reformulate the model in terms of cumulative exploratory drilling footage x. If depth per well (d) is constant, then equation (2.1) can be rewritten as

$$F_i(x) = F_i^* \left[1 - \exp(-\tau_i x)\right], \qquad (2.2)$$

where $\tau_i = C A_i/d$. Suppose fields of class i contain B_i barrels of oil. Letting Q_i represent the cumulative reserve additions to class i fields, we have

$$Q_i = B_i F_i(x) \qquad (2.3)$$

and

$$\frac{dQ_i}{dx} = \tau_i B_i F_i^* \exp(-\tau_i x). \qquad (2.4)$$

If U_i represents the undiscovered oil in fields of class i, then

$$U_i = B_i F_i^* \exp(-\tau_i x) \qquad (2.5)$$

and

$$\frac{dQ_i}{dx} = \tau_i U_i. \qquad (2.6)$$

The left hand side of equation (2.6) represents the finding rate for fields of class i. The equation states that the finding rate is proportional to the amount of undiscovered oil in fields of that class.

To some extent, similar results hold when we aggregate across field classes. Let $Q = \Sigma Q_i$ and $U = \Sigma U_i$. From equation (2.6) we find that

$$\frac{dQ}{dx} = \Sigma \tau_i U_i. \quad (2.7)$$

This equation may be written as

$$\frac{dQ}{dx} = \tau U \quad (2.8)$$

where

$$\tau = \frac{\Sigma \tau_i U_i}{\Sigma U_i} \quad (2.9)$$

so that τ is no longer constant but is a function of x. In fact, it can be shown that τ falls as x increases. Since data are generally not available by field class size, most studies are forced to assume that τ is approximately constant.

We can derive the relationship between cumulative drilling footage (x) and cumulative reserve additions (Q) from equation (2.8). Let Q^* denote the amount of oil that will ultimately be found in all field classes. Then

$$\frac{dQ}{dx} = \tau (Q^* - Q) \quad (2.10)$$

and, integrating,

$$Q = Q^* [1 - \exp(-\tau x)]. \quad (2.11)$$

Equation (2.11) has all of the basic properties one would desire in a function relating Q and x: as x increases, Q(x) increases but at a decreasing rate so that there are diminishing returns to effort expended; and, as x approaches infinity, Q(x) approaches a finite limit. We can derive an alternative version of (2.10) by inserting the value of Q from (2.11) into (2.10) and taking logarithms:

$$\log (dQ/dx) = \log \tau + \log Q^* - \tau x. \quad (2.12)$$

Most estimates of finding rates are based on equations (2.10), (2.11), or (2.12). The notion of using these equations to estimate finding rates goes back at least to Hubbert [1967]. Because Hubbert wanted to include in oil discoveries for a given year all oil subsequently produced from fields discovered in that year, he had to adjust the data on exploratory discoveries to reflect future reserve additions. Using equation (2.12), he provided estimates of Q^* and τ for the lower 48 states. A similar set of estimates for τ was provided on a regional level by Hoffman and Joel [1980]. Their methodology differed from Hubbert's in that they worked with oil-in-place instead of reserves and in that they did not estimate Q^* but instead used estimates of ultimate oil-in-place based on USGS estimates of recoverable resources. Exploratory finding rates were also estimated on a regional basis by Carlson, et al. [1982] using a version of equation (2.12). Carlson also used oil-in-place, and did not associate discoveries to the year of discovery of the field as suggested by Hubbert. No estimates of the size of ultimate recovery could be given since the analysis focused on exploratory activity only.

Strictly speaking, the analysis developed thus far applies to exploratory finding rates only. Developmental activity is treated only indirectly through the multipliers suggested by Hubbert. It seems to us that much can be gained by treating exploratory and developmental drilling as independent activities and estimating separate finding rates for each based on equations (2.10), (2.11), and (2.12). Although we do not develop a precise mathematical model to justify these equations in the developmental case, we can conceive of an Arps-Roberts-like scenario that should apply. As with exploratory activity, there is a probabilistic aspect to developing reserves. The more promising activities are pursued first; the less promising are pursued later, if at all. This leads to a notion of diminishing returns for developmental drilling with the effectiveness of activities like infill drilling tapering off as more wells are drilled.

We acknowledge that the foundation for estimating developmental finding rates is less firm than that for exploratory finding rates. We also see potential problems in treating exploration and development as independent when dealing with immature petroleum-producing regions. But given the state of advancement of petroleum exploration and development in the lower 48 states, we feel that the approximation is a reasonable one to make in our situation. The alternative is to try to attribute developmental reserve additions to previous exploratory drilling. We are of the opinion that this methodology is more misleading than our own in that the data do not exist to provide the appropriate verification.

There is another problem that surfaces whether one takes Hubbert's approach or our own - the fact that additions to developmental reserves can take place that are not the result of drilling activity. When new technology is developed, for example, there can be a reassessment of the size of proved reserves. This shows up as a higher developmental finding rate in our analysis and as a higher multiplier in Hubbert's.

3. ESTIMATES OF THE FINDING RATE EQUATION

We take two different approaches to estimating the finding rate equation. Consider first the relationship between the finding rate and cumulative reserve additions. According to equation (2.10), the finding rate should decline linearly as cumulative reserve additions increase. We can estimate this relationship directly using classical least squares methods. Figure 1 shows the actual and fitted values for the case of developmental drilling in the West Texas region.

Alternatively, we can focus on the relationship between cumulative drilling and cumulative reserve additions as suggested by equation (2.11). This form can be fit using nonlinear least squares methods, as illustrated for West Texas in Fig. 2. From the fitted values, the finding rate for each value of cumulative drilling can be determined.

