Quantum Spread Spectrum Communication

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ABSTRACT

We show that communication of single-photon quantum states in a multi-user environment is improved by using spread spectrum communication techniques. We describe a framework for spreading, transmitting, despreading, and detecting single-photon spectral states that mimics conventional spread spectrum techniques. We show in the cases of inadvertent detection, unintentional interference, and multi-user management, that quantum spread spectrum communications may minimize receiver errors by managing quantum channel access.

Keywords: quantum communication, spread spectrum communication

1. INTRODUCTION

Photons are convenient for realizing a variety of quantum communication protocols. This includes demonstrations of quantum key distribution, quantum teleportation, and other quantum information networks.[1] As the encoded quantum states typically do not decohere significantly over a transmission link, photons are naturally well-suited for the timely transmission of quantum information. Moreover, substantial progress has been made in generating the variety of photonic entangled states necessary for implementing different quantum communication protocols.[2] These features have justified the continued development of photon-based quantum communication and suggest its likely use for future quantum network technologies.

Maturing quantum communication outside of a laboratory setting will require techniques for managing quantum channel access and quality of service (fidelity). For example, quantum communication may expect to encounter both loss and interference effects arising from secondary absorbers and emitters operating within the same transmission environment. This may include intentional and unintentional receivers, such as eavesdroppers and absorptive impurities in the channel, as well as intentional and unintentional emitters, such as jammers and competing transmitters.

Traditionally, quantum communication has mitigated information loss using error-correction techniques, which improve quantum channel capacity by correcting errors at the receiver after judicious encoding in a photon-environment subspace.[1] However, additional techniques are likely necessary for managing the loss and interference present in an asynchronous, multi-user quantum network. In particular, methods for enabling simultaneous but uncoordinated quantum communication among multiple parties accessing the same transmission channel would be useful for scaling quantum network design.

In this contribution, we consider spread spectrum communication for managing quantum channel access and quality of service. Spread spectrum (SS) communication is a well-developed classical strategy for improving channel capacity in the presence of narrowband interferers.[3] It operates by spreading the spectral bandwidth of the photon carrier well beyond the encoded information bandwidth. Although spreading implies a lower signal-to-noise ratio (SNR), a constant channel capacity is maintained by the competing gain in channel bandwidth. Moreover, a gain in SNR against narrowband interference is achieved by despreading the signal at the receiver. Consequently, SS is frequently used for managing multi-user access, i.e., as a channel access method for multiplexing communication. A prominent example is radio communications using either direct-sequence or frequency-hopping SS to share bandwidth between multiple transmitters.

In this work, we emphasize the use of quantum spread spectrum for managing multi-user quantum communication environments. We emphasize three cases: inadvertent detection, unintentional interference, and multi-user management, to demonstrate the potential for implementing quantum medium access control. The latter MAC protocol may prove useful for managing multi-user QKD networks operating over shared infrastructure.

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The paper is organized as follows: in Sec. II, we describe the basic elements of spreading, despreading, and detection of the single-photon spectral state; in Sec. III, we present three use cases germane to multi-user quantum networks; and in Sec. IV, we offer conclusions based on the present work.

2. QUANTUM SPREAD SPECTRUM COMMUNICATION

We begin by describing the modulation and demodulation of a pure single-photon, spectral state in terms of the random spreading sequence applied at the transmitter and the synchronized despreading sequence applied at the receiver.

2.1 Spreading the Single-Photon Spectral State

Consider the single-photon spectral state

$$|\psi_0\rangle = \int \alpha_0(\omega)\hat{a}^\dagger(\omega)\,|\mathrm{vac}\rangle\, d\omega, \tag{1}$$

where the normalized spectral probability amplitude

$$\alpha_0(\omega) = |\alpha_0(\omega)|e^{i\phi_0(\omega)} \tag{2}$$

has phase $\phi_0(\omega)$, and $\hat{a}^\dagger(\omega)$ is the creation operator for a photon at plane-wave frequency ω. We introduce the Fourier transforms

$$\hat{a}(\omega) = \int \hat{a}(t)e^{i\omega t}dt \tag{3}$$

and

$$\alpha_0(\omega) = \int \mathrm{A}_0(t)e^{i\omega t}d\omega \tag{4}$$

to express the multi-mode state of Eq. (1) in the time domain as

$$|\psi_0\rangle = \int \mathrm{A}_0(t)\hat{a}^\dagger(t)\,|\mathrm{vac}\rangle\, dt, \tag{5}$$

where the temporal amplitude

$$\mathrm{A}_0(t) = |\mathrm{A}_0(t)|e^{i\Phi_0(t)} \tag{6}$$

has phase $\Phi_0(t)$, and $\hat{a}^\dagger(t)$ represents the creation of a photon at time t. Our nomenclature for the time t and the frequency ω will distinguish the temporal mode operator $\hat{a}(t)$ from the frequency mode operator $\hat{a}(\omega)$.

