Lecture 9

Synchrotron Radiation

USPAS, Nashville
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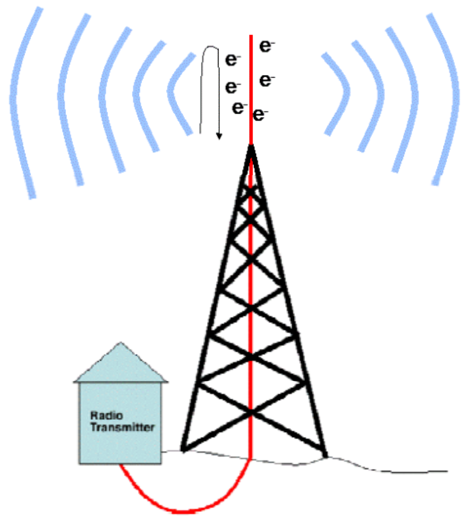
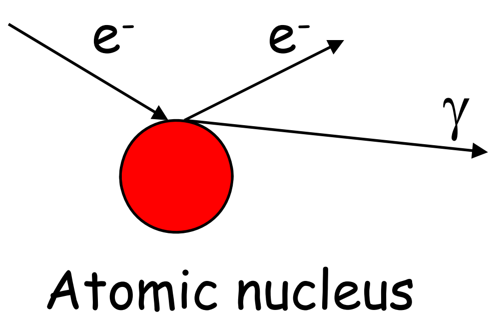
Radiation from an Accelerated Charge

- A charge that is accelerated emits electro-magnetic radiation
- Examples you may be familiar with:

  - EM radiated from an antenna: time-varying current runs up and down the antenna, and in the process emits radio waves

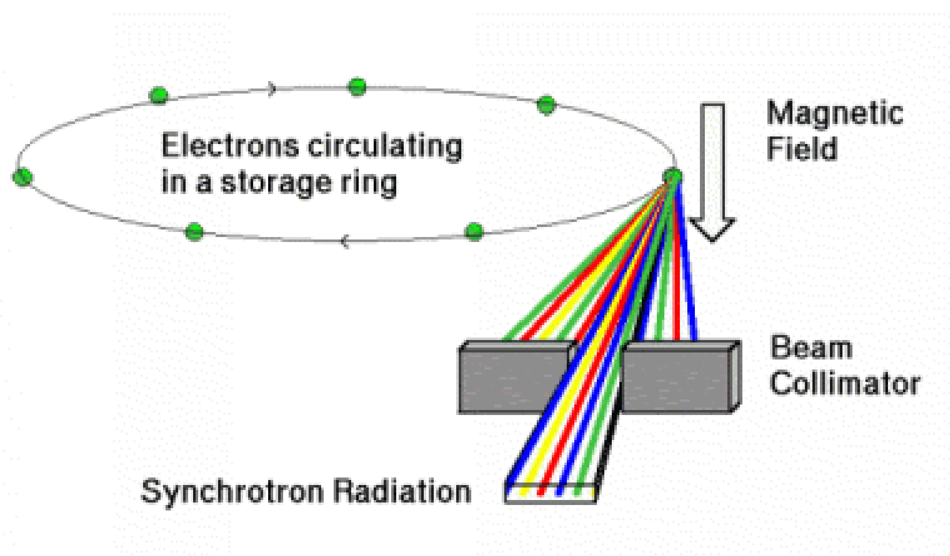


  - Bremsstrahlung: (braking radiation). An electron is accelerated when it collides with an atomic nucles, emitting a photon