In terms of a continuous time formulation, the finding rate is the derivative of the function relating cumulative reserve additions to cumulative drilling. Thus we can refer to the first approach as a "derivative approach" and the second as an "integral approach." The finding rates produced by the two approaches can be different. In Fig. 3 we show the finding rates predicted for developmental drilling in West Texas. For each year of our sample, we compute the estimated finding rate by each of the two methods according to the reserve additions or drilling that actually took place. Although both sets of predictions decline monotonically, those associated with the integral approach take on a narrower range of values.

Let us examine the derivative approach in more detail. We work with the finding rate as a function of cumulative discoveries and estimate the following relationship:

$$R_t = A + B\,Q_t + u_t \tag{3.1}$$

Here Q_t denotes cumulative reserve additions (measured from the beginning of 1970), R_t denotes the finding rate (i.e. the ratio of reserve additions to drilling footage in year t), and u_t is an error term. A and B are the parameters to be estimated. Using the notation of equation (2.10), $A = \tau Q^*$ and $B = \tau$. We estimate this equation separately for each region and for each kind of drilling (exploratory and developmental) using the data given in the Appendix.

The parameter A represents the finding rate in 1970 (when $Q_t = 0$), and B gives a measure of how the finding rate changes as reserves are added. We expect B to be negative. If Q^* is the amount of oil that will ultimately be recovered in a region, then the finding rate is zero when $Q = Q^*$. Thus -A/B is a measure of the ultimate recovery as of 1970.

Our estimates are presented in the following table. The units of B are 1/millions of feet.

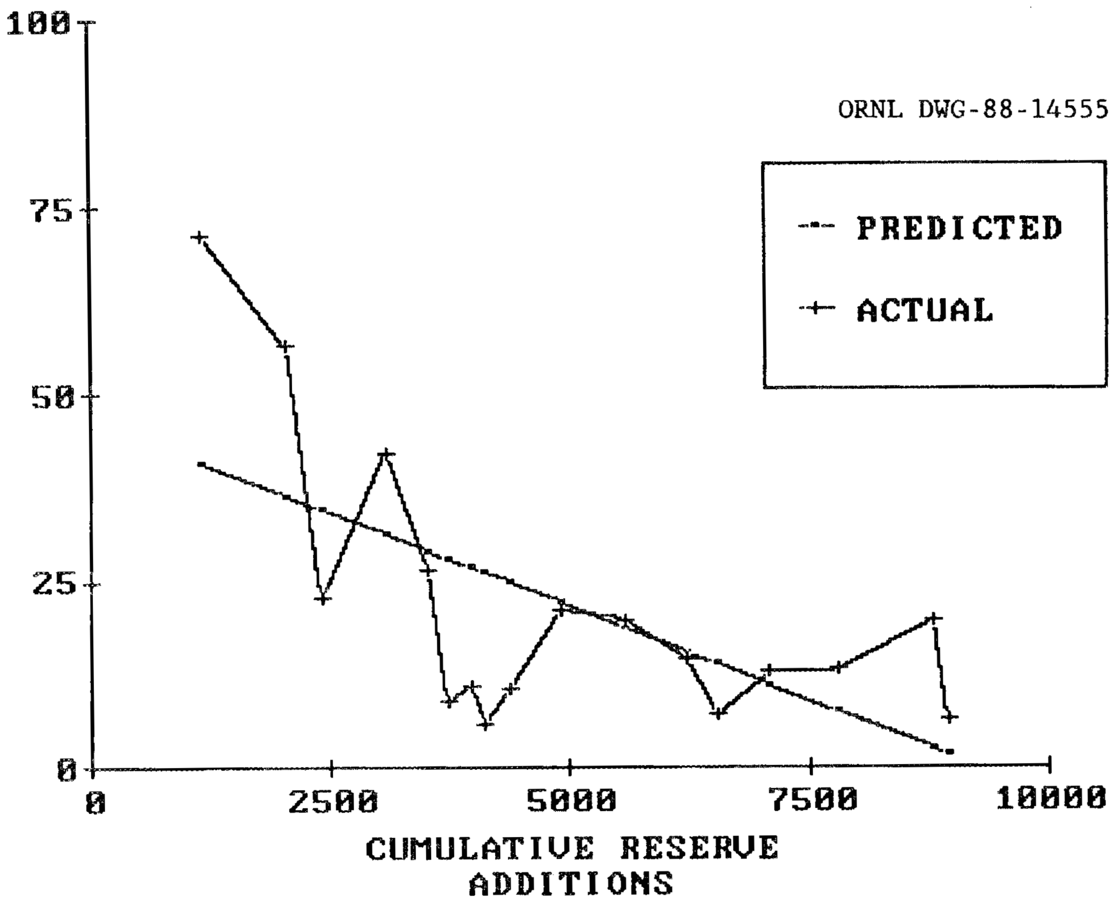

Fig. 1. Developmental Finding Rate in West Texas (Barrels Per Foot and Millions of Barrels).

