An Electric Field Diagnostic for ICRF Antennas in a Plasma Edge Environment

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Introduction

The dynamic Stark effect is a phenomenon in which photon(s) associated with an oscillating electric field are absorbed or emitted with the photon associated with an electronic transition. This multi-photon process leads to the formation of satellites in the spectrum at integer multiples of the frequency associated with the dynamic electric field. Utilizing the dynamic Stark effect the electric field parameters can be determined from the time-averaged and phase resolved optical emission spectra. Currently the OES diagnostic is being developed/benchmarked on a small scale laboratory experiment, DStarVE. The resulting verified diagnostic will be implemented to measure the near field associated with a mock-up ICRF antenna in a plasmas environment similar to that of NSTX ICRF antenna.

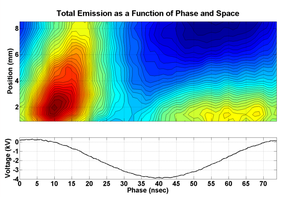
ICRF Antenna Near Field Physics

The primary objective of this research is to gain insight on the physics associated with ICRF antenna near field - plasma interactions.

- What is electric field dynamic and topology?
- How does the electric field dynamic and topology effect local power absorption?

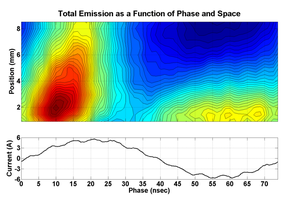
Local Power Absorption/Heating Mechanisms

- **Fermi Acceleration**
- **Two Stream Instability**
- **Compression/Rarefaction of Sheath**
- **Ohmic**



electron beam generated by sheath expansion

--- Fermi Acceleration ---

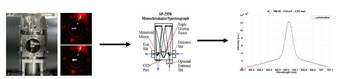


strong oscillation at 163 MHz in current and total emission

--- Two Stream Instability ---

OES – Dynamic Stark Effect

The dynamic electric field associated with the RF sheath is experimentally measured by utilizing time averaged and phase resolved optical emission spectroscopy.

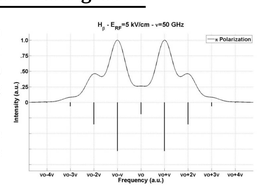


$$\overline{E}(t)=\overline{E}_o+\sum_{n=1}^{\infty}\overline{E}_n\cos(n\omega t+\phi_n)$$

Dynamic Stark Effect - Background

The dynamic Stark effect is a **multi-photon process**. Photons associated with the atomic transitions are emitted/absorbed with photons associated with the oscillating electric field!

Satellite spacing is dictated by the frequency at which the electric field is oscillating.



Line Profile Calculations - Methods

The line profile is calculated utilizing one of two methods based on the validity criteria:

$$\Delta=\frac{1}{\Omega T}\approx\frac{2\pi\lambda_{jk}}{\omega}\qquad \Omega=\left(n(n_1-n_2)-n'(n_1'-n_2')\frac{3h|\overline{E}(t)|}{2m_e ec}\omega^2\right)^{1/3}\qquad T=\frac{\tau_{life}+\tau_{obs}}{\tau_{life}\tau_{obs}}$$

Floquet Method $\Delta<<1$

Quasi-static Method $\Delta>>1$

The validity criteria associated with the Balmer series transitions in the RF range of frequencies, **10's MHz**, is on the order of unity.

Quasi-static Method

In the limit Δ>>1 the **quantum states respond instantaneously** to the electric field and the quasi-static method can be utilized to line profile.

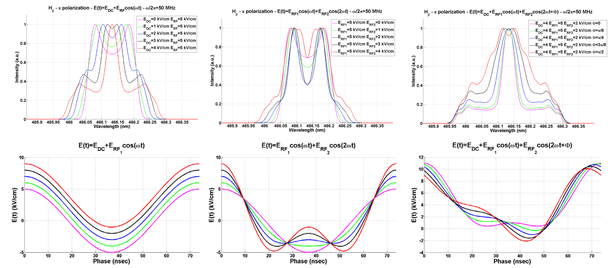
$$i\hbar\frac{\partial\Phi}{\partial t}=H^*\Phi\qquad \overline{E}_i=\overline{E}_o+\sum_{n=1}^{NH}\overline{E}_n\cos(n\omega t_i+\phi_n)\quad t_i\in[0,\ \tau]$$

$$H^*=H^o+\frac{\mu_B}{\hbar}\overline{B}\cdot\left[\overline{J}+\overline{S}\right]+e\overline{E}_i\cdot\overline{r}$$

The dynamic electric field is discretized.

