COMPARISON OF HYBRID METHODS FOR GLOBAL VARIANCE REDUCTION IN SHIELDING CALCULATIONS

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ABSTRACT

For Monte Carlo shielding problems that calculate a mesh tally over the entire problem, the statistical uncertainties computed for each voxel can vary widely. This can lead to unacceptably long run times in order to reduce the uncertainties in all areas of the problem to a reasonably low level. Hybrid methods – using estimates from deterministic calculations to create importance maps for variance reduction in Monte Carlo calculations – have been successfully used to optimize the calculation of specific tallies. For the global problem, several methods have been proposed to create importance maps that distribute Monte Carlo particles in such a way as to achieve a more uniform distribution of relative uncertainty across the problem. The goal is to compute a mesh tally with nearly the same relative uncertainties in the low flux/dose areas as in the high flux/dose areas. Methods based on only forward deterministic estimates and methods using both forward and adjoint deterministic methods have been implemented in the SCALE/MAVRIC package and have been compared against each other by computing global mesh tallies on several representative shielding problems. Methods using both forward and adjoint estimates provide better performance for computing more uniform relative uncertainties across a global mesh tally.

Key Words: Monte Carlo, hybrid, variance reduction, FW-CADIS

1. INTRODUCTION

Hybrid methods use quick forward and/or adjoint flux estimates from a deterministic code to create biasing parameters for a detailed Monte Carlo (MC) calculation. One way to incorporate the deterministic estimate is through an importance map that defines target weights, $\overline{w}$, for MC weight windows.

Importance maps can be developed to optimize the MC calculation over different spatial regions. For a source/detector problem, the goal is to maximize the figure-of-merit (FOM) of a single tally at a specific location. For a source/region problem, the goal is to compute several tallies or a mesh tally over a large portion of the problem with uniform relative uncertainties. For the global problem, the goal is to compute a mesh tally over the entire problem space with uniform relative uncertainties – to converge every voxel at the same rate – regardless of the magnitude of the tally in the voxel.

For the source/region and global cases, there are different quantities that the user may wish to optimize with the importance map. These include the spatial flux, $\phi(\vec{r})$; the space/energy flux, $\phi(\vec{r},E)$; or an integrated response, $R(\vec{r}) = \int f(E)\ \phi(\vec{r},E)dE$, where $f(E)$ is a response function (an interaction cross section or flux-to-dose-rate conversion factors).

Several methods for creating importance maps to optimize the MC calculation of a global quantity have been developed recently and their effectiveness on several representative shielding problems will be compared in this paper. These methods are based on either a forward flux estimate or a combination of forward and adjoint flux estimates.

2. METHODS BASED ON FORWARD FLUX ESTIMATES

2.1. Cooper's Method

The use of hybrid deterministic/Monte Carlo methods, particularly for global variance reduction, has been an active area of research by the radiation transport group at the University of Michigan for a over a decade. One of the first approaches studied was a way to develop Monte Carlo weight window target values that were proportional to deterministically estimated values of the forward flux [1]. For global problems using isotropic weight windows, Ref. 1 argues that in order to obtain uniform relative uncertainties in the Monte Carlo calculation, the weight windows should be set such that the number density of Monte Carlo particles, $m(\vec{r})$, is constant. The physical particle density, $n(\vec{r})$, is related to the Monte Carlo particle density by the average particle weight, $\overline{w}(\vec{r})$, such that $n(\vec{r}) = \overline{w}(\vec{r})m(\vec{r})$. To make $m(\vec{r})$ constant over the geometry, the weight window targets $\overline{w}(\vec{r})$ must be proportional to the physical particle density. Cooper and Larsen calculate the weight window targets, $\overline{w}(\vec{r})$, from an estimate of the forward scalar flux $\phi(\vec{r})$ to be $\overline{w}(\vec{r}) = \phi(\vec{r})/\max\big(\phi(\vec{r})\big)$. Note that Cooper's method is a space-only method.

2.2. Global Flux/Response Weight Windows

Two extensions of Cooper's Method have been developed [2] based on what global information the user desires from the simulation: global flux weight windows, for obtaining energy-dependent flux at every spatial location, and global response weight windows, for obtaining an energy-integrated response at every location. Both of these methods are designed for calculating the "global solution" – everywhere in the geometry of the problem – with nearly uniform uncertainties.

