Spectral Effects in Quantum Teleportation

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Abstract: Quantum teleportation is analyzed in the context of multi-mode interference effects. The teleportation fidelity depends on the spectral relationship between the entangled photons and on the spectral overlap between the photons in the Bell-state measurement.
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Quantum teleportation [1] is an integral component of many developing applications in quantum information science, such as quantum computation, quantum communication, etc. Teleportation using photons is of particular interest for the purposes of developing quantum channels over long distances using, for example, quantum repeaters [2]. In the early descriptions of quantum teleportation, as well as in nearly all subsequent treatments, the photons are treated as single-mode (hence, monochromatic) entities. In actual experiments, however, the photons do not have infinitesimal bandwidths. In fact, if the photons are created via type-II down-conversion pumped by a short-pulse laser, then they may carry spectral features that have been shown to compromise certain quantum interference effects [3]. In this paper, we analyze the effects of these spectral features on the fidelity of quantum teleportation.

The experimental setup under consideration is shown in Fig. 1. Photons 2 and 3 are initially entangled, while photons 1 and 2 combine at a lossless 50:50 beamsplitter and are then directed to polarization beamsplitters. As shown by Braunstein and Mann [4], certain coincidence measurements (V1 and H2, for example) correspond to Bell-state measurements of the composite state and result in the teleportation of the state of photon 1 onto photon 3. The state of the first photon may be written as

$$\left|\psi_1\right\rangle = \int d\omega_1 s(\omega_1)\left[a\left|h_1(\omega_1)\right\rangle + b\left|v_1(\omega_1)\right\rangle\right]. \tag{1}$$

For photons 2 and 3, we first consider the case in which the photons have spectral differences that are correlated with polarization. This corresponds to the usual experimental configuration in which the photons are taken directly from the down-conversion crystal with only birefringent delay compensation. In this case, the entangled state is

$$\left|\varphi_{23}\right\rangle = \iint d\omega_H d\omega_V f(\omega_H,\omega_V)\left[\left|h_2(\omega_H),v_3(\omega_V)\right\rangle - \left|v_2(\omega_V),h_3(\omega_H)\right\rangle\right]. \tag{2}$$

Following detections of photons with frequencies ω_1 and ω_2 at detectors V1 and H2, respectively, the density matrix of photon 3 is

$$\sigma_3 = \iint d\omega_1 d\omega_2 \left|\left\langle v_1(\omega_1),h_2(\omega_2)\right|\left[\left|\psi_1\right\rangle\left|\varphi_{23}\right\rangle\right]\right|^2. \tag{3}$$

However, since it is the polarization degree of freedom that is to be teleported, the more relevant quantity is the polarization density matrix, which is given by $\tilde{\sigma}_3 = \int d\omega' \left\langle\omega'\right|\sigma_3\left|\omega'\right\rangle$, with matrix elements

$$\tilde{\sigma}_{3HH} = |a|^2, \quad \tilde{\sigma}_{3VV} = |b|^2, \quad \tilde{\sigma}_{3HV} = \tilde{\sigma}^*_{3VH} = ab^* \int d\overline{\omega} \int d\omega\, s_1(\omega) f(\omega,\overline{\omega})^* \int d\omega'\, s_1(\omega')^* f(\overline{\omega},\omega'). \tag{4}$$

From this expression, the roles of the spectral properties of the three photons are more evident. In particular, the teleportation fidelity, which is defined as $F = \mathrm{Tr}(\tilde{\sigma}_1\tilde{\sigma}_3)$, is simply

$$F = 1 - 2|a|^2|b|^2\left(1 - \int d\overline{\omega} \int d\omega\, s_1(\omega) f(\omega,\overline{\omega})^* \int d\omega'\, s_1(\omega')^* f(\overline{\omega},\omega')\right). \tag{5}$$

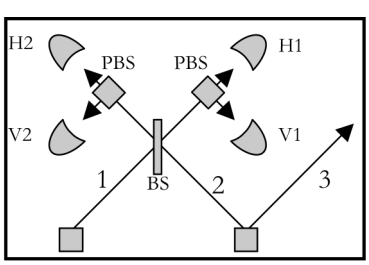

Fig.1. A schematic of the photon teleportation experiment under consideration.