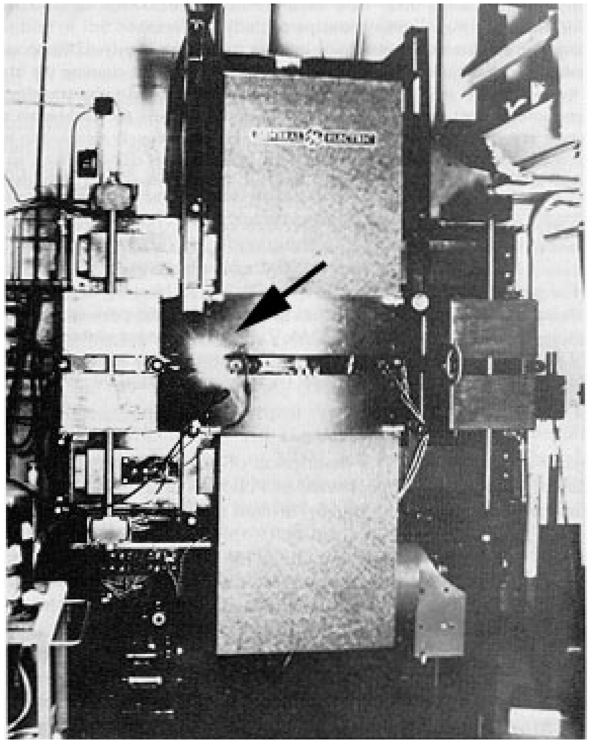


Synchrotron Radiation

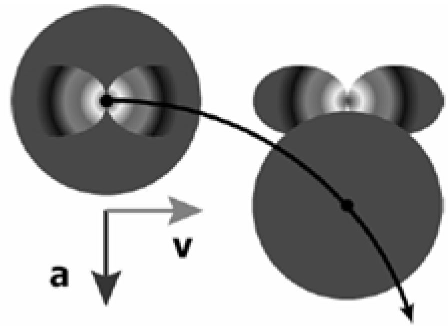
- **Synchrotron radiation is electromagnetic radiation emitted when charged particles are radially accelerated (moved on a circular path).**



•Synchrotron radiation was first observed in an electron synchrotron in 1947: the 70 MeV synchrotron at General Electric Synchrotron in Schenectady, New York

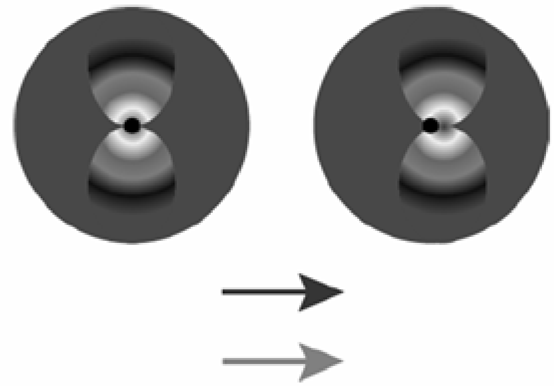


Longitudinal vs. Transverse Acceleration



Radiation field quickly separates itself from the Coulomb field

$$P_\perp = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}\gamma^2\left(\frac{d\mathbf{p}_\perp}{dt}\right)^2$$



Radiation field cannot separate itself from the Coulomb field

$$P_\parallel = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}\left(\frac{d\mathbf{p}_\parallel}{dt}\right)^2$$

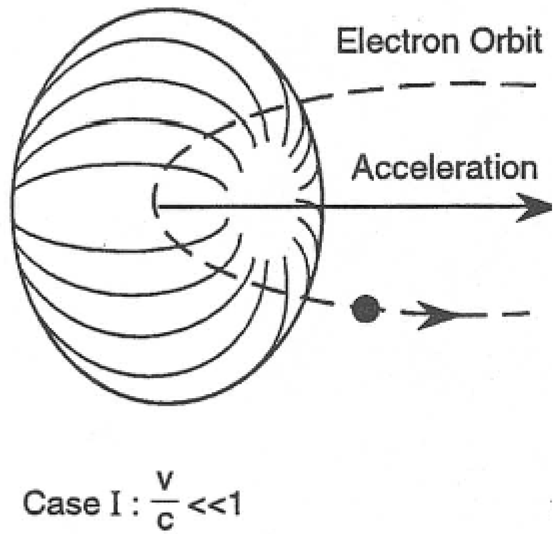
negligible!

$$P_\perp = \frac{c}{6\pi\varepsilon_0} q^2 \frac{(\beta\gamma)^4}{\rho^2} \qquad \rho = curvature \;\; radius$$