The line profile associated with each discretized electric field value is calculated utilizing the static Stark operator.

The line profiles associated with the discretized electric field are summed to give the quasi-static line profile.



Floquet Method

In the limit Δ<<1 the **quantum states do not respond instantaneously** to the electric field and the Floquet method must be utilized to line profile.

$$i\hbar\frac{\partial\Phi}{\partial t}=H^*\Phi+\sum_{n=1}^{NH}H_n^{RF}\left[e^{i(n\omega t+\phi_n)}+e^{-i(n\omega t+\phi_n)}\right]\Phi$$

$$H^*=H^o+\frac{\mu_B}{\hbar}\overline{B}\cdot\left[\overline{J}+\overline{S}\right]+e\overline{E}_o\cdot\overline{r}\qquad \overline{E}(t)=\overline{E}_o+\sum_{n=1}^{NH}\overline{E}_n\cos(n\omega t+\phi_n)$$

$$H_n^{RF}=\frac{e\overline{E}_n\cdot\overline{r}}{2}$$

$$\Phi(t)=T(t)e^{-i\frac{E}{\hbar}}\rightarrow\Phi_k(t)=\sum_{j=1}^{N}\tau(t)_{kj}\Psi_j e^{-i\frac{E_k}{\hbar}t}\rightarrow\Phi_k(t)=\sum_{s=-\infty}^{\infty}\sum_{j=1}^{N}c_{kj}^s\Psi_j e^{-i\left(\frac{E_k}{\hbar}+s\omega\right)t}$$

Substituting this result into the Schrödinger equation we perform the following integral using the orthonormality of our basis set:

$$\left\langle\Psi_l\,|\,i\hbar\frac{\partial\Phi_k}{\partial t}\right\rangle=\left\langle\Psi_l\,|\,H^*\Phi_k\right\rangle+\left\langle\Psi_l\,|\,H^{RF}e^{i\omega t}\Phi_k\right\rangle+\left\langle\Psi_l\,|\,H^{RF}e^{-i\omega t}\Phi_k\right\rangle$$

Equating equal powers of $e^{-is\omega t}$ yields:

$$\left(E_k+sh\omega\right)c_{kl}^s=\sum_{j=1}^{N}c_{kj}^{s-1}W_{lj}^{RF}+\sum_{j=1}^{N}c_{kj}^s W_{lj}^*+\sum_{j=1}^{N}c_{kj}^{s+1}W_{lj}^{RF}\qquad W_{lj}^{RF}=\left\langle\Psi_l\,|\,H^{RF}\,|\Psi_j\right\rangle\qquad W_{lj}^*=\left\langle\Psi_l\,|\,H^*\,|\Psi_j\right\rangle$$

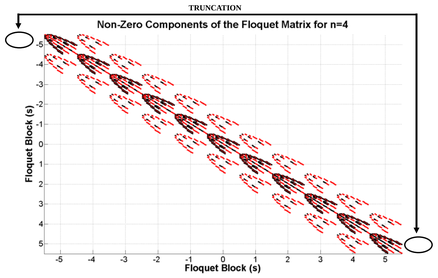
In order to solve this system of algebraic equations we truncate the set such that:

$$c_{kj}^s=0\qquad k=1..N,\quad j=1..N,\quad |s|>s_m$$

The truncated set of algebraic equations can be written in matrix form:

$$\begin{bmatrix}\overline{W}^* & \overline{W}^{RF} & \overline{0} & \overline{0}\\ \overline{W}^{RF} & \overline{W}^*+\overline{I}(s_m-1)\hbar\omega & \overline{W}^{RF} & \overline{0}\\ \overline{0} & \bullet & \bullet & \bullet\\ \overline{0} & \overline{0} & \overline{W}^{RF} & \overline{W}^*-\overline{I}s_m\hbar\omega\end{bmatrix}\begin{bmatrix}\overline{c}_k^{-s_m}\\ \overline{c}_k^{-s_m+1}\\ \bullet\\ \overline{c}_k^{s_m}\end{bmatrix}-\overline{I}E_k\begin{bmatrix}\overline{c}_k^{-s_m}\\ \overline{c}_k^{-s_m+1}\\ \bullet\\ \overline{c}_k^{s_m}\end{bmatrix}=0$$

$$\overline{W}^*=\begin{bmatrix}W_{11}^* & \bullet & W_{1N}^*\\ \bullet & \bullet & \bullet\\ W_{N1}^* & \bullet & W_{NN}^*\end{bmatrix}\qquad \overline{W}^{RF}=\begin{bmatrix}W_{11}^{RF} & \bullet & W_{1N}^{RF}\\ \bullet & \bullet & \bullet\\ W_{N1}^{RF} & \bullet & W_{NN}^{RF}\end{bmatrix}\qquad \overline{c}_k^s=\begin{bmatrix}c_{k1}^s\\ \bullet\\ c_{kN}^s\end{bmatrix}$$



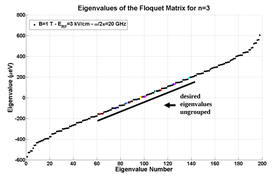
2sm+1 equivalent basis sets can be formed out of the **N(2sm+1)** eigenvectors found from diagonalizing the Floquet matrix.