We describe spreading of the spectral state (1) as modulation of the temporal amplitude $\mathrm{A}(t)$ by a train of pulses

$$\Pi(t) = \sum_{n=-\infty}^{\infty} p_n P_T(t - nT). \tag{7}$$

In Eq. (7), the spreading sequence $\{p_n\}$ represents a random binary sequence with each $p_n \in \{\pm 1\}$. The time T represents the duration of each pulse, while we choose the pulse function to take the form

$$P_T(t) = \begin{cases} 1 \text{ if } |t| \leq T/2 \\ 0 \text{ otherwise} \end{cases}, \tag{8}$$

i.e., a rectangular pulse centered at time zero. Modulation of the state (1) is then represented in the temporal domain as

$$|\psi_\Pi\rangle = \int \mathrm{A(t)}\Pi(t)\hat{a}^\dagger(t)\,|\mathrm{vac}\rangle\, dt, \tag{9}$$

where the subscript Π denotes the applied modulation. The corresponding state in the spectral domain takes the form

$$|\psi_\Pi\rangle = \int d\omega \int d\omega' \alpha(\omega)K_\Pi(\omega,\omega')\hat{a}^\dagger(\omega')\,|\mathrm{vac}\rangle, \tag{10}$$

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where the kernel

$$K_\Pi(\omega,\omega') = \int \Pi(t) e^{i(\omega'-\omega)t} dt \tag{11}$$

signifies a Fourier transform of the temporal modulation with respect to the frequency difference $\omega - \omega'$. Substituting the modulation $\Pi(t)$ as defined in Eq. (7) yields

$$K_\Pi(\omega,\omega') = \sum_{n=-\infty}^{\infty} p_n \int_{-\infty}^{\infty} P_T(t-nT) e^{i(\omega'-\omega)t} dt. \tag{12}$$

As the Fourier transform of the rectangle function $P_T(t)$ is

$$\int_{-\infty}^{\infty} P_T(t) e^{i(\omega'-\omega)t} dt = T \mathrm{sinc}\left[(\omega-\omega')T/2\right] \tag{13}$$

with $\mathrm{sinc}(x) = \sin(x)/x$, the kernel (12) evaluates to

$$K_\Pi(\omega,\omega') = T \mathrm{sinc}\left[(\omega-\omega')T/2\right] \sum_{n=-\infty}^{\infty} p_n e^{i(\omega'-\omega)nT}. \tag{14}$$

The spectral amplitude of the modulated state

$$\alpha_\Pi(\omega') = \int_{-\infty}^{\infty} \alpha_0(\omega) K_\Pi(\omega,\omega') d\omega \tag{15}$$

is then given explicitly as

$$\alpha_\Pi(\omega') = \sum_{n=-\infty}^{\infty} T p_n \int_{-\infty}^{\infty} \alpha_0(\omega) \mathrm{sinc}\left[(\omega-\omega')T/2\right] e^{i(\omega'-\omega)nT}. \tag{16}$$

For the trivial case the spreading sequence is $\{p_n = 1 : \forall n \in \mathbb{Z}\}$, no spreading of the spectral state occurs. This behavior is derived from Eq. (16) by recalling the Poisson summation formula for a Dirac comb

$$\frac{1}{f} \sum_{n=-\infty}^{\infty} e^{i2\pi\omega n/f} = \sum_{k=-\infty}^{\infty} \delta(\omega - kf) \tag{17}$$

with spacing $f = 2\pi/T$, and the Whittaker-Shannon interpolation theorem

$$\alpha_0(\omega) = \sum_{k=-\infty}^{\infty} \alpha_0(kf) \,\mathrm{sinc}\left[(\pi(\omega/f - k)\right]. \tag{18}$$

More generally, the spreading sequence is a random binary sequence whose autocorrelation function $\mathcal{R}(t,\tau) = E[p(t)p(t+\tau)]$ is given as

$$\mathcal{R}(t,\tau) = \Lambda\left(\frac{t}{T}\right), \tag{19}$$

where the triangle function $\Lambda(t)$ is defined as

$$\Lambda(t) = \begin{cases} 1-|t| & \text{if } |t| \le 1 \\ 0 & \text{otherwise} \end{cases}. \tag{20}$$

According to the Wiener-Khinchin theorem, the power spectrum of a wide-sense stationary process (such as a random binary sequence) is provided by the Fourier transform of its autocorrelation function. For the triangle function, this result is proportional to $\mathrm{sinc}^2(\omega T)$, as expected from Eq. (16). Consequently, we expect modulation of the spectral state with a random binary sequence to spread the spectral *amplitude* by the factor $1/T$.

