Multidimensional Coupled Photon-Electron Transport Simulations Using Neutral Particle S_N Codes

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Outline

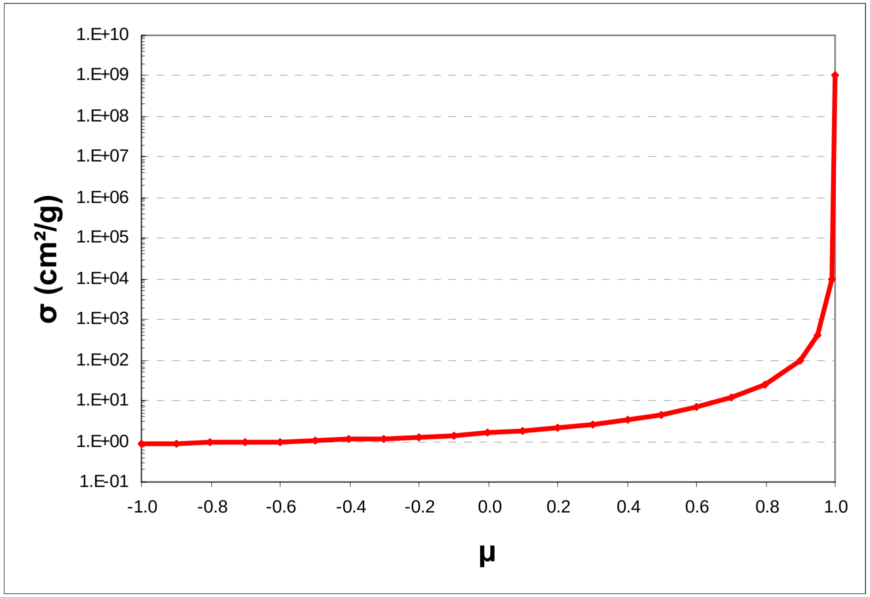
- **Short History**
- **Electron Transport**
  - **Cross Section Preparation**
- **Results**
  - **2-D Results**
  - **3-D Results**
- **Conclusions**

Short History

- **Boltzmann Equation**

$$\hat{\Omega}\cdot\vec{\nabla}\psi(\vec{r},E,\hat{\Omega})+\sigma_t(\vec{r},E)\psi(\vec{r},E,\hat{\Omega})=\int_0^\infty dE'\int_{4\pi} d\hat{\Omega}'\sigma_s(\vec{r},E'\rightarrow E,\hat{\Omega}\cdot\hat{\Omega}')\psi(\vec{r},E',\hat{\Omega}')+Q(\vec{r},E)$$

  - **S_N methods – maturity**
  - **Low order Legendre expansion for scattering kernel, flux and source**
    - **Difficult to represent for high anisotropy**
- **Electron cross sections ⇨ highly anisotropic**



Short History

- **Boltzmann-Fokker-Planck Equation:**

$$\hat{\Omega}\cdot\vec{\nabla}\psi\left(\vec{r},E,\hat{\Omega}\right)-\frac{\partial}{\partial E}\left[S(\vec{r},E)\psi\left(\vec{r},E,\hat{\Omega}\right)\right]-$$

$$\frac{1}{2}T(\vec{r},E)\left\{\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)\frac{\partial\psi\left(\vec{r},E,\hat{\Omega}\right)}{\partial\mu}\right]+\frac{1}{1-\mu^2}\frac{\partial^2\psi\left(\vec{r},E,\hat{\Omega}\right)}{\partial\varphi^2}\right\}+\sigma_t(\vec{r},E)\psi\left(\vec{r},E,\hat{\Omega}\right)=$$

$$\int_0^\infty dE'\int_{4\pi}d\hat{\Omega}'\sigma_{s,reg}\left(\vec{r},E'\to E,\hat{\Omega}\cdot\hat{\Omega}'\right)\psi\left(\vec{r},E',\hat{\Omega}'\right)+Q(\vec{r},E)$$

  – **Obtained from Boltzmann equation by isolating the singularity**

$$\sigma_s\left(\vec{r},E\to E',\hat{\Omega}\to\hat{\Omega}'\right)=\sigma_{s,reg}\left(\vec{r},E\to E',\hat{\Omega}\to\hat{\Omega}'\right)+\sigma_{s,sing}\left(\vec{r},E\to E',\hat{\Omega}\to\hat{\Omega}'\right)$$

- **requires modifications to the existing standard discrete ordinates codes ⇨ non-desirable**