Only **N** of these eigenvalue-eigenvector pairs forms a complete solution to the Schrödinger equation.

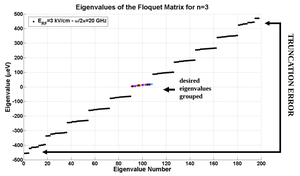
The following relationship exists between the eigenvalue-eigenvector pairs:

$$\overline{c}_k^s=\overline{c}_k^{s+m}\qquad E_k=E_k+m\hbar\omega$$

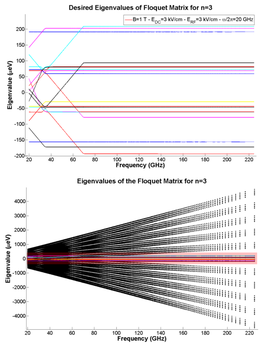
Due to truncation error these relationships do not seem to be useful.

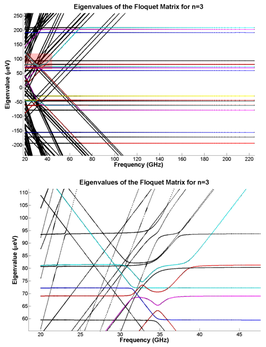


For the frequencies and field strengths of interest the eigenvalues are typically not 'naturally' grouped.



Desired eigenvalues become 'naturally' grouped as the frequency associated with the dynamic field is increased.





Due to **truncation error** the **only*** method to select a consistent set of **N** out of the **N(2sm+1)** eigenvalue-eigenvector pairs is to track the eigenvalues as frequency is decreased – **starting at a frequency where the eigenvalues are 'naturally' grouped!**

The frequency at which the eigenvalues are 'naturally' grouped can be estimated by:

$$\overline{E}(t)=\overline{E}_o+\sum_{n=1}^{NH}\overline{E}_n\cos(n\omega t+\phi_n)$$

$$\downarrow$$

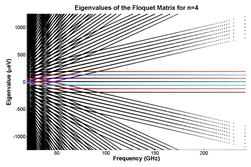
$$\overline{E}=\left|E_{o,x}\right|\hat{x}+\left|E_{o,y}\right|\hat{y}+\left|E_{o,z}\right|\hat{z}+\sum_{n=1}^{NH}\left|E_{n,x}\right|\hat{x}+\left|E_{n,y}\right|\hat{y}+\left|E_{n,z}\right|\hat{z}$$

$$\downarrow$$

$$H^*=\frac{\mu_B}{\hbar}\overline{B}\cdot\left[\overline{J}+\overline{S}\right]+e\overline{E}\cdot\overline{r}$$

$$\downarrow$$

$$v_{estimate}=\frac{E^1_{1,max}-E^1_{1,min}}{\hbar}$$

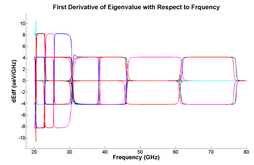


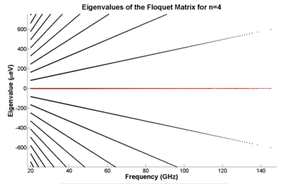
The eigenvalues are tracked by taking small steps in frequency such that the eigenvalues can be predicted by linear extrapolation.

The first derivative of the eigenvalue with respect to frequency is used to control the magnitude of the frequency step.