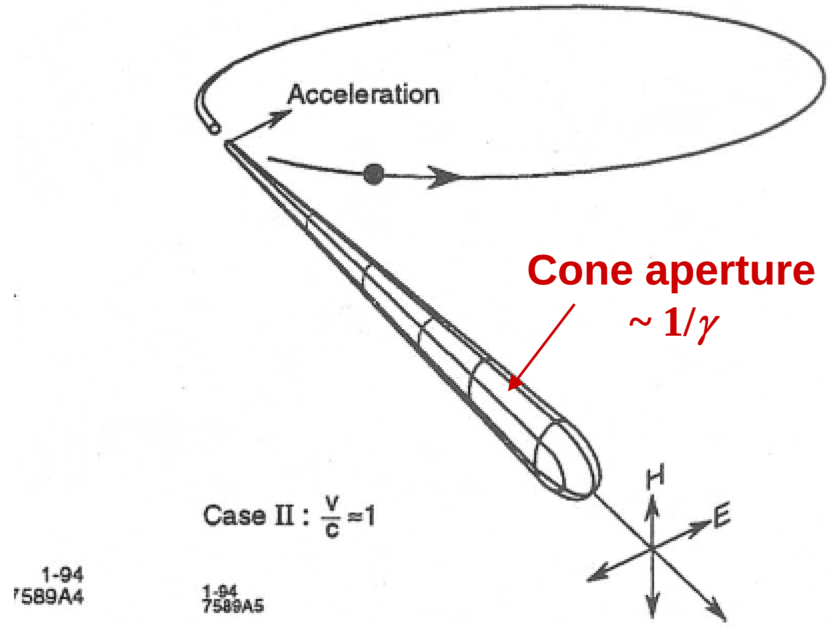
- **Radiated power for transverse acceleration increases dramatically with energy. This sets a practical limit for the maximum energy obtainable with a storage ring, but makes the construction of synchrotron light sources extremely appealing!**

Properties of Synchrotron Radiation: Angular Distribution

- **Radiation becomes more focused at higher energies.**

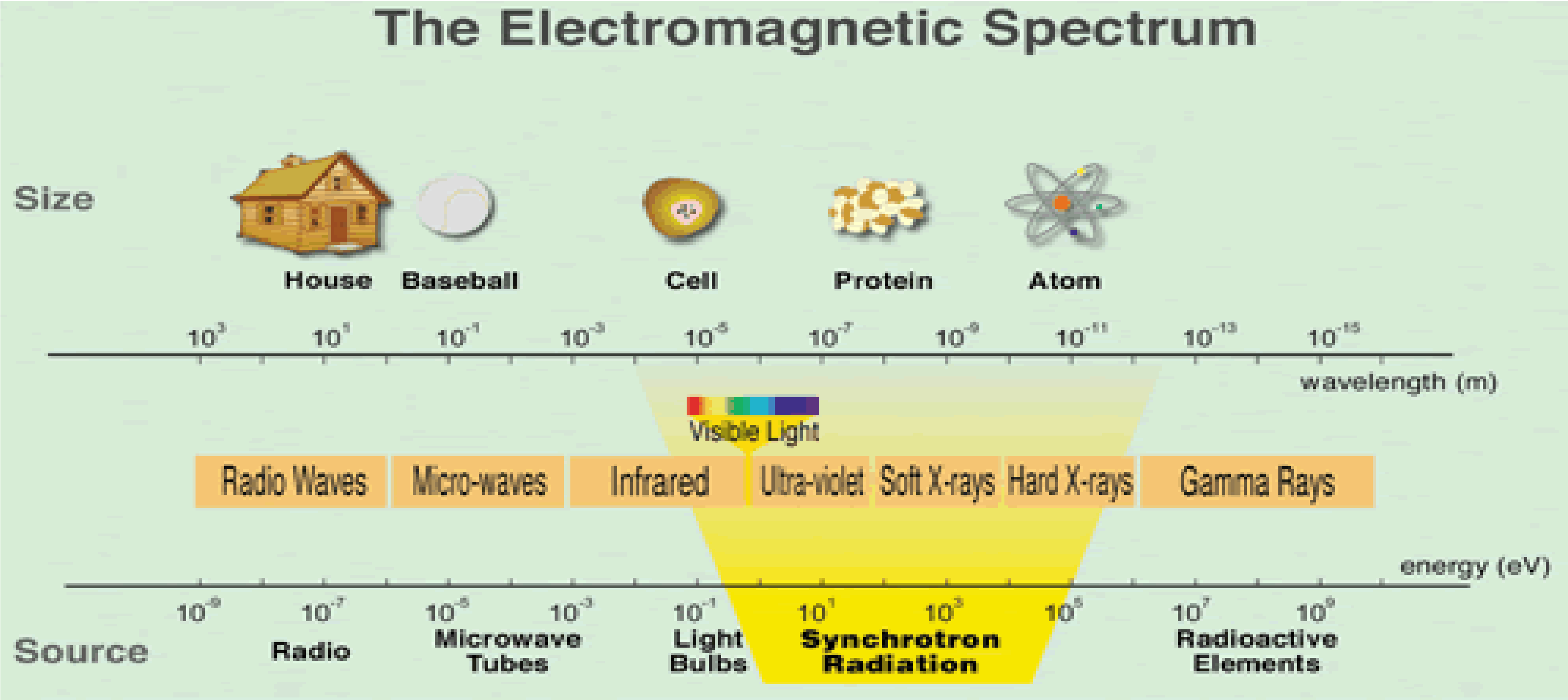


At low electron velocity (non-relativistic case) the radiation is emitted in a non-directional pattern