The global flux weight windows (GFWW) method keeps the Monte Carlo particle distribution uniform in space and energy. This is a space/energy version of the original space-only Cooper's Method. The target weight windows, $\overline{w}(\vec{r},E)$, should be proportional to the estimate of the forward scalar flux, $\overline{w}(\vec{r},E) = c\,\phi(\vec{r},E)$. When an energy-integrated response (such as dose) is desired at all spatial locations, the global response weight windows (GRWW) method keeps the Monte Carlo particle distribution proportional to the integral of the product of the response function, $f(E)$, and the estimate of the forward flux, $\phi(\vec{r},E)$. The energy-integrated response, $D(\vec{r})$, is found from $D(\vec{r}) = \int_E f(E)\,\phi(\vec{r},E)dE$. The target weight windows, $\overline{w}(\vec{r},E)$, are then $\overline{w}(\vec{r},E) = cD(\vec{r})/f(E)$.

These two methods develop weight windows as a function of both space and energy. Becker and Larsen state that the constant c in either method is usually set to unity. In practice, normalizing the target weights such that they are equal to unity in the source region produces higher FOMs.

2.3. Van Wijk's Methods

In an iterative scheme using the MCNP weight window generator, van Wijk et al. [3] developed weight window targets based on previous MC estimates of either the spatial flux or the relative uncertainty in the spatial flux. The target weights were then normalized using the maximum flux or the minimum relative uncertainty, respectively. The authors state that normalization to the highest flux or lowest uncertainty typically corresponds to the source area of the problem.

The first method (called flux-based) constructs weight window targets based on a previous Monte Carlo estimate of the forward flux using $\overline{w}(\vec{r}) = \phi(\vec{r})/\max\big(\phi(\vec{r})\big)$. Van Wijk et al. argue that because the goal of global variance reduction is to reduce the uncertainty in the computed flux, an alternative method (called Re-based) is to use the relative error, $Re(\vec{r}) \propto 1/\sqrt{\phi(\vec{r})}$ of a previously computed Monte Carlo estimate of the forward flux and define the target weight windows to be $\overline{w}(\vec{r}) = \min\big(Re(\vec{r})\big)/Re(\vec{r})$.

Like the approaches outlined in the previous sections, weight window targets based on the flux or the relative uncertainty in the flux could also be generated from deterministic estimates of those quantities. Note that van Wijk's flux-based method would then be the same as Cooper's method. A potential downfall of the Re-based method is that for a problem where the flux spans n orders of magnitude, the weight window targets will only span $n/2$. Particles would have to survive many successive roulettes in order to populate the lowest flux areas of the problem.

3. METHODS BASED ON BOTH FORWARD AND ADJOINT ESTIMATES

3.1. Becker's Methods

Becker [4] has also proposed three methods for developing weight windows based on both forward and adjoint deterministic solutions. These three methods correspond to the portion of the phase space over which uniform relative uncertainties are desired: a small "detector" region, a region comprising a significant portion of the entire problem, and the global problem. Each of the three methods can be used in one of two modes: to provide uniform uncertainties in flux across space and energy or provide uniform uncertainties for an energy-integrated response across space. A brief outline of Becker's method for the global problem is given below:

Compute the forward flux estimate $\phi(\vec{r},E)$

Construct the adjoint source -

for optimizing flux use

$$q^{+}(\vec{r},E) = {}^{1}\!/_{\phi(\vec{r},E)} \tag{1}$$

or

for optimizing response use

$$q^{+}(\vec{r},E) = {}^{f(E)}\!\Big/_{\int_0^\infty f(E)\,\phi(\vec{r},E)\,dE} \tag{2}$$

