Laser Range Camera Calibration

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ABSTRACT

This paper describes an imaging model that was derived for use with a laser range camera (LRC) developed by the Advanced Intelligent Machines Division of Odetics. However, this model could be applied to any comparable imaging system. Both the derivation of the model and the determination of the LRC's intrinsic parameters are explained. For the purpose of evaluating the LRC's external orientation (extrinsic parameters), a transformation of the LRC's imaging model into a standard camera's (SC) pinhole model is derived. By virtue of this transformation, the evaluation of the LRC's external orientation can be found by applying any SC calibration technique. Experimental results prove the accuracy of the derived camera model as well as the proposed calibration procedure.
1. INTRODUCTION

LRC's are attractive sensor systems for the purpose of 3-D machine vision tasks because they acquire dense range (and reflectance) images. These range images contain the coordinates of surfaces with respect to the sensor's (internal) coordinate system. If such a sensor system is designated for world modeling, e.g., for navigation or manipulation tasks, the transformation of the sensor data into a world coordinate system is necessary, and requires the calibration of the sensor system. The process of camera calibration consists of determining the camera's imaging model, intrinsic parameters, and external orientation (extrinsic parameters) which consist of rotational translation with respect to a world coordinate system.

When first comparing the LRC's reflectance image with the illumination images of SCs, the image generating process seems to be similar. This might lead to the conclusion that the application of SC calibration techniques for the LRC is sufficient. However, the different distortions of a grid imaged by a SC (Fig. 1) and by a LRC (Fig. 2) reveal differences in the imaging processes between the two cameras.

![Grid imaged by a SC with a wide-angle lens, the grid plane is parallel to the image plane.](image)

Unlike the radial distortion of SCs images\(^*\):\(^2\)

\[
X_{SC} = f_{SC}(X_{grid}^2 + Y_{grid}^2) \cdot X_{grid} \\
Y_{SC} = f_{SC}(X_{grid}^2 + Y_{grid}^2) \cdot Y_{grid}
\]

\(^*\) Center of the coordinate systems is in the middle of Fig. 1
the distortion of the LRC's image can be modeled as follows:

\[
X_{LRC} = X_{grid}
\]
\[
Y_{LRC} = f_{LRC}(X_{grid}) \cdot Y_{grid}
\]

For high accuracy LRC calibration, any approach has to take into consideration the specific imaging process of the LRC.

The organization of the paper is as follows. In Section 2, the LRC's imaging model is derived and the intrinsic parameters are determined. In Section 3, the transformation of the LRC's imaging model onto the SC pinhole model is presented. By virtue of this transformation, calibration of the LRC's external orientation is accomplished by applying a SC calibration technique. In Section 4 the accuracy of the calibration results are discussed. The paper concludes with a summary.

† Center of the coordinate systems is in the middle of Fig. 2.
2. THE LASER RANGE CAMERA  
MODEL AND INTRINSIC PARAMETERS

The Odetics LRC\textsuperscript{1} transmits an amplitude modulated laser beam. The power of the returned signal is a function of the target reflectance, and the phase shift is a function of the round trip travel time, i.e., target range. While scanning the laser beam horizontally and vertically, the range and reflectance data are spatially registered in separate images.

The scan mechanism consists of a rotating polygon mirror for the horizontal deflection, and a planar mirror, rotating up and down, for the vertical deflection of the laser beam (Fig. 3). The laser beams scan pattern is comparable to that of a television's electron beam.

![Fig. 3. Setup of the LRC scan mirrors.](image)

![Fig. 4. Coordinate system of the LRC.](image)
The horizontal scan mirror is rotating with a constant angle velocity of \( \dot{\alpha}_{vm} \); the rotation of the vertical scan mirror \( \alpha_{vm} \) is governed by a rotating cam driven by the horizontal scan mirror. The horizontal and vertical angle velocities of the deflected beam are \( \dot{\alpha}_h = 2\dot{\alpha}_{hm} \) and \( \dot{\alpha}_v = 2\dot{\alpha}_{vm} \). In the sensors coordinate system the beam can be described as a vector pointing to an object point \( X_{op} \).

For convenience the camera coordinates of an imaged object point \( X_{op} \) are expressed by spherical coordinates \( \alpha_v, \alpha_h, \lambda \). The spherical coordinates \( \alpha_v, \alpha_h \) are equal to the angles of the deflected laser beam. \( \lambda \) is the length of the vector \( X_{op} \), it equals the measured range (Fig. 4).