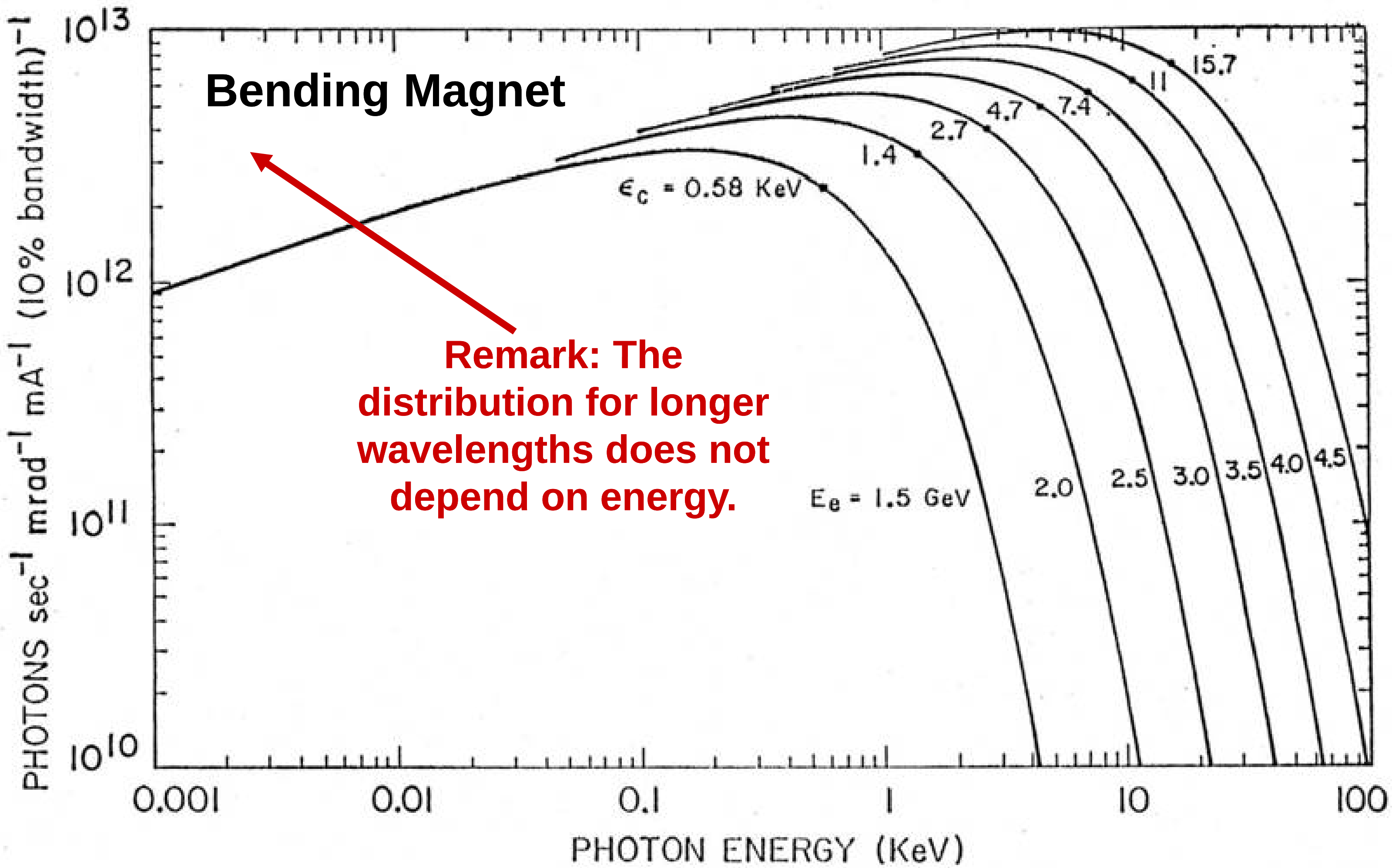
When the electron velocity approaches the velocity of light, the emission pattern is folded sharply forward.