Compute the adjoint flux estimate $\phi^{+}(\vec{r},E)$

Construct the contributon flux

$$\phi^{c}(\vec{r},E) = \phi(\vec{r},E)\,\phi^{+}(\vec{r},E) \tag{3}$$

Find the space-only contributon flux

$$\phi^{c}(\vec{r}) = \int_0^\infty \phi^{c}(\vec{r},E)\,dE \tag{4}$$

Construct the weight windows

$$\bar{w}(\vec{r},E) = \frac{\phi^c(\vec{r})}{\phi^+(\vec{r},E)} \tag{5}$$

3.2. FW-CADIS

Wagner and Haghighat [5,6] developed the Consistent Adjoint Driven Importance Sampling (CADIS) method to develop weight window target values that were consistent with a biased source to optimize the MC calculation of an integrated response R at one detector location. Using an adjoint source equal to the response function, $q^+(\vec{r},E) = f(E)$, located at the detector, the adjoint flux, $\phi^+(\vec{r},E)$, is computed and an estimate of the response, $R = \iint q(\vec{r},E)\,\phi^+(\vec{r},E)\,dE\,dV$, is made using the true source, $q(\vec{r},E)$. Then the target weights, $\bar{w}(\vec{r},E)$, and biased source, $\hat{q}(\vec{r},E)$, can be developed as

$$\bar{w}(\vec{r}) = \frac{R}{\phi^+(\vec{r},E)}, \tag{6}$$

$$\hat{q}(\vec{r},E) = \frac{q(\vec{r},E)\,\phi^+(\vec{r},E)}{R}. \tag{7}$$

Though the idea of using adjoint information to accelerate Monte Carlo was well known for several decades before Ref. 6 was published, the unique feature of the CADIS method is that it ensures that the biased source and importance map are consistent with each other, so that computational time is not wasted splitting or rouletting particles right after birth.

This method has been extended for optimizing source/region and global problems by incorporating the forward flux estimate into the definition of the adjoint source. The forward-weighted CADIS method (FW-CADIS) [7,8] uses an adjoint source of $q^+(\vec{r},E) = g(\vec{r})/\phi(\vec{r},E)$ for optimizing the space/energy flux, $q^+(\vec{r},E) = g(\vec{r})/\int \phi(\vec{r},E)\,dE$ for optimizing the total flux, and $q^+(\vec{r},E) = f(E)\,g(\vec{r})/\int f(E)\,\phi(\vec{r},E)\,dE$ for optimizing a spatially-dependent response. The FW-CADIS method can be used for source/region problems or global problems by changing the extent of the volume where the adjoint source is defined (expressed as $g(\vec{r}) = 1$, where the mesh tally is to be optimized and 0 otherwise). From the resulting adjoint fluxes, the weight window targets and biased source are developed similar to CADIS. Also like CADIS, the normalization of the target weights is based on the biased source.

4. IMPLEMENTATION IN SCALE/MAVRIC

4.1. Methods

Both SCALE 6.0 and 6.1 [9] contain the MAVRIC sequence, a user-friendly approach to applying CADIS or FW-CADIS to challenging shielding problems. MAVRIC uses the Denovo S_N code [10] for the forward and adjoint flux estimates, creates the importance map (target weights) and biased source, and then uses them in Monaco [11], a fixed-source multi-group Monte Carlo code. The methods in this summary were added as alternatives to the CADIS/FW-CADIS processing step of the MAVRIC sequence in an internal development version of SCALE.

The default in MAVRIC for Cooper's, van Wijk's, GFWW/GRWW and Becker's forward/adjoint methods is to develop and use a biased source and normalize the computed importance map to that biased source. Keywords are available to turn the biased source creation and normalization off, so that the user can use the methods as described in their original references. In that case, further normalization of the target weights is left to the user. An option for the forward flux-based methods is to normalize the target weight windows to the maximum flux value (like the description of Cooper's method). In most cases, this would set the target weight to 1 at the center of the source, which is a reasonable approach. In this paper, the forward-based methods will be run in a manner that corresponds to their original references – no biased sources and the target weights are normalized using the maximum forward flux estimate. Becker's forward/adjoint method will use the biased source, because normalizing adjoint-based importance maps is difficult without the biased source.

4.2. Mesh Tally Metrics

Because a mesh tally consists of many actual tallies, the statistical tests are more complex than for a single tally. Several statistical quantities and tests are used in Monaco, similar to those described in a recent study [12], that assess the distribution of relative variances over the mesh tally. In Monaco, the basis of the statistical tests centers on the distribution of relative uncertainties in the V voxels that have a non-zero score. The mean of the distribution, $\bar{r}$, is simply $\bar{r} = \sum r_i/V$, where r_i is the relative uncertainty of the flux or dose in voxel i. If every voxel has been sampled well and its relative uncertainty is decreasing with the square root of the number of MC histories N ($r_i \propto 1/\sqrt{N}$), then the mean relative uncertainty of the voxels should also behave as $1/\sqrt{N}$. The variance of $\bar{r}$ can also be calculated, as well as a figure-of-merit for the mesh tally using $\mathrm{FOM} = 1/(\bar{r}^2 T)$, with the time T in minutes.