Short History

- **Use of standard codes – redefine cross sections:**
  - **P_{N+1} transport-corrected P_N expansion**

$$\sigma_l^* = \sigma_l - \sigma_{N+1} < \sigma_l$$

$$\bar{\mu}^* < \bar{\mu}$$

  - **Redefine in terms of (reduced) stopping power and momentum transfer ⇨ CPEXS/ONELD**
    - **Works well in 1-D with Gauss quadrature**

Short History

- **Higher-dimensional calculations**
  - **Drumm (1997)-based on a Goudsmit-Saunderson approach to prepare multigroup Legendre cross sections ⇨ CEPXS-GS**
    - **limited to infinite-medium problems**
    - **neglects the hard-inelastic (wide angle) scattering**
    - **works for non-Gauss quadrature**

Cross Section Preparation

- **CEPXS-BFP code – contributed to ORNL by Russian Academy of Sciences**
- **Options:**
  - **“S_N-BFP” ⇨ restricted stopping powers and restricted momentum transfer coefficients**
  - **“S_N-CSD” ⇨ restricted stopping powers, angular operator indirectly**
  - **“S_N-Indirect” ⇨ data for transport codes for indirectly treatment**

Cross Section Preparation

- **“S_N-CSD” used (40 electron groups)**

$$\tilde{\sigma}_t^g = \sigma_{t,CSD}^g + \frac{S_{g+1/2}}{\Delta E_g}\left(1+\frac{P_g}{2}\right)$$

$$\tilde{\sigma}_{s,l}^{g'\to g} = \sigma_{s,l,CSD}^{g'\to g} + \sigma_S^{g'\to g}, \quad \text{for } g' = g-1, g-2$$

where

$$\sigma_S^{g-1\to g} = \frac{\left(S_{g-1/2}\left(1+\frac{P_{g-1}}{2}\right)+S_{g+1/2}\frac{P_g}{2}\right)}{\Delta E_{g-1}}, \qquad \sigma_S^{g-2\to g} = -\frac{P_{g-1}}{2\Delta E_{g-2}}S_{g-1/2}$$

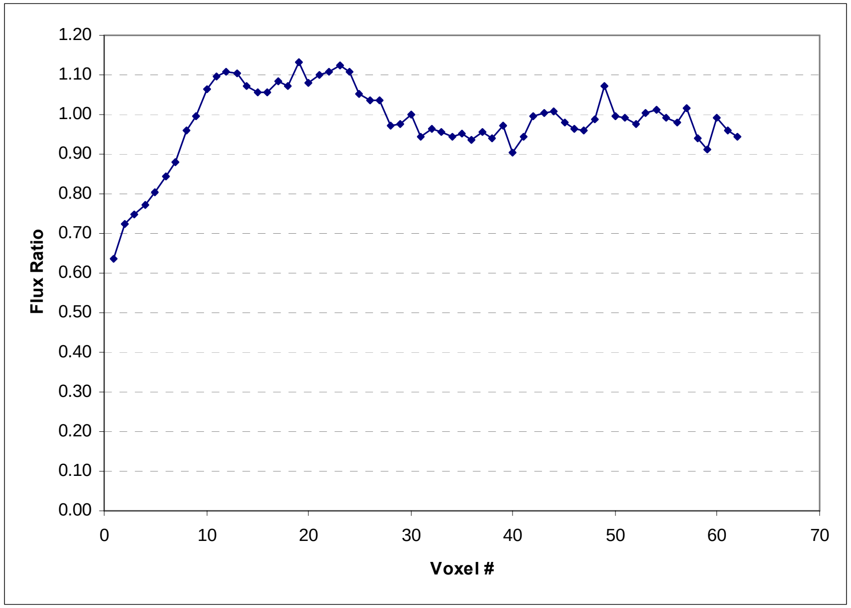
$$\sigma_{t,CSD}^g = \sigma_{t,BFP}^g + \frac{T^g}{2}L(L+1)$$

$$\sigma_{s,l,CSD}^{g'\to g} = \sigma_{s,l,BFP}^{g'\to g} + \frac{T^g}{2}\left[L(L+1)-l(l+1)\right]\delta_{g'g}$$