Properties of Synchrotron Radiation: Radiation Spectrum



Synchrotron Radiation Spectrum

- **The Synchrotron radiation spectrum depends on a single parameter, *the critical energy***



Bend Magnet Synchrotron Radiation Spectrum

Spectrum:

$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

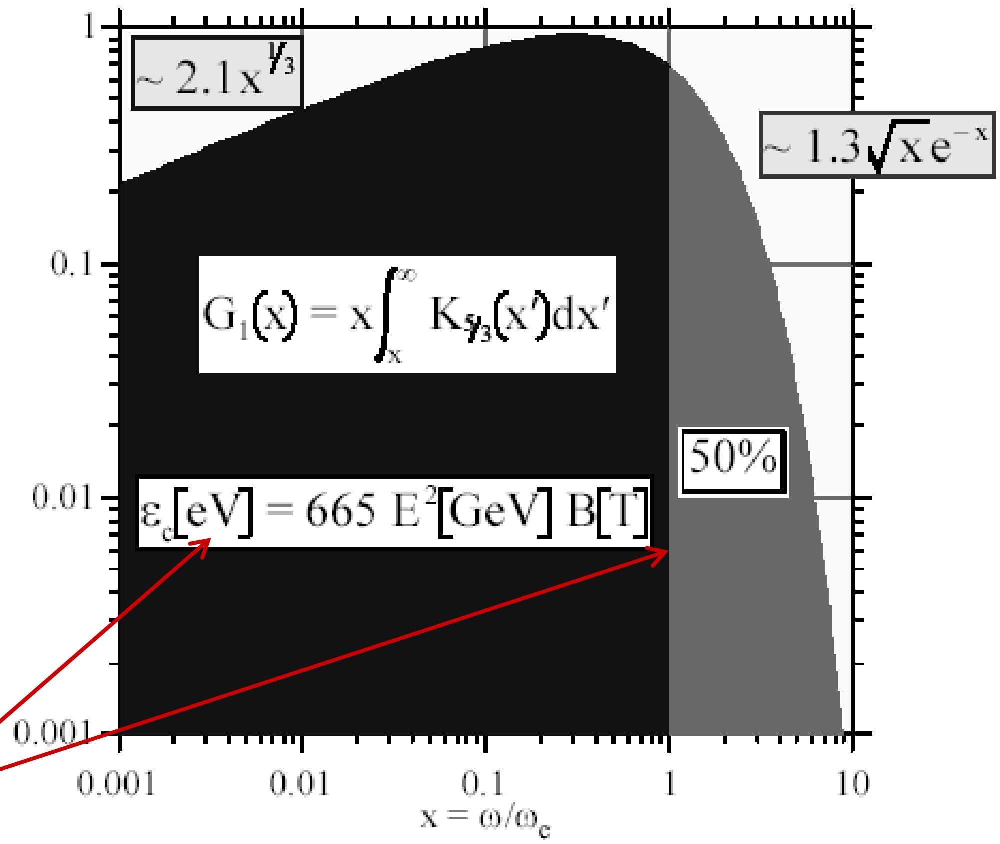
Universal function

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')dx' \qquad \int_0^\infty S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3}\hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$

$$\varepsilon_c[\mathrm{keV}] = 2.22 \frac{E^3[\mathrm{GeV}^3]}{\rho[\mathrm{m}]}$$



Critical frequency

SR Power and Energy loss for Electrons

- Instantaneous Synchrotron Radiation Power for a single electron

$$P_\gamma[\mathrm{GeV/s}] = \frac{cC_\gamma}{2\pi}\frac{E^4[\mathrm{GeV}^4]}{\rho^2[\mathrm{m}^2]}$$

$$C_\gamma = 8.8575\times10^{-5}\,\frac{\mathrm{m}}{\mathrm{GeV}^3}$$

- Energy loss per turn for a single particle in an isomagnetic lattice with bending radius ρ

$$\Delta E[\mathrm{GeV}] = C_\gamma \frac{E^4[\mathrm{GeV}^4]}{\rho[\mathrm{m}]}$$

- Radiated Power

$$P_\gamma[\mathrm{MW}] = 8.8575\times10^{-2}\,\frac{E^4[\mathrm{GeV}^4]}{\rho[\mathrm{m}]}\,I[\mathrm{A}]$$

