Accelerator Fundamentals Problem Set Friday Week 1

1) Consider a quadrupole doublet consisting of two identical thin quadrupoles, the first horizontally defocusing, and the second horizontally focusing, with individual focal lengths |f1|=|f2|=f. The distance between the quads is L.

a) Schematically draw the doublet with the right symbols. In terms of the commonly used acronyms for drifts and quads – O D F – what is this sequence called? What would it be called if we added another drift at the end?

b) Write out the matrix for each element in the sequence, using a thin lens approximation for the quadrupoles. In what order should you multiply these matrices in order to come up with the total transfer matrix?

c) Find the total transfer matrix for the sequence.

d) Show that the doublet is horizontally focusing by showing that a particle entering the doublet which is horizontally displaced but moving parallel to the axis, e.g., with initial phase coordinates ($x=x_o$, $x'=0$), crosses the axis after leaving the doublet. (Hint: It's enough to find the final coordinates, (x, x'), and based on their signs argue that the particle should eventually cross the zero axis).

e) For quadrupoles with focal lengths 2.0 meters, and a distance of L=1 meters between the quadrupoles, find the distance d after the doublet where the particle crosses the axis. (Hint: Transport your result from above through a drift of length d, and then solve for d).

2) In class we found the transfer matrix for a FODO cell with focal lengths that were equal in magnitude.

a) Suppose that instead of drifts between the quadrupoles we have rectangular dipole magnets. Find the sequence FBDB, where B represents the rectangular dipole magnet. Assume that the quadrupoles are short enough to use a thin lens approximation, but the dipoles are not. (Hint: Compare the matrix for a drift of length L with that of a rectangular magnet. You should not have to recalculate any matrices to do this!)

b) If we had sector dipoles instead of drifts, what would be the transfer matrix for the first three elements of the sequence, FBD?

3) The beam ellipse at a certain point s in the accelerator is defined by the Twiss parameter set (β_o, α_o)=(10 m, 0.1 radians).

a) What is the emittance of the beam if the coordinates of a particle at the edge of the beam are x=10 mm, x'=1 mrad?

b) Propagate the Twiss parameters through a horizontally focusing quadrupole with focal length f=10 m (you can use a thin lens approximation).

c) Propagate the particle coordinates through the same quadrupole.

d) Calculate the beam emittance after the quadrupole. Did it change?