The four statistical tests in Monaco examine whether, over the last half of the MC simulation, the:

1) fraction of voxels with score, ζ, is constant (within ±0.10);
2) mean relative uncertainty, $\bar{r}$, is decreasing as $1/\sqrt{N}$ (as measured by the coefficient of determination, R^2, of a fit to a curve with a power of -0.5, $R^2 > 0.99$);
3) variance of the mean relative uncertainty is decreasing with $1/N$ (with an $R^2 > 0.95$ of a fit to a curve with a power of -1) ; and
4) FOM is constant (within ±0.10).

For non-uniform meshes (especially cylindrical), these tests may not be the best measure of performance because different size voxels will have a wider variety of relative uncertainties. The user is also cautioned that if there are individual voxels within the mesh tally that have relative uncertainties that are not decreasing as $1/\sqrt{N}$, then the mesh tally statistical tests will not be meaningful. Note that the FOM computed by Monaco is a measure of the average relative uncertainty, not a measure of the uniformity in relative uncertainties which is the goal of this comparison study. To compare uniformity, the entire relative uncertainty distribution must be examined. It is ultimately up to the user to decide if the mesh tally is performing well (is the goal of the mesh tally to calculate the dose rate, not the flux?; are all spatial areas of the mesh tally equally important?; are all magnitudes of the flux or response values equally important?; etc.).

5. EXAMPLE PROBLEMS

5.1. Simple Shielding Problem

The first example problem is a simple shielding problem based on an example problem from van Wijk [3]. This problem consists of a 1 Ci 14.1 MeV neutron point source at the center of a 40 cm cube of paraffin encased in 10 cm of lead. This source box is located in a two-room building constructed of lead walls 10 cm thick. The rooms are separated by a borated glass window. The outer dimensions of the problem, shown in Fig. 1, are 420×220×240 cm. The objective of this problem is to compute the total flux everywhere. This problem is not computationally very challenging — the ratio of the maximum to minimum fluxes is 1.6×10^6. If all of the materials were voids instead, that ratio would be 1.2×10^4, showing that most of the variation in flux is due to spatial attenuation, not material attenuation.

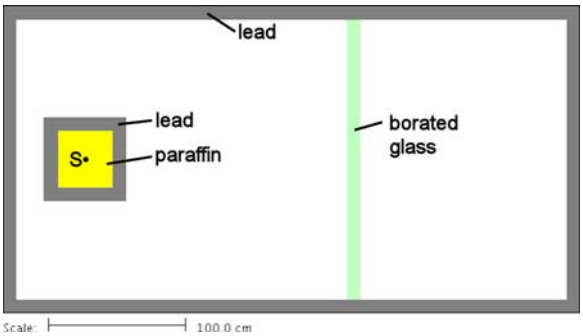

Figure 1. Geometry of the simple shielding problem.

Table I. Run times for the simple shielding problem

	Time (min)			
	for	adj	MC	Total
Analog			420	420
Cooper	8.9		50.9	60
van Wijk (Re)	8.9		50.1	59
GRWW	8.9		51.0	60
Becker	8.9	7.4	40.8	57
FW-CADIS	8.9	7.4	40.3	57

The analog (with implicit capture) simulation was run for 7 hours so that the mesh tally statistical tests could all be satisfied. The target time for each hybrid calculation was a total of one hour on a single processor, as shown in Table I. The deterministic calculations used a non-uniform 51×35×37 mesh, 27 energy groups, a level-symmetric S_8 quadrature and P_3 Legendre scattering expansion. Because this problem involves a point mono-energetic source, no biasing of the source distribution in space or energy is possible. Results for the final MC mesh tally are summarized in Table II, showing that all of the hybrid methods performed well relative to the analog simulation. For the statistical tests, "X" is passing, "-" is not. Figure 2 shows the cumulative distributions of the voxel relative uncertainties for the different methods. This shows that the analog calculation has the smallest fraction of voxels below a given relative uncertainty and FW-CADIS has the largest fraction.