Examples

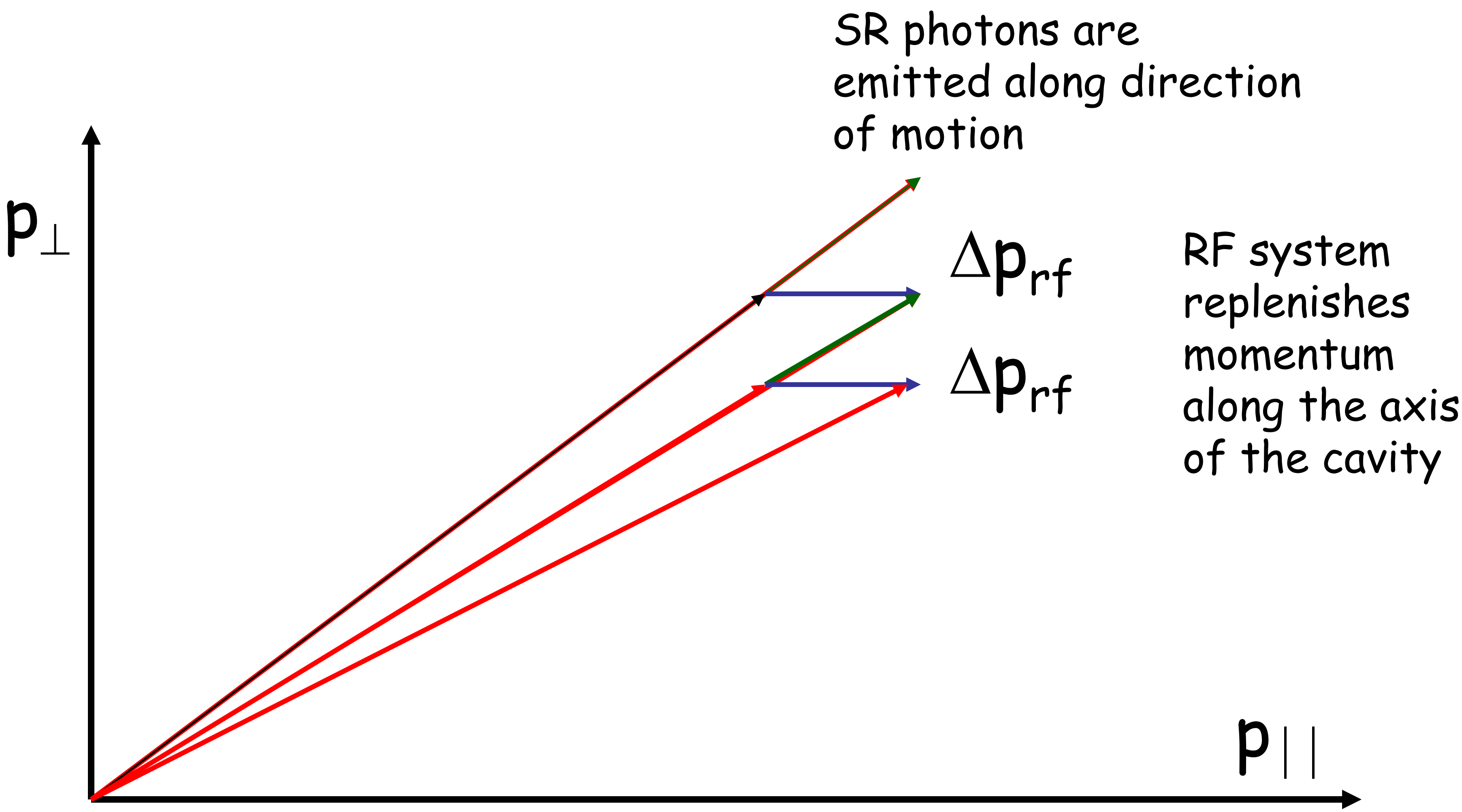
- Calculate SR radiated power for a 100 mA electron beam of 3 GeV in a storage ring with circumference 1 km (typical light source)
- Calculate SR radiated power for a 1 mA electron beam of 100 GeV in a storage ring with circumference 27 km (LEP storage ring)