In general, the fidelity depends on the state to be teleported, as well as the spectral features of all three photons. For the remainder of the analysis, we restrict attention to the cases for which $|a| = |b| = 1/\sqrt{2}$, which corresponds to the lowest fidelity for given functions $s(\omega_1)$ and $f(\omega_H,\omega_V)$. The dependence on the spectral functions is governed by a number of parameters, but space restrictions preclude a full analysis here. It is instructive, however, to consider a few limiting cases.

Anti-correlated spectra (strong spectral entanglement): In this case, photons 2 and 3 may each have fairly large bandwidths, but the photon energies are constrained to be strongly anti-correlated. This type of spectral

entanglement would be obtained from a source with a large down-conversion bandwidth but with a narrow pump bandwidth. One might expect that this type of spectral entanglement would work well in quantum teleportation, since it has the high degree of spectral symmetry that is needed in many two-photon experiments. This turns out not to be the case, however, as can be seen in Fig. 2(a), which shows a plot of the fidelity versus the bandwidth of photon 1. The fidelity reaches maximum value when the bandwidth of photon 1 matches that of photon 2, but the peak value is still quite low.

Uncorrelated spectra (no spectral entanglement): Here, the energies of the horizontally and vertically polarized photons are assumed to be completely uncorrelated, *i.e.*, the energy of one polarization yields no information about the energy of the other. Figure 2(b) shows plots of the teleportation fidelity versus the bandwidth of photon 1. The different curves correspond to different bandwidths for the vertically polarized photon (the bandwidth of the horizontally polarized photon was held fixed). The uncorrelated spectra yield higher fidelities in general, and the fidelity reaches unity for the case in which the bandwidths of the two polarizations are identical. The different curves reach their maxima for different values of the bandwidth of photon 1. The reason for this is that photon 2 can be regarded as having two bandwidths: one for horizontal polarization and one for vertical polarization. The optimum bandwidth is somewhere between the two.

Spectral characteristics correlated with path: The analysis thus far has been limited to the case in which spectral differences are correlated with polarization. While this is the typical case for type-II down-conversion photons, it is also possible to generate photon pairs for which the spectral features are correlated with path [5]. In this case, the two-photon state would be written

$$\left|\varphi_{23}\right\rangle_{PATH} = \iint d\omega_2 d\omega_3 g(\omega_2,\omega_3)\left[\left|h_2(\omega_2),v_3(\omega_3)\right\rangle - \left|v_2(\omega_2),h_3(\omega_3)\right\rangle\right]. \tag{6}$$

The analysis proceeds as above, with the resulting fidelity given by

$$F = 1 - 2|a|^2|b|^2\left(1 - \int d\overline{\omega}\left|\int d\omega\, s_1(\omega) g(\omega,\overline{\omega})^*\right|^2\right). \tag{7}$$

Note that the ordering of the frequency arguments is slightly different. This, in turn, results in slightly different results for the limiting cases considered above. Most notable is the case in which the spectra are uncorrelated, i.e., the energy of photon 2, which is independent of polarization, carries no information whatsoever about the energy of photon 3, also independent of polarization. The curve in Fig. 2(c) shows the teleportation fidelity versus the bandwidth of photon 1. It is interesting to note that the fidelity is completely independent of the bandwidth, and even the center wavelength, of photon 3.

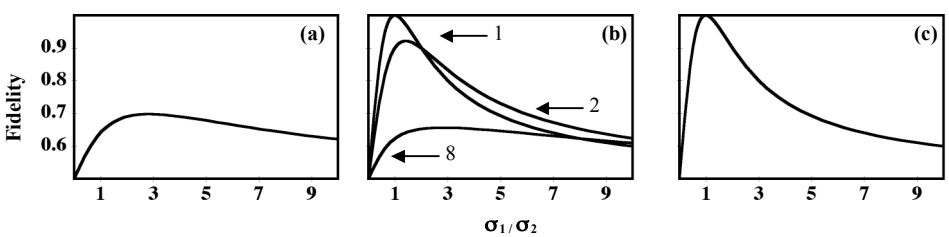

Fig. 2. Plots of fidelity versus the bandwidth of photon 1, normalized to the bandwidth of photon 2. In 2(a) and 2(b), the spectral differences between the entangled photons are correlated with polarization. In 2(a), the photon energies are strongly correlated with each other, whereas in 2(b) there is no spectral entanglement. The three curves correspond to three different spectral widths for the vertically polarized photon. In 2(c), the spectral differences are correlated with path.

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