5.2. Deep Penetration Shielding Problem

As an example of a deep penetration shielding problem, consider a simplified, yet representative, problem involving an 860 × 460 × 460 cm concrete structure holding nuclear fuel [8], as shown in Fig. 3. The fuel is represented as an 80 × 80 × 80 cm cube of homogenized uranium dioxide and water, surrounded by a 10 cm layer of steel. The concrete walls are 30 cm thick, spaced every 1 meter in the x-direction, leaving air regions that are 70 cm thick. The source is modeled

uniformly over the central cube, emitting neutrons from a Watt fission spectrum, with a total strength of 1 Ci. In this example, the goal is to optimize the calculation of the space/energy neutron flux, $\phi(\vec{r}, E)$, over the entire problem.

Table II. Statistical tests for the simple shielding problem

	ζ	$\bar{r}$	FOM (/min)	Stat. Test 1	2	3	4
Analog	1.0000	1.07E-01	0.21	X	X	X	X
Cooper	1.0000	5.23E-02	6.11	X	-	X	-
van Wijk (Re)	1.0000	4.61E-02	7.97	X	X	X	X
GRWW	1.0000	3.38E-02	14.60	X	-	-	-
Becker	1.0000	3.77E-02	12.32	X	-	X	-
FW-CADIS	1.0000	2.03E-02	42.83	X	X	X	X

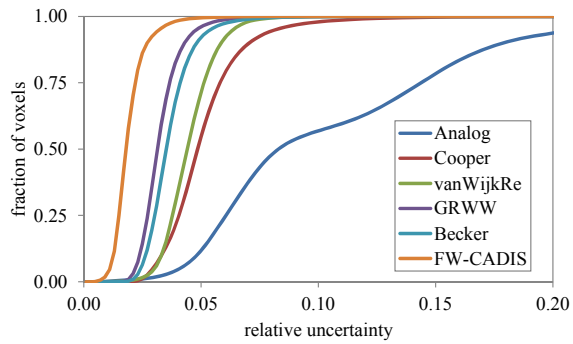

Figure 2. Cumulative distribution of relative uncertainties for the simple shielding problem.

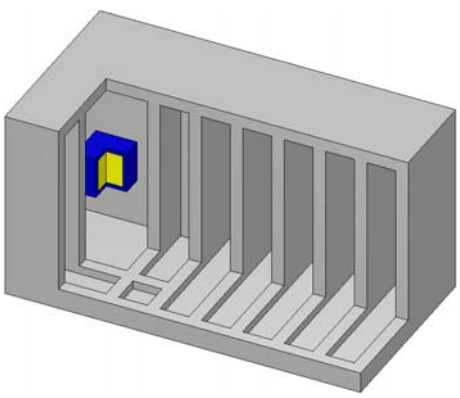

Figure 3. Geometry of the deep penetration shielding problem.

Table III. Run times for the deep penetration shielding problem

	Time (min) for	adj	MC	Total
Analog			1446	1446
Cooper	29.2		1416	1445
van Wijk (Re)	29.2		1416	1445
GFWW	29.2		1416	1445
Becker	29.2	23.7	1386	1439
FW-CADIS	29.2	23.6	1392	1445

An analog calculation (with implicit capture) and five hybrid methods were all used to calculate a mesh tally (86 × 46 × 46 uniform spatial mesh, 27 groups) of the neutron flux. The run times for each, including the forward and adjoint discrete ordinates calculations and the final Monte Carlo calculations are shown in Table III. The deterministic estimates used a non-uniform spatial mesh of 106 × 58 × 50, 27 energy groups, S_8 quadrature and a P_3 Legendre expansion of the scattering. The mesh tally statistical test results for each method are shown in Table IV. After 24 hours of run time, the analog calculation has non-zero scores in only 45% of the voxels. Cooper's method and GFWW do a better job at getting scores to all voxels/groups but van Wijk's Re-based method does not, because the importance map spans only half the range that the forward flux does. The adjoint-based methods do better at achieving a more uniform distribution of relative uncertainties across the 4.9 million tallies within the mesh tally. As shown in Fig. 4,

for a given level of relative uncertainty, r_{max}, the FW-CADIS method has a larger fraction of voxels with relative uncertainties less than or equal to r_{max}.