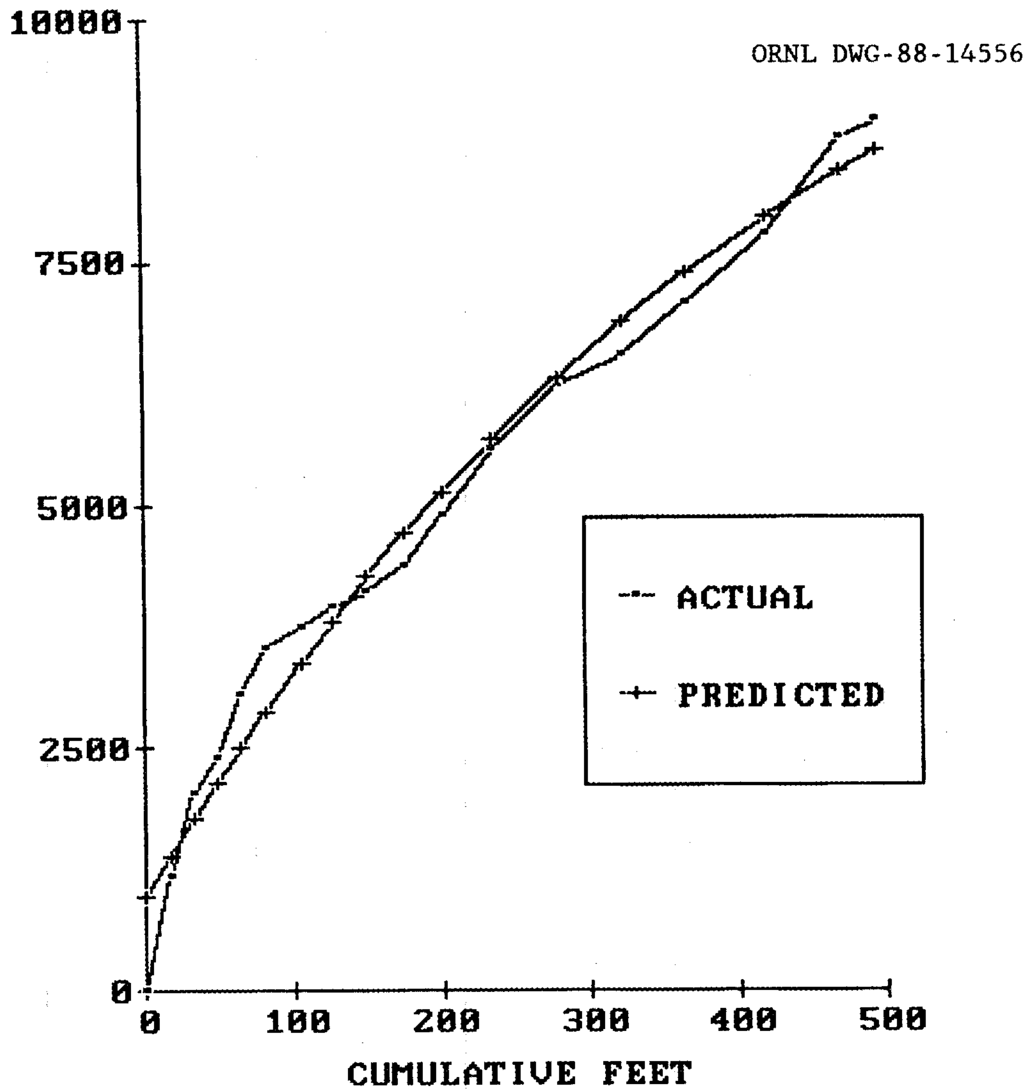

Fig. 2. Cumulative Developmental Reserve Additions in West Texas (Millions of Barrels and Millions of Feet).