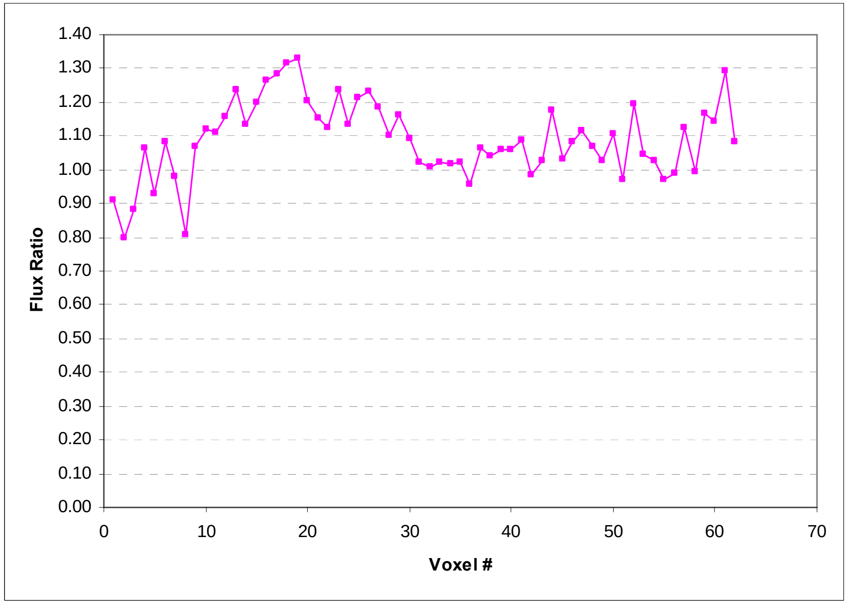
Results – 2D

- **DORT calculations on a 1-D-like water phantom**
  - **volumetric isotropic source at top**
  - **S_{16}FS quadrature**

Photon flux

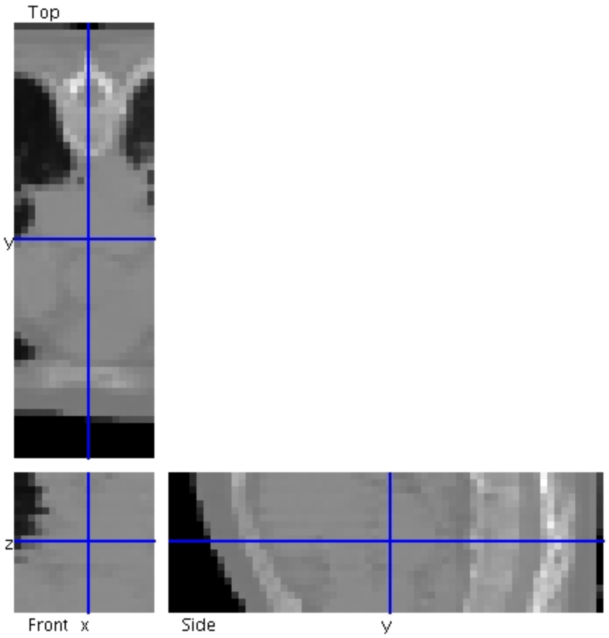


Electron Flux



Results – 3D

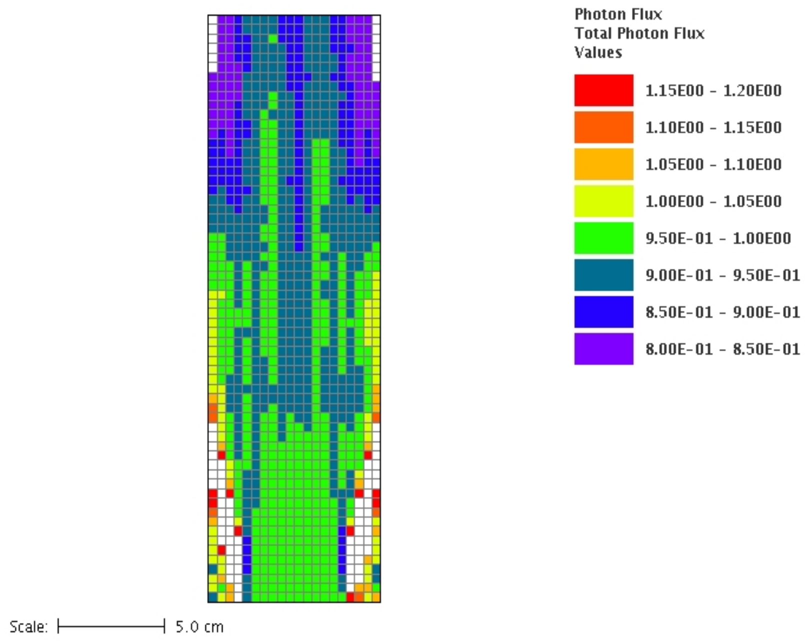
- **realistic, fully 3D phantom**
- **photon source modeled as a surface source**
  - **spectrum typical of a radiotherapy source**
  - **emits photons within a 30^{o} cone of directions**