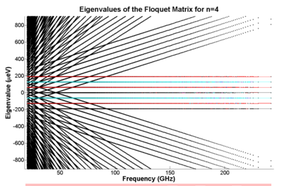
$$\left|\frac{E_k^1}{dv}_{min,f-df}-\frac{E_k^1}{dv}_{min,f}\right|\le\left|\frac{E}{dv}_{max}\right|\quad pass$$

$$\left|\frac{E}{dv}_{max}\right|\quad fail,\ decrease\ df$$

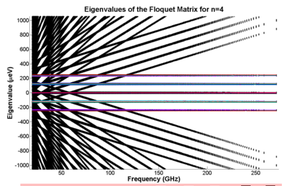




$\overline{E}(t)=\overline{E}_1\cos(\omega t)\qquad \overline{B}=0$



$\overline{E}(t)=\overline{E}_o+\overline{E}_1\cos(\omega t)\qquad \overline{E}_o\parallel\overline{E}_1\qquad \overline{B}=0$



$\overline{E}(t)=\overline{E}_o+\overline{E}_1\cos(\omega t)\qquad \overline{E}_o\parallel\overline{E}_1\parallel\overline{B}_o\qquad \overline{B}=\overline{B}_o$

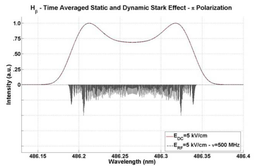
The eigenvalues are **independent of frequency** when all of the field components are parallel and the 'fine structure' is not considered.

Convergence of Quasi-static and Floquet Method

When the eigenvalues are **independent of frequency** the line profile calculated from the quasi-static and Floquet method converge in the RF range of frequencies.

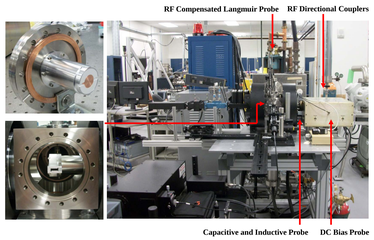
Due to finite instrument response function!

$FWHM\equiv0.025\ nm\longrightarrow 5\ to\ 15\ GHz$

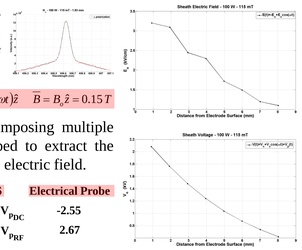


Due to **anti-level crossing** the line profile calculated from the quasi-static and Floquet method may not converge.

Initial Results - DStarVE



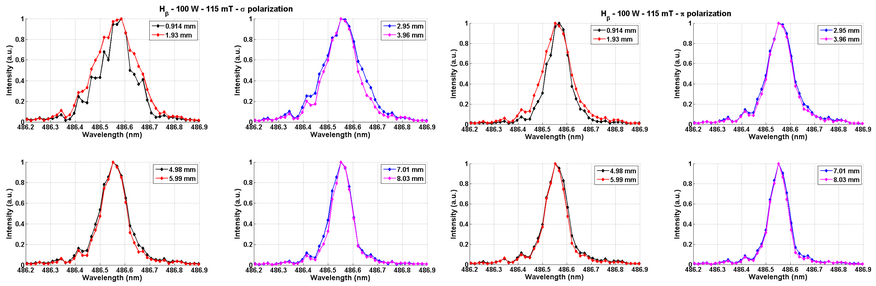
A simple fitting routine for the H_α line profile is currently utilized based off the quasi-static method.



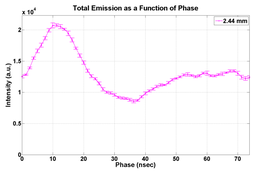
Model $\longrightarrow$ $\overline{E}(t)=E_o\hat{z}+E_o\cos(\omega t)\hat{z}\qquad \overline{B}=B_o\hat{z}=0.15\,T$

A robust fitting algorithm imposing multiple constraints is being developed to extract the exact time dependence of the electric field.

	OES	Electrical Probe
Electrode DC Voltage (kV):	-2.38 + V_{PDC}	-2.55
Electrode RF Voltage (kV):	2.38 + V_{PRF}	2.67



Total emission has a strong time dependence!



Future Work

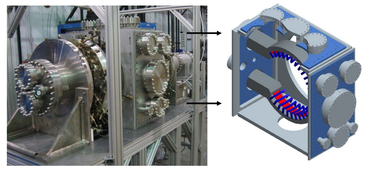
DStarVE

Determine which method – **quasi-static or Floquet** – approximates the atomic physics well in the RF range of frequencies.

Develop robust line fitting algorithm based on the following constraints:

- Multiple line profiles
- Two polarizations: σ π
- Time dependence of transition intensity

ORNL RFTF



Utilizing the developed OES diagnostic, characterize ORNL mock-up ICRF antenna in a plasma environment similar to that of NSTX ICRF antenna.

$P_{\mu\text{-wave}}$ = 30 kW

$f_{\mu\text{-wave}}$ = 18 GHz

P_{RF} = 20 kW

f_{RF} = 40 – 50 MHz

B = 1 – 2 kG

$n_e \sim 10^{12}\ cm^{-3}$

$kT_e \sim 5$ eV

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