H33A-0964: Constructing Design Rainfall Hyetographs Using Trivariate Plackett Family of Copulas

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Introduction

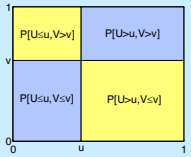
Current risk-based hydrologic design is based on univariate analysis of rainfall, where durations are artificially prescribed. Multivariate stochastic analyses via copulas are receiving increasing attention for constructing joint distributions. However, there are limitations to extending copulas to trivariate and higher dimensions:

- Difficulties in preserving all lower-level mutual dependencies
- Compatibility problem in multivariate statistics

In this study, the Plackett family of copulas is examined and found to be a potential trivariate dependence model. It is applied to construct design rainfall hyetographs for several stations in Indiana.

Plackett Family of Copulas

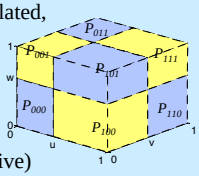
Constant Cross Product Ratio Theory:



- Bivariate $\psi_{UV} = \dfrac{P[U \le u, V \le v]P[U > u, V > v]}{P[U > u, V \le v]P[U \le u, V > v]}$

$$C_{UV}(u,v) = \frac{[1+(\psi_{UV}-1)(u+v)] - \sqrt{[1+(\psi_{UV}-1)(u+v)]^2 - 4uv\psi_{UV}(\psi_{UV}-1)}}{2(\psi_{UV}-1)}$$

$\Psi = 1$ => independent, $\Psi > 1$ => positively correlated,
$\Psi < 1$ => negatively correlated.

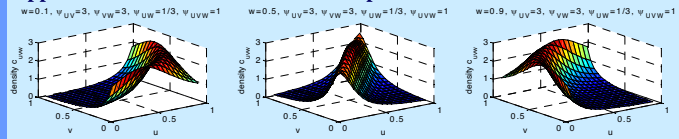


- Trivariate $\psi_{UVW} = \dfrac{P_{000}P_{011}P_{101}P_{110}}{P_{111}P_{100}P_{010}P_{001}}$

$$\psi_{UVW}(a_1 - z)(a_2 - z)(a_3 - z)(a_4 - z) - z(z - b_1)(z - b_2)(z - b_3) = 0$$

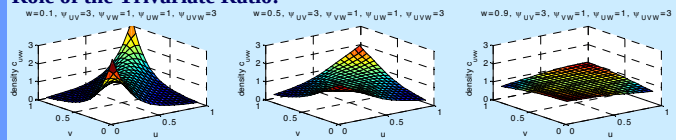
(Needs solution of 4th order polynomial, not intuitive)

Appearance of Trivariate Plackett Copulas:

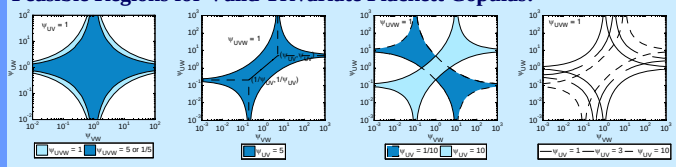


copula density in terms of marginals u, v

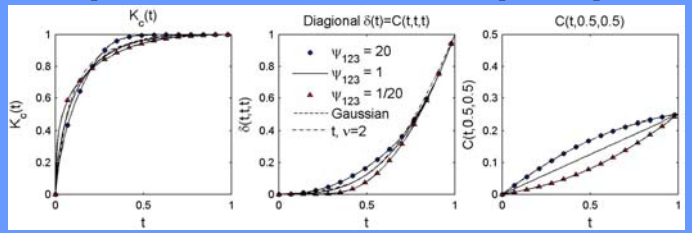
Role of the Trivariate Ratio:



Feasible Regions for Valid Trivariate Plackett Copulas:



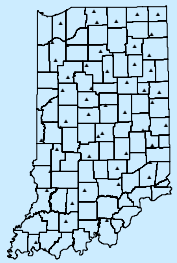
Comparison between Plackett and Meta-elliptical Copulas



Construction of Trivariate Extreme Rainfall Distribution

Study Area and Date Source:

- 53 COOP hourly precipitation stations in Indiana with record length greater than 50 years
- Minimum rainfall hiatus: 6 hours
- Approximately 4800 events per station



Definition of Extreme Rainfall:

- Annual maximum cumulative probability (AMP) definition for selecting annual series (Kao and Govindaraju, 2007)

Selected Variables for Analysis:

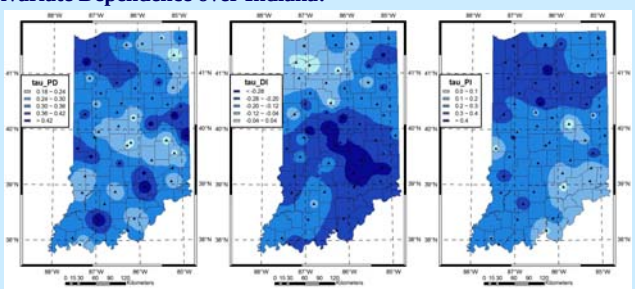
- Depth, P (mm); Duration, D (hour); Peak intensity, I (mm/hour)
- Percent. time to peak, T_p (%)
- Percent. accumulation at each 10% temporal ordinates, A_{10}, …, A_{90} (%)

Marginal Distribution:

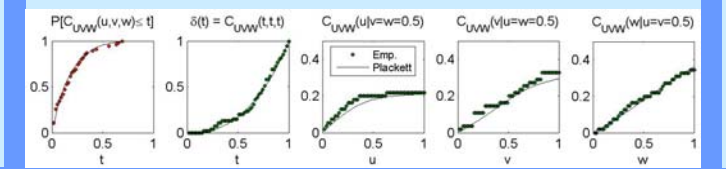
- P, D, I: Log Normal
- T_p, A_k: Beta

	Number of stations with the minimum AIC				Number of stations with the minimum BIC			
	GEV	LN	P3	LP3	GEV	LN	P3	LP3
Depth, P	7	38	1	7	4	47	0	2
Duration, D	1	46	6	0	1	50	2	0
Intensity, I	9	41	2	1	2	49	1	1

Bivariate Dependence over Indiana:



Trivariate Dependence Structure:



Design Rainfall Hyetographs

Given design depth (P) and duration (D), the conditional expectations are computed to get the design hyetographs (rainfall temporal distributions)

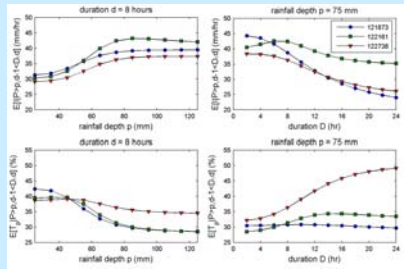
Rainfall Peak Attributes:

Conditional expectation of peak intensity (I)

$$E[I \mid P > p_D, d_D - 1 < D < d_D]$$

Conditional expectation of percentage time to peak (T_p)

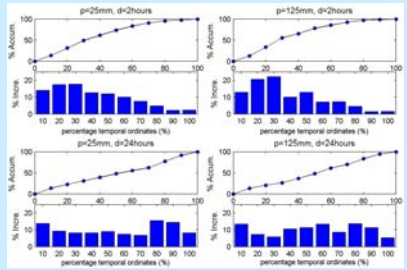
$$E[T_p \mid P > p_D, d_D - 1 < D < d_D]$$



Temporal Accumulation Curves:

Conditional expectation of percentage accumulations at each 10% temporal ordinates (A_{10}, A_{20}, …, A_{90})

$$E[R_k \mid P > p_D, d_D - 1 < D < d_D]$$



- Peak intensity increases with total depth, decreases with duration
- Time to peak increases with duration, decreases with total depth

Summary and Conclusions

- Plackett family of copulas, along with the underlying cross product ratio theory, is found to be a suitable trivariate dependence model in constructing rainfall temporal distribution.
- The feasibility region for Plackett parameters that would result in valid 3-copulas has been identified numerically in this study.
- Not every set of given bivariate dependencies have a corresponding valid 3-copula. The compatibility of given bivariate dependencies needs to be investigated.

References

Kao, S.-C., and R. S. Govindaraju (2007a), A Bivariate Frequency Analysis of Extreme Rainfall with Implications for Design, *Journal of Geophysical Research*, 112, D13119.

Kao, S.-C. and R. S. Govindaraju (2007b), Trivariate Statistical Analysis of Extreme Rainfall Events via Plackett Family of Copulas, *Water Resources Research*, accepted for publication.

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