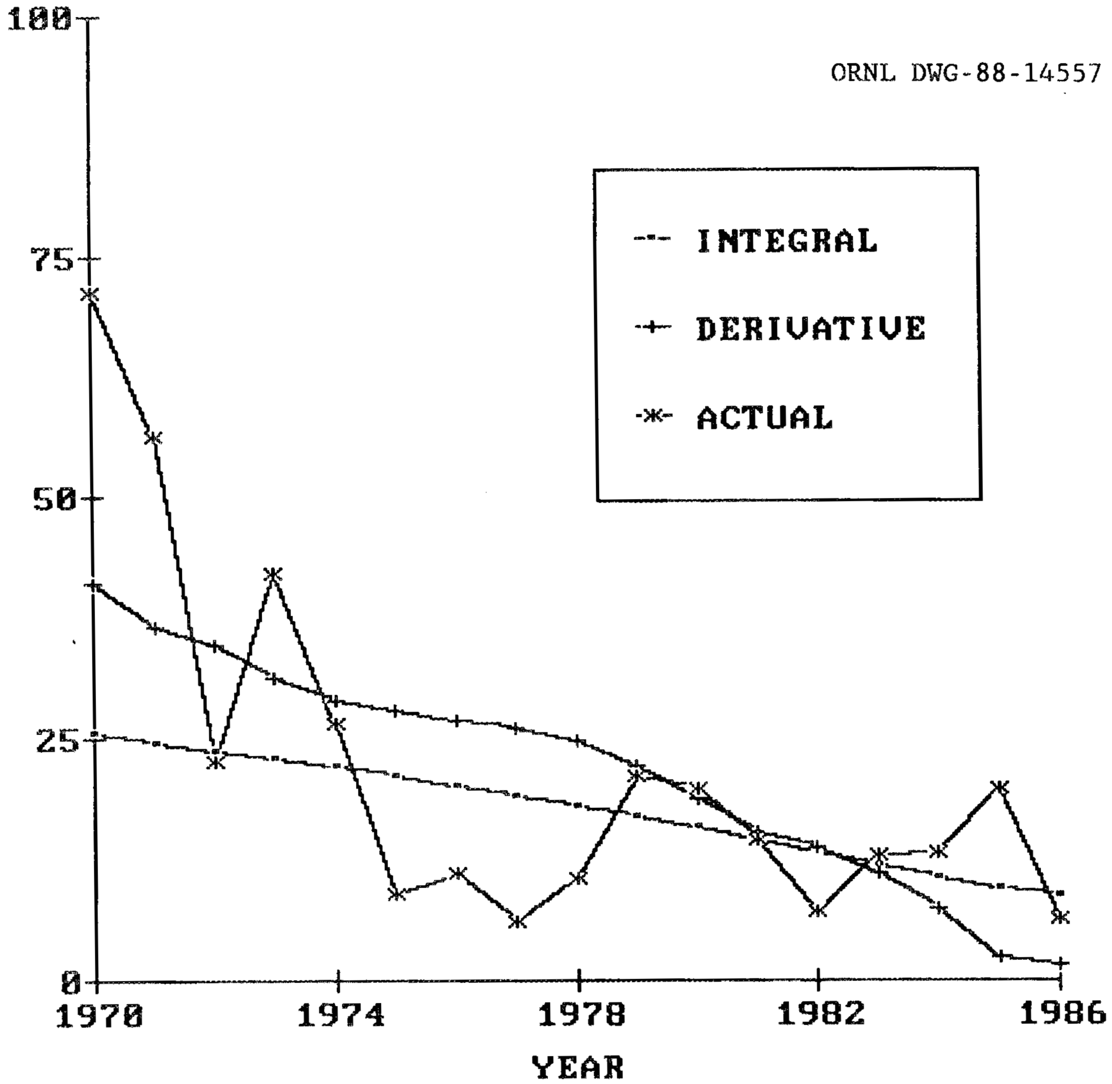

Fig. 3. Predicted Finding Rates for Developmental Drilling in West Texas (Barrels Per Foot).

Table 3.1. Estimates Obtained Using the Derivative Approach

Exploratory Drilling

Region	A	SE of A	B	SE of B	-A/B	R^2
West Coast	2.6	1.7	+.0704	.0429	-	.15
Rockies	3.4	.7	+.0015	.0028	-	.02
Mid Contin.	6.0	1.2	-.0101*	.0057	594	.17
West Texas	7.1	.6	-.0169*	.0030	420	.68
Gulf Coast	13.5	1.7	-.0096*	.0032	1406	.38
Appalachia	28.1	8.4	-.1068*	.0465	263	.26
US Lower 48	7.9	.6	-.0021*	.0004	3781	.64

Developmental Drilling

Region	A	SE of A	B	SE of B	-A/B	R^2
West Coast	49.2	17.3	+.0051	.0074	-	.03
Rockies	26.1	4.4	-.0028	.0023	9321	.09
Mid Contin.	17.7	2.8	-.0058*	.0016	3052	.47
West Texas	46.8	8.6	-.0051*	.0016	9176	.41
Gulf Coast	18.3	11.1	-.0045	.0054	4067	.04
Appalachia	10.7	2.0	-.0072*	.0030	1486	.27
US Lower 48	29.1	4.3	-.0010*	.0003	29100	.44

Note that we have placed an asterisk by those estimates of B which are significantly less than zero at the 5% level. Note also that we have not calculated -A/B when B is positive.

Consider now the integral approach. Let x denote cumulative drilling (measured since the beginning of 1970). If the finding rate is given by equation (2.10), the relationship between cumulative reserve additions and cumulative footage is given by equation (2.11):

$$Q = D + E \exp(Bx). \tag{3.2}$$

This is the equation we estimate. Because the choice of 1970 as a starting point was arbitrary, we chose not to impose the condition that $Q = 0$ when $x = 0$ (the condition requires that $D = -E$). Thus we present separate estimates for D and E. When x is infinity and B is negative, $Q = D$. Thus D is an estimate of the ultimate recovery in the region. When $x = 0$, $Q = D + E$. We expect this to be close to zero; alternatively, we expect D and -E to be about the same. B has the same interpretation as before ($B = \ r$).

Taking the second approach, we now estimate the relationship

$$Q_t = D + E \exp(Bx_t) + u_t. \tag{3.3}$$

We expect D to be positive and E and B to be negative. We estimate this equation for each region and each type of drilling using values for Q_t and x_t calculated from the tables given in the Appendix. In order to capture the effects of 1970, the first year in our sample, we include the pair ($x_0 = 0$, $Q_0 = 0$) in our data set. We used a nonlinear least squares procedure to estimate the parameters. Our results are given in the following table.

Table 3.2. Estimates Obtained Using the Integral Approach

Exploratory Drilling

Region	D	SE of D	E	SE of E	B	SE of B	R^2
West Coast	-43	27	45	25	+.0542	.0197	.99
Rockies	-2940	3228	2921	3221	+.0013	.0013	.99
Mid Contin.	615	100	-605	96	-.0089*	.0021	.99
West Texas	433	18	-421	16	-.0138*	.0011	1.00
Gulf Coast	1370	134	-1356	126	-.0092*	.0014	1.00
Appalachia	311	28	-285	27	-.0512*	.0100	.99
US Lower 48	3877	173	-3831	166	-.0019*	.0001	1.00

Developmental Drilling

Region	D	SE of D	E	SE of E	B	SE of B	R^2
West Coast	-1031075	50921563	1030959	50921493	+.0001	.0028	.99
Rockies	9080	2767	-9006	2734	-.0026*	.0010	.99
Mid Contin.	3307	254	-3101	229	-.0038*	.0007	.98
West Texas	12304	1897	-11336	1838	-.0023*	.0006	.98
Gulf Coast	3355	620	-2903	565	-.0051*	.0021	.84
Appalachia	1474	138	-1479	116	-.0075*	.0013	.99
US Lower 48	32647	4268	-30909	4044	-.0007*	.0001	.98

Once again we have used an asterisk to denote values of B that are significantly less than zero at the 5% level.