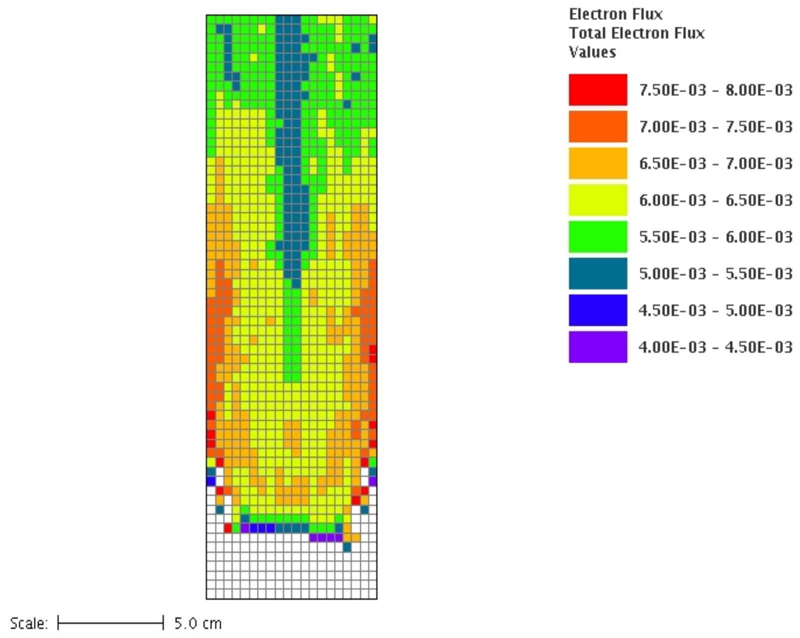
Results – 3D

- ratios to EGSnrc results
- 2 iterations per group, 630-direction forward biased quadrature