Circular vs. Linear Electron Accelerators

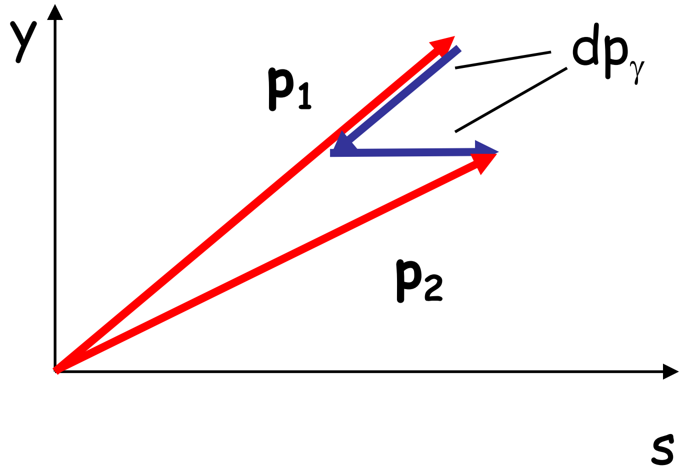
- At high enough electron energy, the radiated synchrotron power becomes impractical.
- Say you want to build the International Linear Collider as a circular collider, using the LEP tunnel
  - E=500 GeV, I=10 mA
- Gives P=13 GW!! This is ten times the power capacity of a commercial nuclear power plant
- Using two linacs avoids the necessity of bending these high energy beams, so synchrotron radiation is nearly eliminated

Consequences of Sychrotron Radiation: Radiation Damping

- Consider betatron motion in the vertical plane



Radiation Damping



$$\vec{p}_2 = \vec{p}_1 - d\vec{p}_\gamma + |dp_\gamma|\hat{s}$$

$$p_{2,\perp} = p_{1,\perp} - |dp_\gamma|\frac{p_{1,\perp}}{|\vec{p}_1|} = p_{1,\perp}\left(1 - dp_\gamma / |\vec{p}_1|\right)$$

$$p_{2,\parallel} = p_{1,\parallel} - |dp_\gamma|\frac{p_{1,\parallel}}{|\vec{p}_\parallel|} + |dp_\gamma| = p_{1,\parallel}\left(1 - dp_\gamma / |\vec{p}_1| + dp_\gamma / p_{1,\parallel}\right)$$

$$y_2' = \frac{p_{2,\perp}}{p_{2,\parallel}} = \frac{p_{1,\perp}}{p_{1,\parallel}} \frac{\left(1 - dE_\gamma / E\right)}{\left(1 - dE_\gamma / E + dE_\gamma / cp_s\right)}$$

$$y_2' \approx y_1'\left(1 - dE_\gamma / E\right)$$

- The rate of change of slope with s is

$$y'' = \frac{dy'}{ds} = \frac{y_2' - y_1'}{ds} = \frac{y_1'\left(1 - dE_\gamma / E\right) - y_1'}{ds}$$

$$y'' = -y' \frac{1}{E}\frac{dE_\gamma}{ds}$$

Radiation Damping

- We see now another new term in the equation of motion, one proportional to the instantaneous slope of the trajectory y′:

$$y'' + y' \frac{1}{E}\frac{dE_\gamma}{ds} + ky = 0$$

- This looks like the damped harmonic oscillator equation from classical mechanics:

$$m\ddot{x} + b\dot{x} + kx = 0$$

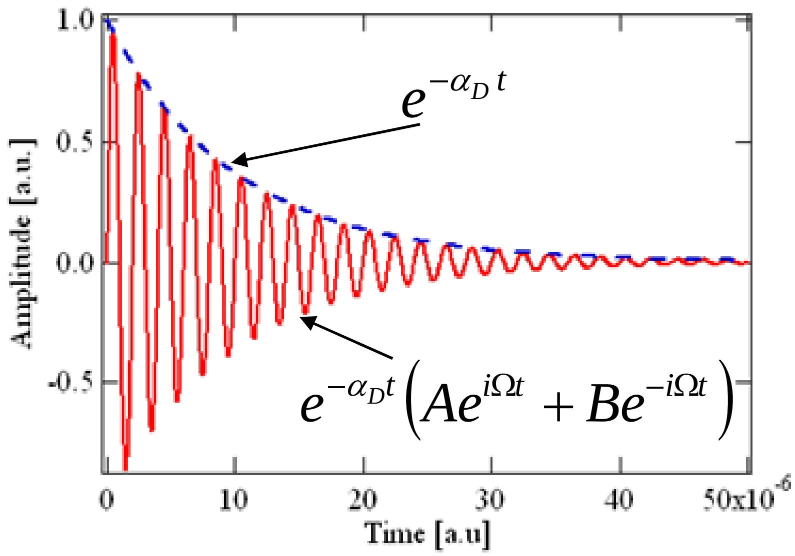
- Which is often written like this

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 = 0$$

- With

$$\alpha = \frac{b}{2m}$$

- The solution is a damped harmonic oscillator



$$x = Ae^{-\alpha t}\cos(\omega_1 t + \phi_0) \qquad \omega_1 = \sqrt{\omega_0^2 - \alpha^2}$$

