Accelerator Fundamentals Problem Set Monday Week 1

1) A rod of length Δs = 2m is traveling in the s direction with velocity v= 0.6c (c the speed of light). a) How much is the relativistic factor β? b) How much is the relativistic factor γ? c) How much does the length of the rod change in the frame of reference moving with the rod? d) How much does the length of the rod change in the frame of reference of a stationary observer (lab frame)?

2) A pion has a rest energy of E0 = 139.568 MeV and a mean lifetime of τ0=26.029 ns in its rest frame. What are the life-times of a pion accelerated to kinetic energies T1 = 20 MeV and T2 = 100 MeV? Assuming the pions decay exponentially according to exp(-t/τ), at what distance from the source will a population of N pions with kinetic energies T1 and T2 have fallen to 50% of its original value?

3) A proton with longitudinal momentum 500 GeV/c moves through a 10 meter long magnet. The field in the magnet is uniform in the positive vertical direction with strength 5 T. Calculate: a) The force on the proton (magnitude and direction), b) the change in momentum (magnitude and direction), and c) the angular deflection in radians (Hint: $\theta \cong \Delta p/p_s$). What is the radius of curvature of the proton's orbit in the magnet (use Wiedemann Eq. 4.26 and recall that $\theta \approx l/\rho$)?

4) a) Both electric and magnetic fields can be used to bend particles. If you had a 1.5 meter length space available in your beamline, what would be the magnitude of the dipole field required to bend a 1 GeV proton by 10 degrees? What would be the magnitude of the electric field necessary to produce the same amount of bend (Hint: The amount of force needed is the same regardless of whether it comes from an E or a B field)? Based on your answer, why do accelerators usually use magnets and not electric fields to bend high energy beams? (Keep in mind that air begins to spark at 1 kV/cm).
b) Repeat this calculation for a 50 keV beam. Is it feasible to use an electric field for bending at this energy?

5) For a 1 meter long dipole, what is the field necessary to bend a 3 GeV proton by 10 degrees? What about a 3 GeV electron?

6) In the SNS accumulator ring, the necessary bending of the proton beam in the arcs is provided by 32 dipoles with a 17 cm gap and a magnetic length of 1.5 m. If the dipole coils are built with 20 turns, how much should the power supply current be in order to give the necessary field for 1 GeV proton beam operation? What about 1.3 GeV? The rest energy of the proton is 938.273 MeV.

7) Derive the pole profile (aperture radius r = 1cm) for a combined function magnet including a vertical dipole field to produce a radius of ρ=300m, a normal quadrupole field with focusing strength k=0.45 m^{-2}, and a normal sextupole strength of m=23.0 m^{-3}. (Hint: Use the magnetic potential expansion found in lecture, and evaluate the equation at a known, conveniently chosen (x,y) point on the equipotential surface to find the constant potential line.)