There is another way we can estimate equation (3.2) without using nonlinear least squares methods. This involves approximating the exponential function with a quadratic:

$$\exp(Bx) = 1 + Bx + 1/2\ B^2\ x^2 \qquad (3.4)$$

we can write (3.2) as

$$Q = (D + E) + (EB)\ x + (1/2\ EB^2)\ x^2. \qquad (3.5)$$

We then estimate

$$Q_t = F + G\,x_t + H\,x_t^2 + u_t \qquad (3.6)$$

with estimates of B, D and E given by

$$B = 2H/G \qquad (3.7)$$

$$E = G^2 \;/\; (2H) \qquad (3.8)$$

$$D = F - E. \qquad (3.9)$$

Our estimates of F, G, and H are reported in Table 3.3.

Table 3.3. Estimates Obtained Using the Quadratic Approximation

Exploratory Drilling

Region	F	SE of F	G	SE of G	H	SE of H	R^2
West Coast	1.33	2.04	2.39	.60	+.0918	.0324	.98
Rockies	-18.32	9.98	3.67	.38	+.0024	.0029	.99
Mid Contin.	12.17	5.38	5.10	.29	-.0157	.0031	.99
West Texas	19.00	5.11	4.97	.26	-.0188	.0025	.99
Gulf Coast	23.01	12.71	11.57	.61	-.0350	.0060	.99
Appalachia	30.53	5.43	12.14	.78	-.1584	.0213	.99
US Lower 48	62.01	17.81	6.83	.19	-.0044	.0004	1.00

Developmental Drilling

Region	F	SE of F	G	SE of G	H	SE of H	R^2
West Coast	-116.11	87.23	59.38	5.87	+.0017	.0795	.99
Rockies	85.08	52.05	22.75	1.54	-.0228	.0086	.99
Mid Contin.	297.53	82.46	9.27	1.06	-.0083	.0023	.97
West Texas	1106.33	236.00	22.75	2.54	-.0149	.0051	.97
Gulf Coast	650.10	218.00	9.49	3.02	-.0082	.0078	.82
Appalachia	10.49	19.25	9.78	.61	-.0212	.0033	.99
US Lower 48	2166.03	608.50	17.38	1.88	-.0032	.0011	.98

We report our estimates of B, obtained from equation (3.7), in Table 3.4. Making use of an approximation formula for the quotient of two random variables (Mood, Graybill, and Boes [1974, p.181]), we calculate the standard errors of B as well. Since this formula involves the covariance between G and H, the covariances are also reported.

Table 3.4. Quadratic Estimates of B

Exploratory Drilling

Region	COV(G,H)	B	SE of B
West Coast	-.0188	+.0768	.0459
Rockies	-.0011	+.0013	.0017
Mid Contin.	-.0009	-.0062*	.0009
West Texas	-.0006	-.0076*	.0006
Gulf Coast	-.0036	-.0061*	.0007
Appalachia	-.0161	-.0261*	.0019
US Lower 48	-.0001	-.0013*	.0001

Developmental Drilling

Region	COV(G,H)	B	SE of B
West Coast	-.4498	+.0001	.0027
Rockies	-.0128	-.0020*	.0006
Mid Contin.	-.0024	-.0018*	.0003
West Texas	-.0125	-.0013*	.0003
Gulf Coast	-.0229	-.0017	.0011
Appalachia	-.0019	-.0043*	.0004
US Lower 48	-.0019	-.0004*	.0001

We have used an asterisk to denote values of B that are significantly less than zero at the 5% level.

4. DISCUSSION

We have estimated the finding rate equation using two different approaches - a derivative approach and an integral approach. In fact, we have provided two different methods for using the integral approach - a nonlinear least squares estimation and an estimation via an approximation to a quadratic function. It is interesting to compare the results produced by each method. Table 4.1 reproduces the estimates of B and denotes with an asterisk those significantly less than zero at the 5% level. The units of B are 1/millions of feet.

Table 4.1. Comparison of Estimates of B

Exploratory Drilling

Region	Derivative	Integral Nonlinear	Integral Quadratic
West Coast	+.0704	+.0542	+.0768
Rockies	+.0015	+.0013	+.0013
Mid Contin.	-.0101*	-.0089*	-.0062*
West Texas	-.0169*	-.0138*	-.0076*
Gulf Coast	-.0096*	-.0092*	-.0061*
Appalachia	-.1068*	-.0512*	-.0261*
US Lower 48	-.0021*	-.0019*	-.0013*

Developmental Drilling

Region	Derivative	Integral Nonlinear	Integral Quadratic
West Coast	+.0051	+.0001	+.0001
Rockies	-.0028	-.0026*	-.0020*
Mid Contin.	-.0058*	-.0038*	-.0018*
West Texas	-.0051*	-.0023*	-.0013*
Gulf Coast	-.0045	-.0051*	-.0017
Appalachia	-.0072*	-.0075*	-.0043*
US Lower 48	-.0010*	-.0007*	-.0004*

Because of the large standard errors, it is difficult to argue that the choice of an approach leads to profound differences in the estimates of B. It does appear, however, that the derivative approach forecasts the fastest decline in the finding rate, and the quadratic method forecasts the slowest. In some cases the differential approach seems to push the finding rate to zero too fast. In West Texas, for example, the predicted finding rate for 1986 is close to zero. In cases like this, we find the predictions given by the integral approach to be much more palatable.