While the beam is scanning, reflectance and depth data are registered. The sample time for a single pixel is \( t_{sp} \); the sample time for an entire scan line, including horizontal retrace is \( t_{sl} \). The spread angles of two consecutive horizontal pixels and two consecutive scan lines are \( \dot{\alpha}_v t_{sp} \) and \( \dot{\alpha}_h t_{sl} \), respectively. \( \dot{\alpha}_{hm}, \dot{\alpha}_{vm}, t_{sp}, t_{sl} \) are intrinsic camera parameters. Their values can be taken from the LRC manual.

The image coordinates \( r \) and \( c \) correspond to the row and column number of image pixels (Fig. 5). The translation of the camera with respect to the image coordinate system is determined by \( r_m \) and \( c_m \), and can be placed in the center of the LRC image. An accurate estimation of \( r_m \) and \( c_m \) can be achieved by applying the experimental techniques for SCs proposed by Lenz.

Once the intrinsic parameters are known, the mapping equations of the camera coordinates into the image coordinates of the range \( D(r, c) \) or reflectance images \( R(r, c) \) can be determined. The measured range value \( D(r, c) \) depends linearly on the target distance, \( \lambda \). The measured reflectance value \( R(r, c) \) is a function of the target surface albedo and orientation.

\[ \text{D}=\text{rnd} \ (p,2) \]

The coefficient \( p \) is dependent on the laser beams modulation frequency, its value can be formed in the LRC manual.
If the vertical scan mirror is not moving within a scan line, the mapping of image coordinates onto camera coordinates is given by the following equations:

\[
\alpha_h(c) = \dot{\alpha}_h t_{sp}(c - c_m) \\
\alpha_v(r) = \dot{\alpha}_v t_{sl}(r_m - r)
\]  

Because of the coupling between the vertical scan mirror and the horizontal scan mirror due to the cam, the scan lines are inclined. The vertical scan mirror is moving within each scan line (Fig. 6) and an offset, depending on the horizontal beam angle, has to be added:

\[
\Delta \alpha_v(\alpha_h) = \pm \alpha_h \frac{\dot{\alpha}_v}{\dot{\alpha}_h}
\]  

Fig. 6. Inclination of the LRC scan lines.

When the vertical scan mirror is moving up or down the sign in Eq. (5) is positive or negative, respectively. Combining Eq. (4) and Eq. (5) leads to the final transformation equations:

\[
\alpha_h(c) = \dot{\alpha}_h t_{sp}(c - c_m) \\
\alpha_v(r, c) = \dot{\alpha}_v t_{sl}(r_m - r) \pm \alpha_h \frac{\dot{\alpha}_v}{\dot{\alpha}_h} \\
= \dot{\alpha}_v t_{sl}(r_m - r) \pm \dot{\alpha}_v t_{sp}(c - c_m)
\]  

The inversion of Eq. (6) and Eq. (7) defines the transformation of camera into image coordinates:
Equations (6), (7), (8), and (9) express the relationship between the horizontal and vertical beam angles (spherical camera coordinates) and the image coordinates.

\[ c(\alpha_h) = r \cdot n \left( c_m + \frac{\alpha_h}{\dot{\alpha}_h t_{sp}} \right) \quad (8) \]

\[ r(\alpha_v, \alpha_h) = r \cdot n \left( r_m - \frac{1}{\dot{\alpha}_v t_{sl}} \left( \alpha_v + \dot{\alpha}_v t_{sp}(c - c_m) \right) \right) \]

\[ = r \cdot n \left( r_m - \frac{1}{\dot{\alpha}_v t_{sl}} \left( \alpha_v + \alpha_h \frac{\dot{\alpha}_v}{\dot{\alpha}_h} \right) \right) \quad (9) \]
3. TRANSFORMATION OF THE LASER RANGE CAMERA MODEL INTO A STANDARD CAMERA PINHOLE MODEL

Once the imaging model and its intrinsic parameters are known, the external orientation of the LRC remains to be determined. It defines the camera’s rotation and translation with respect to a world coordinate system.

\[ X_w = R \cdot X_c - T \]

Several calibration techniques for SCs have been reported. Therefore, it is convenient to simply derive a transformation from the LRC imaging model onto the SC pinhole model. By virtue of such a transformation, any of the SC calibration techniques can be utilized for the evaluation of the LRC external orientation.