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2.2 Despreading the Single-Photon Spectral State

Following transmission of the spread spectral state defined in Eqs. (9) and (10), the receiver applies a demodulation to the state prior to detection or coherent storage. When applied synchronously, demodulation reverses the modulation applied by the transmitter and despreads the spectral state. In this subsection, we ignore any effects of the transmission channel on the spectral state for the purpose of exposition. This implies that the state arriving at the receiver is defined by $|\psi_\Pi\rangle$.

Demodulation of the spectral state is described as applying $\Pi^{-1}(t)$, the inverse of the modulation $\Pi(t)$. For the binary modulation presented in Eq. (7), $\Pi^{-1}(t) = \Pi(t)$ such that

$$\Pi^{-1}(t)\Pi(t) = \sum_{n=-\infty}^{\infty} P_T(t - nT) \tag{21}$$

since $p_n^2 = 1\,\forall n \in \mathbb{Z}$. Demodulation of the state $|\psi_\Pi\rangle$ defined in Eq. (9) is then represented as

$$|\psi_{\Pi^{-1}\Pi}\rangle = \int A(t)\Pi^{-1}(t)\Pi(t)\hat{a}(t)\,|\text{vac}\rangle\,dt = |\psi_0\rangle\,, \tag{22}$$

where the last equality follows from Eq. (21) and represents successful despreading of the transmitted state.

The exact despreading represented by Eq. (22) results only when the demodulation is perfectly synchronized with the modulation. More generally, there may be an offset in the spreading and despreading sequences due, e.g., from improper synchronization of the transmitter and receiver. In that case, demodulation does not despread the spectral state. Rather,

$$\Pi(t)\Pi(t+\tau) = \sum_{m,n=-\infty}^{\infty} p_n p_m P_T(t - nT)P_T(t + \tau - mT) = \sum_{n=-\infty}^{\infty} p_n p_{n+k} P_T(t - nT) \tag{23}$$

for $\lceil \tau \rceil = kT$. This represents the modulation of the spectral state by a new random sequence; consequently, despreading requires the transmitter and receiver be synchronized to within one symbol duration T.

2.3 Measuring the Spectral State

We consider reception of the single-photon state following demodulation. In general, a quantum receiver may projectively measure the quantum state, store the quantum state for additional processing, or retransmit the state. In this work, we will focus on a receiver that implements a projective measurement of the quantum state; e.g., using a single-photon detector.

We model the measurement as a narrowband spectral projective measurement described by the operator

$$\mathcal{M} = \int \eta(\omega)\,|\omega\rangle\,\langle\omega|\,d\omega, \tag{24}$$

where the bandwidth of the spectral window $\eta(\omega)$ is taken to be narrow relative to the modulation bandwidth $1/T$. For simplicity, we set the detector efficiency to unity. For this measurement device, the probability for detecting a generic, mixed quantum state ρ is defined as

$$P_{\mathcal{M}}(\rho) = \text{Tr}\,[\mathcal{M}\rho] = \int \eta(\omega)\,\langle\omega|\,\rho\rangle\,d\omega, \tag{25}$$

where Tr denote the trace with respect to all degrees of freedom. Measurement of the despread state $|\psi_0\rangle$ then occurs with probability

$$P_0 = \int \eta(\omega)\,|\alpha_0(\omega)|^2\,d\omega; \tag{26}$$

this probability will represent the optimal probability of detection for the communication of the state $|\psi_0\rangle$. In contrast, the probability of detection for the state $|\psi_\Pi\rangle$ is

$$P_\Pi = \int \eta(\omega)\,|\alpha_\Pi(\omega)|^2\,d\omega. \tag{27}$$

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3. QUANTUM MEDIUM ACCESS CONTROL

We describe the use of quantum spread spectrum communication for managing channel access in a mult-user quantum communication network. We specify the protocol in terms of step-wise encoding, transmission, and decoding of the single-photon spectral state using the composition of operators

$$(\mathcal{M} \circ \mathcal{D} \circ \mathcal{T} \circ \mathcal{S}) : \rho \to P_{det}, \tag{28}$$

where P_{det} is the probability to detect the state ρ transmitted though a channel $\mathcal{T}$ using the spreading operator $\mathcal{S}$, the despreading operator $\mathcal{D}$, and the measurement operator $\mathcal{M}$. In general the channel operator $\mathcal{T}$ is responsible for the introduction of noise, errors, and decoherence into the received state. We will explore the effects of these phenomena in subsequent work. At present, we consider the role of QSSC in mitigating various problems that may arise in multi-user communication environments.