Table IV. Statistical tests for the deep penetration shielding problem

	ζ	$\bar{r}$	FOM (/min)	Stat. Test 1	2	3	4
Analog	0.4527	2.14E-01	0.0151	-	-	-	-
Cooper	0.9914	2.18E-01	0.0146	X	-	-	-
van Wijk (Re)	0.7677	2.68E-01	0.0096	-	-	-	-
GFWW	1.0000	1.54E-01	0.0292	X	-	X	-
Becker	0.9997	1.34E-01	0.0387	X	-	-	-
FW-CADIS	1.0000	9.84E-02	0.0715	X	X	X	X

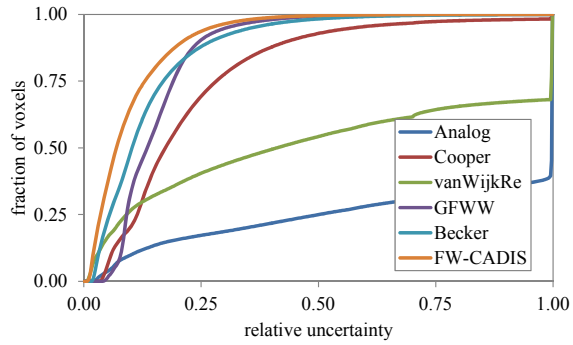

Figure 4. Cumulative distribution of relative uncertainties for the deep penetration shielding problem.

5.3. Becker's Challenge Problem

The third example problem is from Becker [4]. The model consists mostly of nested cubes, with a volumetric UO_2 fission neutron source (10^{16} n/s) surrounded by layers of steel, concrete and steel. This is located in a large water region, surrounded by concrete, lead, concrete and lead. Several concrete pillars are also included in the large water region. The overall size of the problem is 300×300×300 cm. The objective of this problem is to compute the total flux everywhere. This is a more difficult shielding problem than the ones previously described, covering 14 orders of magnitude in total flux, with only 3 orders of magnitude due to spatial attenuation. Figure 5 shows a portion of the problem geometry (with water made transparent) and Table V lists the problem run times.

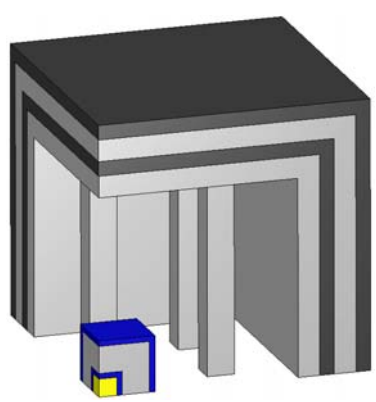

Figure 5. Geometry of Becker's challenge problem (one octant) with the water removed.

Table V. Run times for Becker's challenge problem

	Time (min) for	adj	MC	Total
Analog			1203	1203
Cooper	38.6		1173	1212
van Wijk (Re)	38.6		1163	1202
GRWW	38.6		1184	1222
Becker	38.6	48.6	1118	1205
FW-CADIS	38.6	48.7	1117	1204

The analog calculation and the five hybrid methods were each run for 20 hours, with the forward and adjoint S_N calculations (non-uniform 74×74×74 mesh, S_8, P_3) taking about 40 and 50 minutes, respectively. The mesh tally results, listed in Table VI, show that the forward-based methods did improve the mesh tally by getting particles to every voxel or nearly every voxel. The adjoint-based methods do much better at reducing the mean relative uncertainty and converging the mesh tally. The cumulative distributions of the voxel relative uncertainties for all of the methods are shown in Fig. 6.

Table VI. Statistical tests for Becker's challenge problem

	ζ	$\bar{r}$	FOM (/min)	Stat. Test 1	2	3	4
Analog	0.3428	2.64E-01	0.012	-	-	-	-
Cooper	1.0000	3.49E-01	0.007	X	-	-	-
van Wijk (Re)	0.9675	3.54E-01	0.007	X	-	-	-
GRWW	1.0000	3.58E-01	0.007	X	-	-	-
Becker	1.0000	5.26E-02	0.323	X	X	X	X
FW-CADIS	1.0000	3.92E-02	0.582	X	X	X	X

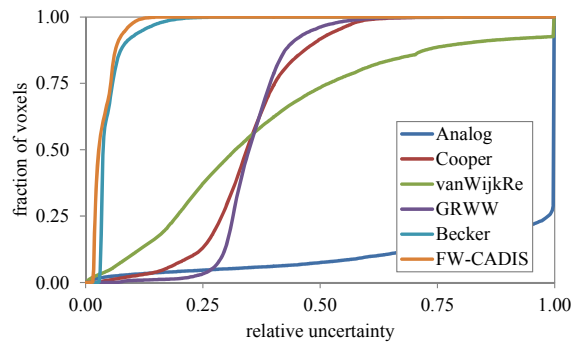

Figure 6. Cumulative distribution of relative uncertainties for Becker's challenge problem.