In general, we are more comfortable using the integral approach than we are using the derivative approach. Although it is the structure of the finding rate (i.e. the derivative) that we ultimately want to estimate, the large variations in this variable from year to year make it difficult to estimate directly. The long run relation between cumulative reserve additions and cumulative feet (i.e. the integral) naturally has less variability and appears to be a more reasonable function to estimate. Note the dramatic difference between the values of the R^2 coefficients obtained using the first approach and those obtained using the second.

Overall, our study does provide evidence for the proposition that finding rates fall as more drilling takes place. With the exception of five cases out of twelve using the derivative approach (West Coast Exploratory and Development, Rockies Exploratory and Development, and Gulf Coast Development) and three or four cases out of twelve using the integral approach (West Coast Exploratory and Development and Rockies Exploratory for both methods and Gulf Cost Development for the quadratic method), the parameter B is significantly less than zero. In all cases B is significantly less than zero for the Lower 48 United States in total.

None of our estimates for the West Coast conforms to the theory developed in Section 2; instead, our best fits show finding rates that increase with cumulative drilling. To some extent, there is a distortion in the exploratory data in this region due to high finding rates between 1983 and 1985. Before 1983 the exploratory finding rate actually had a slightly negative trend. We suspect that the developmental finding rate in the West Coast behaves perversely due to the significant presence of heavy oil in California. Because heavy oil requires special recovery methods, reserves can change without drilling taking place as technology advances or prices change. Our developmental estimates for this region were also affected by the large negative adjustments in 1978 which produced negative developmental reserve additions.

Our estimates of the coefficient B for exploratory drilling in the Rockies, though positive, are not significantly different from zero at the 5% level. Except for statistical fluctuation and the possibility that finding rates are relatively constant over long periods of time (which is consistent with the Arps-Roberts framework), we can offer no good reasons why the finding rate does not fall here. All of our developmental estimates for B in the Rockies are negative; two are statistically significant while one is not. Likewise, one of our estimates for developmental drilling in the Gulf Coast has a negative sign but is not significantly less than zero. We should add that there were unusually large negative revisions in this region in 1979 which produced negative reserve additions and substantially affected our developmental figures.

There are also differences in the coefficient B as we move across regions. The exploratory finding rate seems to fall at about the same rate in the Mid Continent, West Texas, and Gulf Coast regions. It falls faster in Appalachia and slower in the West Coast and Rockies. The Appalachia figures are somewhat distorted by relatively large discoveries in 1970, the first year in our sample. Similar conclusions hold for the developmental finding rate except that the Rockies and Appalachia are less obviously out of line.

The finding rate is, of course, a function. Finding rates can thus differ because of different values for the parameter B, different values for other parameters, or different values for the dependent variables. It is useful to look at the average finding rate by region over the sample period. These finding rates are computed by dividing total reserves added over the period by total feet drilled and are reported in the following table. The units are barrels per foot.

Table 4.2. Average Finding Rates 1970-86

Region	Exp. Rate	Dev. Rate
West Coast	4.2	59.3
Rockies	3.7	19.6
Mid Contin.	3.8	6.4
West Texas	3.3	18.1
Gulf Coast	8.6	8.8
Appalachia	7.5	5.9
US Lower 48	4.9	13.6

If there are no differences in cost, we would expect finding rates to be uniform across regions. If not, economic forces should drive drilling resources from less productive regions to more productive regions, thus bringing finding rates more in line with one another. The data in the Appendix (summarized, in a sense, by Table 4.2) suggest that there are important differences among regions that have not been adequately explained in the literature.

We conclude our discussion with two remarks about finding rates. First, the search for oil is by its very nature a stochastic process, and what is found varies considerably from one trial to the next. Because those who study the subject must deal with a data set that contains a great deal of noise, any estimates of a finding rate function are bound to have a good deal of uncertainty associated with them. Our estimates, which were obtained using seventeen years of data, unquestionably have this feature. Those who attempt to estimate some kind of stable, long-run finding rate on the basis of only a few years of data are apt to be greatly misled.

Second, the notion of a finding rate is a soft concept. The introduction of a new technology (e.g. water flooding) can result in an increase in proved reserves without any drilling taking place and thus bring about an increase in the developmental finding rate. Likewise, higher oil prices can encourage more infill drilling and cause the developmental finding rate to fall. Even for exploratory drilling, the finding rate is a soft concept as there can be lags before reserves are booked. These lags become increasingly important as we extend our study beyond the onshore lower 48 states. Lags might be associated with building platforms offshore or waiting for a connection to a pipeline in a frontier region. As another example, it appears that there might be large amounts of cold oil in Alaska - oil that is prevented from flowing to the surface by a few thousand feet of permafrost. If the price of oil should reach a high enough level, the technology might be developed to inject heat and recover the oil. If so, the finding rate would soar as possibly billions of barrels were added to reserves.

We had difficulty detecting the influence of price in our sample period because of the strong correlation between price and cumulative reserve additions.

5. CONCLUSION

We set out to provide estimates, by region, of a finding rate equation - i.e. a function that can explain the behavior of finding rates. Starting from the Arps-Roberts model, we provided a methodology for estimating equations for both exploratory and developmental drilling.

We pursued two approaches to estimating finding rates. The first approach focused on the relationship between the finding rate and cumulative reserve additions. It was appropriate to call this the "derivative approach." The second approach focused on the relationship between cumulative reserve additions and cumulative drilling. This was the integral approach. We considered two methods for dealing with the inherently nonlinear relationship in the integral approach.