3.1 Inadvertent Detection

The case of inadvertent detection is highlighted in Fig. 1, where the spread state sent by the transmitter is inadvertently detected by another receiver also accessing the channel. This may occur for example, if the transmitter is misaligned with the intended receiver, or if the unintended receiver is not aware of the transmitter's presence.

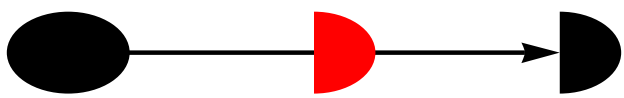

Figure 1. A transmission is inadvertently detected.

We assume the transmitter uses a spreading sequence defined by the modulation operator $\Pi_1(t)$ to prepare the state $|\psi_{\Pi_1}\rangle$. We further assume the inadvertent receiver uses a different or unsynchronized despreading sequence $\Pi_2(t)$. Consequently, the probability for detection is $P_{\Pi_1\Pi_2}$ as defined in Eq. (27). By comparison, an unspread spectral state would be detected with the probability P_0 defined by Eq. (26). This results indicates that the use of quantum spread spectrum techniques minimizes the adverse effects of another user's inadvertent detection.

Assuming the inadvertent receiver blocks a photon that is not detected, the undetected subspace $1 - \mathcal{M}$ is mapped to the vacuum and inadvertent detection appears to the intended receiver as complete loss. However, the inadvertent receiver may forward any undetected spectral amplitude onwards, in which case the loss due to detection appears only as a frequency-selective absorptive channel. We return to this interpretation in our conclusions.

3.2 Unintentional Interference

The case of unintentional interference is exhibited in Fig. 2, where interference in the communication between a transmitter-receiver pair results from a second transmitter. Interference may occur when the transmitter is misaligned with its intended receiver or the line-of-sight paths of the two transmitter intersect at a receiver, with the effect that a single-photon is unintentionally sent to the receiver.

As for the case of inadvertent detection, we assume the transmitter-receiver pair are synchronized to use the spreading sequence Π_1, while the interfering transmitter operates using Π_2. Then, reception of the spread-spectral state $|\psi_{\Pi_1}\rangle$ is correctly demodulated and detected with the maximum probability P_0. In contrast, the state $|\psi_{\Pi_2}\rangle$ is not despread by the receiver, but instead retains its broad bandwidth. The latter state is then detected with probability $P_{\Pi_1\Pi_2}$, which is much less than the probability in the unspread case. As as result, the use of quantum spread spectrum management techniques reduces errors resulting from interference. The same result applies in the case that the interfering signal is narrowband, since demodulation spreads that spectral state prior to detection.

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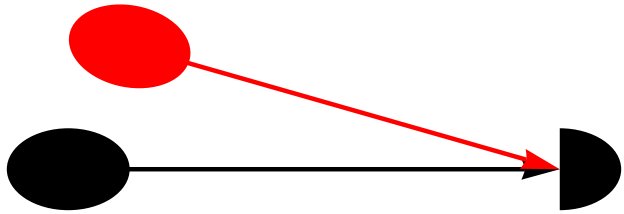

Figure 2. A transmitter unintentionally interferes with a receiver.

3.3 Multi-user Management

The case of multi-user management is depicted in Fig. 3, in which two transmitters intentionally communicate with a single receiver. The present case differs from the unintentional interference of Fig. 2 as now the receiver must differentiate, despread, and detect these asynchronous, potentially interfering transmissions.

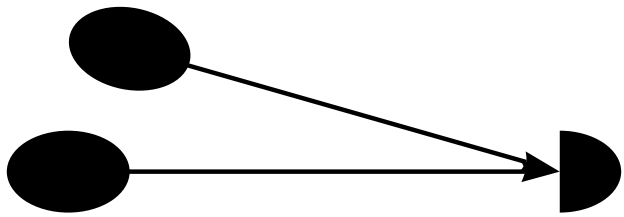

Figure 3. A receiver communicates asynchronously with two transmitters.