In this problem, the three forward-based methods would benefit from using source biasing and normalizing the importance map based on that biased source. Figures of merit for Cooper's and van Wijk's methods improve slightly and the FOM for GRWW improves by a factor of three when using a biased source and normalization.

5.4. Reactor Benchmark Problem

The last example problem is the Monte Carlo Performance Benchmark problem [13]. This problem is designed to be an eigenvalue benchmark but can also be used as a test for global variance reduction methods. For this study, the problem was done in two steps. First, a Monte Carlo eigenvalue calculation was performed to compute the source distribution on a mesh (68 × 68 × 61 over the fuel). The second step used the mesh-source and performed a fixed-source calculation to compute a global mesh tally of total flux. (In the fixed source calculation, fission was treated as absorption.) This study focused on the second step, comparing an analog calculation and the five hybrid methods.

The problem is a simplified PWR core model with 241 fuel assemblies, each with a 17 × 17 array of fuel pins and control rod guide tubes filled with water. The fuel rods consist of two sets of materials – one for the upper half (with less dense water) and one for the lower half. Outside of the fuel assemblies, the simplified model includes water reflectors, a downcomer region, the reactor vessel and several regions above and below the core that represent the nozzle areas. The geometry is shown in Fig. 7. Run times for the analog and five hybrid method fixed-source

calculations are shown in Table VII. Deterministic calculations used a non-uniform 92×92×80 mesh, S_4, and P_3. The computed source distribution is shown in Fig. 8.

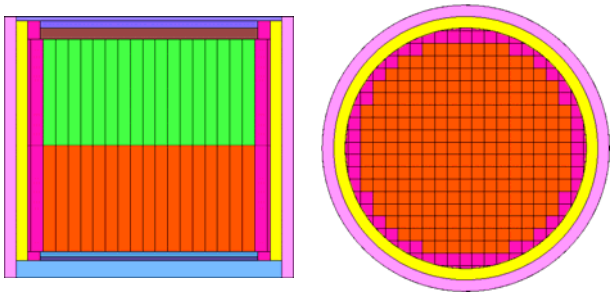

Figure 7. Geometry of the reactor benchmark problem (left – vertical slice, right – horizontal slice).

Table VII. Run times for the reactor benchmark problem

	Time (min)			
	for	adj	MC	Total
Analog			612	612
Cooper	16.9		585	602
van Wijk (Re)	16.9		586	602
GRWW	16.9		584	601
Becker	16.9	31.2	558	606
FW-CADIS	16.9	31.2	558	606