We found that the different approaches produced somewhat different estimates of the parameters. Because we estimated a function with less variability associated with it, the integral approach seemed to provide the most appealing estimates.

For most cases we considered, finding rates could be shown to fall as drilling proceeded. Exploratory drilling in the West Coast and the Rockies and developmental drilling in the West Coast were significant exceptions. It seems likely to us that heavy oil in California is largely responsible for the developmental anomalies. We found that both developmental and exploratory finding rates are falling at roughly the same level in the Mid Continent, West Texas, and the Gulf Coast but are probably falling slower in the West Coast and the Rockies and faster in Appalachia. There seem to be important differences in finding rates among regions that we cannot explain.

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APPENDIX: REGIONAL RESERVE AND DRILLING DATA

In this appendix we present a database consisting of reserve and drilling statistics for the years 1970 to 1986, aggregated by region. We work with six onshore regions: West Coast, Rockies, Mid Continent, West Texas, Gulf Coast, and Appalachia. The regions are similar to those used by the National Petroleum Council and the USGS, and they are chosen so that petroleum-bearing lands in each region are somewhat homogeneous. States and subdivisions are assigned to the various regions as follows:

West Coast: California, Oregon, Washington

Rockies: Arizona, Colorado, Montana, Nevada, New Mexico - West, North Dakota, South Dakota, Utah, Wyoming

Mid Continent: Kansas, Missouri, Nebraska, Oklahoma, Texas Railroad Commission District 10

West Texas: New Mexico - East, Texas Railroad Commission Districts 7B, 7C, 8, 8A, and 9

Gulf Coast: Arkansas, Florida, Louisiana, Mississippi, Texas Railroad Commission Districts 1, 2, 3, 4, 5, and 6

Appalachia: Alabama, Illinois, Indiana, Kentucky, Michigan, New York, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia

We also provide totals for the lower 48 states.

We use annual data on reserve additions which were reported by the American Petroleum Institute (API) [1980] through 1979 and have been reported by the Energy Information Administration (EIA) (Wood, et al. [1987]) since 1977. There is a well-known discrepancy between the two data sets during the overlap years, and we use the EIA data whenever possible. In these data sets there are five categories of additions to proved reserves: revisions, adjustments, extensions, new field discoveries, and new reservoir discoveries in old fields. We assume that revisions, adjustments, and extensions are the result of development activity while new field and new reservoir discoveries are the result of exploratory activity.

The EIA also maintains a database consisting of drilling information gathered from API well tickets and compiled on an "as completed" basis. From the EIA ([Petersen, 1987]) we obtained a historical summary of onshore drilling footage aggregated by state or subdivision, completion year, well type (exploratory or developmental), and well class (oil, gas, or dry). Here we aggregate this data by region. To get a measure of the drilling activity for oil, we include a fraction of the footage assigned to dry holes - the fraction representing the ratio of footage assigned to successful drilling of oil to that assigned to successful drilling of oil and gas.

The data are presented in Tables A.1 through A.7. Reserve additions are measured in millions of barrels and drilling footage is measured in millions of feet. The finding rates are computed as the ratio of reserve additions to drilling feet in the current year. In some cases reserve additions, and hence finding rates as well, can actually be negative because of the presence of large negative adjustments or revisions.

Table A.1. West Coast

	Exploratory Data			Developmental Data		
Year	Reserve Add	Feet	Find Rate	Reserve Add	Feet	Find Rate
70	2	1.058	1.9	105	4.097	25.6
71	1	0.811	1.2	73	3.297	22.1
72	6	0.742	8.1	183	2.834	64.6
73	3	0.528	5.7	228	2.577	88.5
74	2	0.953	2.1	287	3.841	74.7
75	3	0.749	4.0	404	4.254	95.0
76	3	0.794	3.8	260	4.491	57.9
77	4	0.808	5.0	173	3.974	43.5
78	2	1.493	1.3	-65	3.381	-19.2
79	5	1.571	3.2	487	3.487	139.7
80	8	1.709	4.7	387	4.514	85.7
81	6	1.873	3.2	112	5.334	21.0
82	5	1.858	2.7	95	4.900	19.4
83	7	0.500	14.0	229	4.986	45.9
84	9	0.579	15.5	601	6.555	91.7
85	6	0.827	7.3	459	5.480	83.8
86	0	0.154	0.0	229	3.605	63.5

Table A.2. Rockies

Year	Exploratory Data Reserve Add	Feet	Find Rate	Developmental Data Reserve Add	Feet	Find Rate
70	18	7.310	2.4	165	7.768	21.2
71	12	5.851	2.0	120	4.953	24.2
72	24	6.463	3.7	266	6.099	43.7
73	19	5.193	3.6	205	6.451	31.7
74	15	5.269	2.8	202	7.031	28.7
75	7	5.842	1.2	115	9.427	12.2
76	13	5.473	2.4	123	8.290	14.8
77	37	4.956	7.5	167	9.973	16.7
78	28	5.688	4.9	109	8.899	12.2
79	30	6.022	5.0	230	8.848	26.0
80	56	8.010	7.0	343	12.173	28.2
81	51	14.721	3.5	88	17.388	5.1
82	44	11.121	4.0	258	14.608	17.7
83	42	8.425	5.0	384	13.060	29.4
84	24	11.744	2.0	210	19.169	11.0
85	33	8.963	3.7	327	13.396	24.4
86	16	6.157	2.6	101	6.209	16.3