Assuming the transmitters, labeled 1 and 2, use the spectral modulations Π_1 and Π_2, respectively, then the receiver must apply the corresponding demodulations to improve reception of the transmitted signals. A receiver that performs this function is shown in Fig. 4, where spread single-photon state enters from the left to be split by a 50:50 beamsplitter. The outputs of the beamsplitter sample the left (L) and right (R) arms of the receiver. Each arm demodulates the state, using either Π_1 or Π_2, prior to detection. It is assumed that each individual transmitter synchronizes the corresponding spreading sequence with the appropriate arm of the receiver.

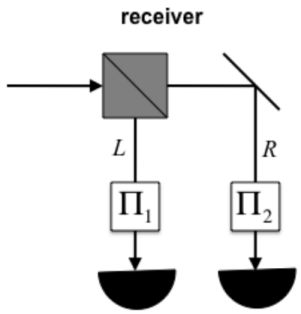

Figure 4. A receiver to communicate asynchronously with two transmitters.

In the case that the incoming spectral state was sent by transmitter 1, then demodulation in the L arm of the receiver despreads the single photon and maximizes the detection probability. In contrast, demodulation of the same spectral state in the R arm produces a negligible probability for detection at the corresponding detector. It is important to note that the beam splitter necessary transmits an equal amount of amplitude to each arm of the receiver (50%). Thus, the maximum probability to detect the despread state is half of P_0; conversely, the probability to make an erroneous detection is also half of $P_{\Pi_2\Pi_1}$. If amplitude outside of the detector bandwidth $\eta(\omega)$ is mapped to the vacuum, then these factors of 0.5 can not be recovered.

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4. CONCLUSIONS

We have presented the development of quantum spread spectrum communication based on the technique of direct-phase modulation of the temporal amplitude of a single photon quantum state. The crucial elements of this presentation include the modulation and spreading of the initially narrowband spectral state, the transmission of the resulting broadband state, the demodulation and despreading of the state by a synchronized receiver, and the subsequent narrowband single-photon detection. Within this framework and under idealized transmission conditions, we have examined the cases of inadvertent detection, unintentional interference, and multi-user management as demonstration of a quantum medium access control protocol. We have found that the probabilities for detection and error to be more favorable relative to the unmanaged transmission of an unspread single-photon state. The reason for these improvements stem from the synchronized spreading and despreading operations performed by a designated transmitter-receiver pair.

Unlike management schemes based on global synchronization, the current MAC protocol limits the synchronization problem to between individual user pairs. This is advantageous from the point of view that quantum communication networks may consist of either ad hoc or oblivious/selfish users that are unwilling or unable to synchronize by global scheduling. The disadvantage is that synchronization must be to within the modulation pulse duration T; for typical pulsed quantum light sources operating in the visible regime, this requirement may translate to picosecond synchronization accuracy.

Our description has avoided complications inherent to noisy and lossy channels (apart from those effects attributed to secondary users). Based on the current results, we anticipate the advantages attributed to quantum spread spectrum communication in the current work should extend to the case of more general channel; this work is ongoing. For the moment, we may draw an analogy with the case of inadvertent detection presented above. In that context, the inadvertent receiver was taken as another user accessing the quantum channel. However, the role of inadvertent detection could also be taken as a narrowband absorptive contaminant occupying the channel, e.g., atmospheric pollutants. For this case, the undetected spectral subspace $1 - \mathcal{M}$ continues onward to the intended receiver. At the receiver, the absorption by the narrowband channel contaminant appears as a frequency-selective loss, but despreading of the spectral state reconstitutes the narrowband spectrum and these losses are averaged out.

Recently, Harris and coworkers have implemented single-photon spread spectrum using synchronized electro-optic modulators driven by a pseudorandom sequence.[4] They demonstrated how to spread and despread a photon and how to mitigate interference from a secondary light source using a narrowband Fabry-Pérot (etalon) filter. Electro-optic modulators represents one means of broadening the spectral state at the transmitter and receiver. Another is suggested in recent work on spread-spectral teleportation, in which a single-photon spectral state is broadened and modulated during frequency up-conversion.[5] Up-conversion of a narrowband photon in a phase-engineered nonlinear optical crystal driven by a phase-modulated pump pulse dilates the spectral quantum state. This form of modulation adapts quantum communication, such as teleportation and entanglement swapping, to be inherently quantum spread spectrum communication.

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