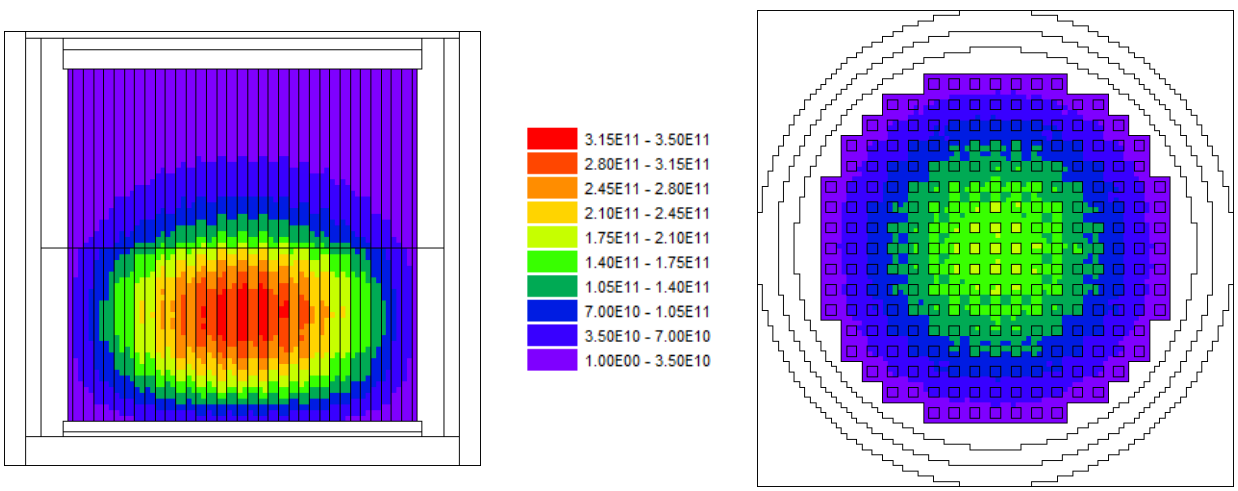

Figure 8. Computed source distribution (neutrons per cm^3) for the reactor benchmark problem (left – vertical slice through center, right – horizontal slice at midplane).

The non-uniform rectangular mesh tally (27 × 27 × 27) used voxels that corresponded to the assembly size in the core region and slightly smaller voxels in the reflector regions. Only 14,823 of the mesh tally voxels fell within the defined cylindrical model, giving a maximum ζ value of 0.753086. For each method, Table VIII lists the statistical results and Fig. 9 shows the cumulative distribution of the relative uncertainties. After ten hours, only the two hybrid methods that used both forward and adjoint deterministic calculations passed all four of the statistical tests, with FW-CADIS having a slightly higher FOM than Becker's method.

Using a biased source and normalizing the importance map to that source would improve the FOM of Cooper's method, van Wijk's Re-based method and the GRWW method by factors of 8, 5 and 13, respectively. The mesh tallies for these methods would then pass the four statistical checks within the 10 hours. But even with the biased source, none of the forward methods would come close to the FOM performance of the adjoint-based methods.

Table VIII. Statistical tests for the reactor benchmark problem

	ζ	$\bar{r}$	FOM (/min)	Stat. Test 1	2	3	4
Analog	0.6203	2.02E-01	0.040	-	-	-	-
Cooper	0.7531	5.79E-02	0.510	X	-	X	-
van Wijk (Re)	0.7524	7.27E-02	0.323	X	-	-	-
GRWW	0.7531	1.40E-01	0.087	X	-	-	-
Becker	0.7531	1.47E-02	8.281	X	X	X	X
FW-CADIS	0.7531	1.27E-02	11.096	X	X	X	X

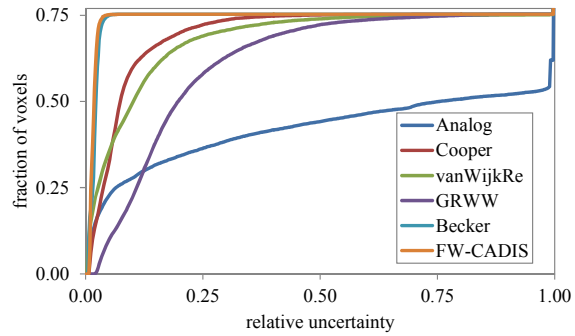

Figure 9. Cumulative distribution of relative uncertainties for the reactor benchmark problem.

6. CONCLUSIONS

This study has shown that the adjoint-based hybrid methods perform better than the forward-based hybrid methods at computing a global mesh tally with more uniform relative uncertainties in four representative shielding problems. Among the two adjoint methods, FW-CADIS performed better than Becker's global on all four example problems. On the more difficult problems, van Wijk's Re-based method had more difficulty than the other forward-based methods at fully populating the mesh tally and achieving uniform relative uncertainties – most likely due to the fact that the importance map spanned only half the range of the estimated flux values.

In the MAVRIC implementations, the FW-CADIS method and Becker's methods are more flexible for semi-global problems. In practice, a global solution is often not required. Typically, a mesh tally is done over a large portion of a problem but some areas are not important. For example, in the reactor benchmark problem, the fluxes may be desired for depletion calculations but are not required in the reactor vessel. Dose-rate maps over buildings may only include the spaces where people could be present – areas inside walls or inside equipment do not require low uncertainties in a dose-rate mesh tally. The adjoint-based methods allow the user to define the important areas within a mesh tally and to optimize relative uncertainties there. The importance map based only on an estimate of forward fluxes will populate the MC problem with particles in all parts of phase space, whether those areas are important to the user or not.

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