Table A.3. Mid Continent

Year	Exploratory Data Reserve Add	Feet	Find Rate	Developmental Data Reserve Add	Feet	Find Rate
70	20	4.246	4.8	225	12.967	17.4
71	38	3.956	9.7	274	11.429	24.0
72	34	3.486	9.9	91	10.252	8.9
73	4	2.651	1.7	176	8.829	19.9
74	6	4.031	1.6	199	12.928	15.4
75	12	4.151	2.9	188	17.275	10.9
76	14	4.147	3.5	161	19.904	8.1
77	29	4.518	6.4	74	20.349	3.6
78	17	4.926	3.5	-4	23.284	-0.2
79	18	4.084	4.4	297	25.895	11.5
80	18	4.939	3.6	49	41.223	1.2
81	31	8.170	3.8	287	61.091	4.7
82	20	7.172	2.8	228	51.579	4.4
83	21	6.845	3.1	136	42.724	3.2
84	23	11.008	2.1	264	44.069	6.0
85	37	8.849	4.2	260	34.290	7.6
86	11	4.812	2.3	6	18.165	0.3

Table A.4. West Texas

Year	Exploratory Data: Reserve Add	Feet	Find Rate	Developmental Data: Reserve Add	Feet	Find Rate
70	30	3.505	8.4	1165	16.341	71.3
71	18	3.216	5.6	871	15.448	56.4
72	18	3.166	5.8	367	16.186	22.7
73	20	2.463	8.3	670	15.868	42.2
74	16	3.548	4.6	450	16.950	26.5
75	22	4.492	4.9	212	23.992	8.8
76	20	4.182	4.8	228	20.928	10.9
77	13	5.532	2.4	142	23.874	5.9
78	13	4.900	2.7	265	25.068	10.6
79	21	5.458	3.8	533	25.413	21.0
80	22	6.920	3.2	675	34.224	19.7
81	23	10.813	2.1	654	45.148	14.5
82	25	10.531	2.4	305	43.223	7.1
83	21	9.975	2.1	545	42.127	12.9
84	28	11.607	2.4	709	53.827	13.2
85	23	10.189	2.3	999	50.478	19.8
86	14	5.863	2.4	160	25.456	6.3

Table A.5. Gulf Coast

Year	Exploratory Data: Reserve Add	Feet	Find Rate	Developmental Data: Reserve Add	Feet	Find Rate
70	90	7.118	12.6	437	16.335	26.7
71	54	5.414	9.9	525	16.089	32.6
72	85	4.525	18.9	182	14.866	12.2
73	58	4.383	13.2	408	11.270	36.2
74	47	4.964	9.5	266	11.818	22.5
75	39	4.842	8.1	7	14.085	0.5
76	31	6.013	5.1	119	16.105	7.4
77	54	5.424	10.0	-21	16.638	-1.3
78	32	5.718	5.6	27	15.127	1.8
79	36	5.228	6.9	-602	16.539	-36.4
80	47	6.611	7.1	367	28.957	12.7
81	57	8.729	6.5	324	39.994	8.1
82	60	6.269	9.6	130	32.467	4.0
83	38	5.421	7.0	346	33.930	10.2
84	33	6.759	4.9	358	38.732	9.2
85	48	5.439	8.8	275	33.511	8.2
86	19	3.455	5.5	127	17.135	7.4

Table A.6. Appalachia

Year	Exploratory Data Reserve Add	Feet	Find Rate	Developmental Data Reserve Add	Feet	Find Rate
70	55	0.813	68.2	43	4.817	8.9
71	11	1.308	8.5	50	4.132	12.2
72	9	1.405	6.1	9	4.677	1.9
73	12	1.340	9.1	44	4.663	9.5
74	20	1.774	11.5	106	6.433	16.5
75	19	1.899	10.1	70	6.713	10.4
76	10	1.804	5.3	122	7.919	15.4
77	18	1.838	9.8	44	9.226	4.8
78	19	1.552	12.2	58	10.249	5.7
79	10	1.729	5.8	-6	10.170	-0.6
80	14	2.646	5.3	138	17.355	8.0
81	21	3.258	6.4	109	21.466	5.1
82	10	3.791	2.6	69	18.846	3.7
83	13	3.697	3.5	116	17.025	6.8
84	11	3.322	3.3	101	16.932	6.0
85	15	2.628	5.7	18	13.543	1.3
86	2	1.212	1.7	-9	8.620	-1.0

Table A.7. Total Lower 48 States

Year	Exploratory Data Reserve Add	Feet	Find Rate	Developmental Data Reserve Add	Feet	Find Rate
70	215	24.049	8.9	2140	62.325	34.3
71	134	20.555	6.5	1913	55.348	34.6
72	176	19.787	8.9	1098	54.914	20.0
73	116	16.557	7.0	1731	49.658	34.9
74	106	20.540	5.2	1510	59.002	25.6
75	102	21.975	4.6	996	75.747	13.1
76	91	22.413	4.1	1013	77.636	13.0
77	155	23.075	6.7	579	84.034	6.9
78	111	24.277	4.6	390	86.009	4.5
79	120	24.092	5.0	939	90.352	10.4
80	165	30.835	5.4	1959	138.446	14.1
81	189	47.563	4.0	1574	190.421	8.3
82	164	40.742	4.0	1085	165.623	6.6
83	142	34.863	4.1	1756	153.853	11.4
84	128	45.019	2.8	2243	179.283	12.5
85	162	36.895	4.4	2338	150.697	15.5
86	62	21.653	2.9	614	79.191	7.8

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