Photon Fluxes



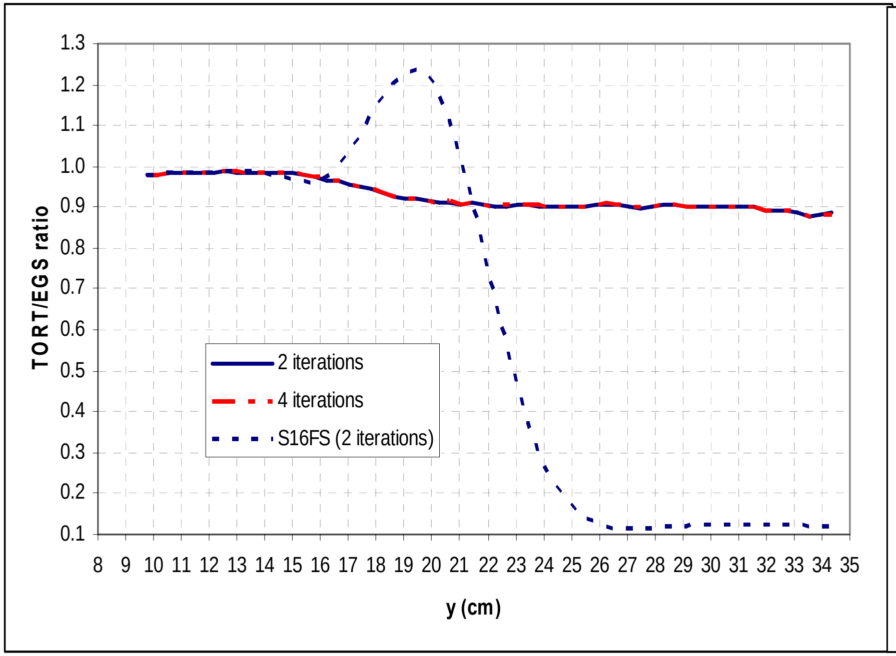
Electron Fluxes



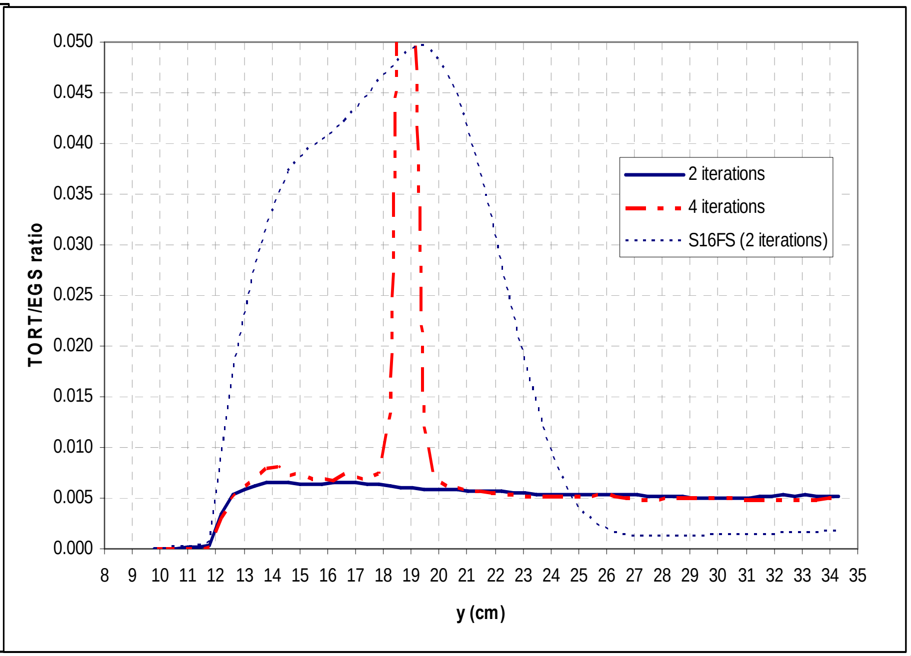
Results – 3D

- **TORT/EGS ratio for fluxes along a central depth line**

Photons

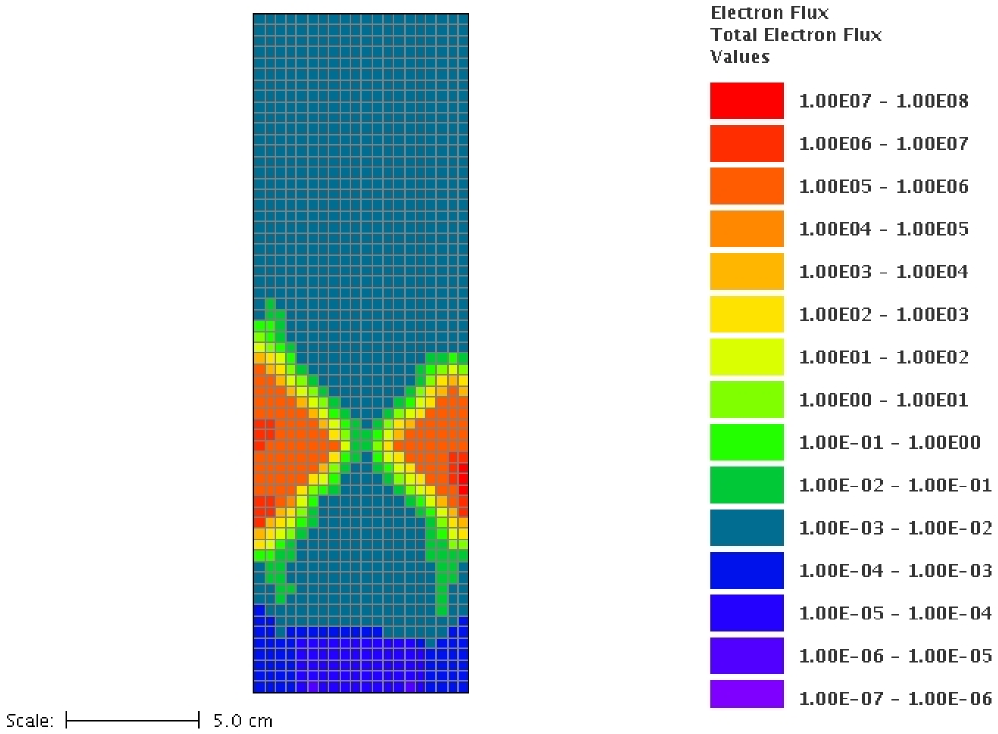


Electrons



Results – 3D

- **Divergence in electron groups with increasing number of inner iterations**



Conclusions

- **reasonable good agreement was obtained for one-dimensional calculations for both photon and electron transport**
- **difficulties in handling electron transport for 2-D and 3-D**
- **difficulties stem from the cross section preparation ⇨ deeper insight into this issue is necessary**
- **computational time for the discrete ordinates calculations compared well against Monte Carlo for the problems considered**
- **explicit (direct) treatment of the Fokker-Planck terms ⇨ the way to go for electron transport ?**