Equation 10 shows the parametric equation of the laser beam in cartesian coordinates as a function of its spherical coordinates.

\[
X_{op} = \begin{pmatrix}
X_c \\
Y_c \\
Z_c
\end{pmatrix} = \lambda \begin{pmatrix}
\sin \alpha_h \cos \alpha_v \\
\sin \alpha_v \\
-\cos \alpha_h \cos \alpha_v
\end{pmatrix}
\]  

(10)

At this point, a virtual image plane is introduced (Fig. 7). The X and Y axes of the virtual image plane coordinate system, \(X_p\) and \(Y_p\), are parallel to the X and Y axes of the LRC coordinate system.

\[ Z_0 \] corresponds to the SC focal length. It can be chosen arbitrarily because it merely determines the scaling on the virtual image plane. For the following calculations \(Z_0 = -1\). The intersection of the laser beam with the virtual image plane is given by:
and

\[ X_p(\alpha_h) = \frac{\sin \alpha_h \cos \alpha_v}{\cos \alpha_h \cos \alpha_v} = \tan \alpha_h \]  \hspace{1cm} (11)

\[ Y_p(\alpha_v, \alpha_h) = \frac{\sin \alpha_v}{\cos \alpha_h \cos \alpha_v} = \frac{1}{\cos \alpha_h} \tan \alpha_h \]  \hspace{1cm} (12)

Since \( X_p \) is a function of \( \alpha_v \) only, Eqs. (11) and (12) can be easily inverted:

\[ \alpha_h(X_p) = \arctan(X_p) \]  \hspace{1cm} (13)

\[ \alpha_v(X_p, Y_p) = \arctan(Y_p \cos(\arctan(X_p))) \]  \hspace{1cm} (14)

Eqs. (11) and (12) describe the mapping of the LRC imaging model into a SC pinhole model. Eqs. (13) and (14) describe the inverse mapping. By inserting Eqs. (6) and (7) into Eqs. (11) and (12) LRC images can be mapped onto the virtual image plane. This was done for the imaged grid Fig. 2. The result, Fig. 8 reveals the compensation of the LRC's distortion and gives a qualitative prove of the accuracy of the derived model.
4. CALIBRATION RESULTS

For the evaluate of the LRC external orientation, a calibration techniques proposed by Lenz was applied. For this algorithm the world coordinates of at least five calibration points and their corresponding coordinates in the image plane have to be known. The calibration result can be improved by using more than five calibration points. In this case the result is optimized by a mean square fit. Fig. 9 shows the LRC reflectance image of a calibration point plane, where each single disc represents a calibration point.

![Fig. 9. LRC's reflectance image of the calibration point plane.](image)

The world coordinates \( P_i \) of each calibration point, i.e. the disc's center, are known in advance. The projection \( P_i' \) in the reflectance image was measured by evaluating the center of gravity for each disc. This procedure allows the estimation of a calibration point projection with sub-pixel accuracy.

By applying Eqs.6, 7, 10 and 11, the measured projections of the calibration points were then mapped onto the virtual image plane. The set of calibration point world coordinates and their corresponding virtual image plane coordinates were then passed to the calibration algorithm. The output of the calibration were then passed to the calibration algorithm. The output of the calibration algorithm is the
external orientation of the sensor’s coordinate system with respect to the world coordinate system, given as the rotation matrix R and the translation vector T. Once the external camera orientation is known, one can map object points into the sensor’s coordinate system and vice versa.

To reveal calibration errors, the projection of the calibration points into the image memory \( P_i' \) were calculated by mapping them into the sensor coordinate system and then applying Eqs. 8, 9, 12 and 13. Fig. 10 shows the displacement between measured (needle head) and calculated (needle tip) projections of calibration points in the image memory.

\[
M_p = \frac{1}{n} \sum_{i=1}^{n} | P_i - p_i' |
\]  

(15)
This is denoted as the mean pixel displacement $M_p$, and its standard deviation is denoted $S_p$.

$$S_p = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (P_i - P'_i)^2}$$  \hspace{1cm} (16)

$M_p$ and $S_p$ were measured for four different orientations of the calibration point plane and averaged. The result was $M_p = 0.95$, and $S_p = 1.22$. In other words, the mean error for calculating the projection of an object point into the image memory was less than a pixel.
5. SUMMARY

With Eqs. (6), (7), (11) and (12), the image coordinates $r$ and $c$ can be mapped into the coordinates of a virtual image plane. By virtue of this transformation, the calibration of the LRC external orientation can be evaluated with calibration techniques for SCs. Experimental results prove the accuracy of the LRC’s derived imaging model as well as the proposed calibration procedure.

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