Radiation Damping

- The resulting betatron motion is damped in time.
- The damping term we derived is in units of m^{-1}. We need the damping rate in sec^{-1}. They are related by velocity: $\alpha[sec^{-1}] = c\beta\ \alpha[m^{-1}]$

$$\alpha = \frac{c\beta}{2E}\frac{dE_\gamma}{ds} = \frac{c\beta}{2E}\frac{dE_\gamma}{c\beta dt} = \frac{1}{2E}\left\langle P_\gamma \right\rangle$$

$$\alpha = \frac{1}{\tau_y} = \frac{1}{2\tau_0}$$

- Where we have defined

$$\tau_0 = \frac{E}{\left\langle P_\gamma \right\rangle}$$

- This is the damping time for vertical betatron oscillations
- Motion in the horizontal and longitudinal planes are damped also, but their derivation is more complex.
- The damping rates are:

$$\alpha_y = \frac{1}{2\tau_0} = \frac{1}{2\tau_0} J_y$$

$$\alpha_x = \frac{1}{2\tau_0}(1-\vartheta) = \frac{1}{2\tau_0} J_x$$

$$\alpha_z = \frac{1}{2\tau_0}(2+\vartheta) = \frac{1}{2\tau_0} J_z$$

- And they are related by *Robinson's damping criterion*

$$\sum_i J_i = 4$$

Radiation Damping

- The damping partition numbers depend on the lattice properties according to

$$\vartheta = \frac{\oint \frac{\eta}{\rho^3}\left(1+2\rho^2 k\right)ds}{\oint \frac{ds}{\rho^2}}$$

- Which, for an isomagnetic lattice (constant bending radius) gives

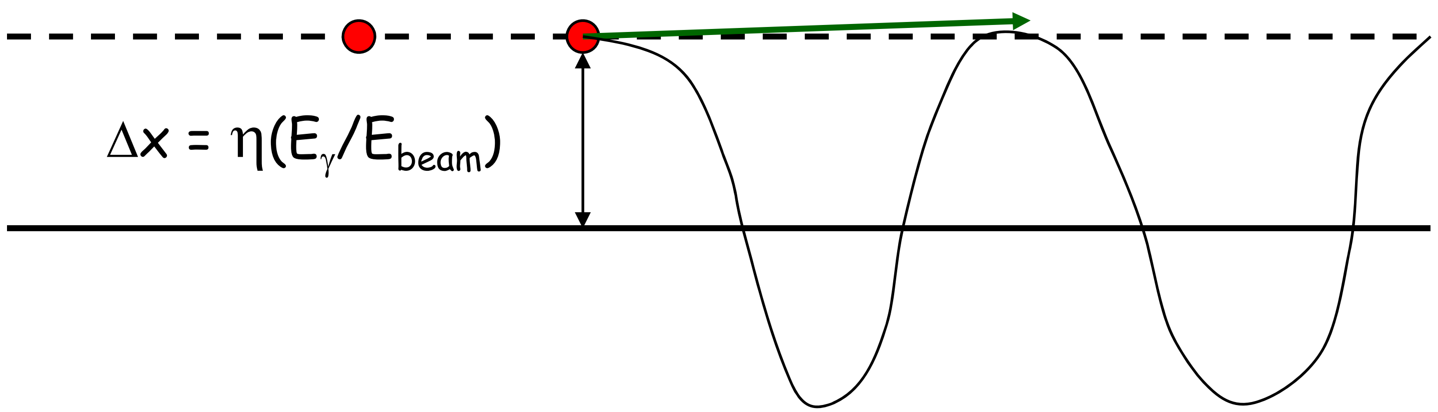
$$\vartheta_{iso} = \frac{\alpha_c L}{2\pi\rho}$$

- You might imagine that oscillations in the beam would eventually be damped to zero, collapsing the beam to a single point in phase space. Is this possible?

Consequences of Synchrotron Radiation: Quantum Excitation

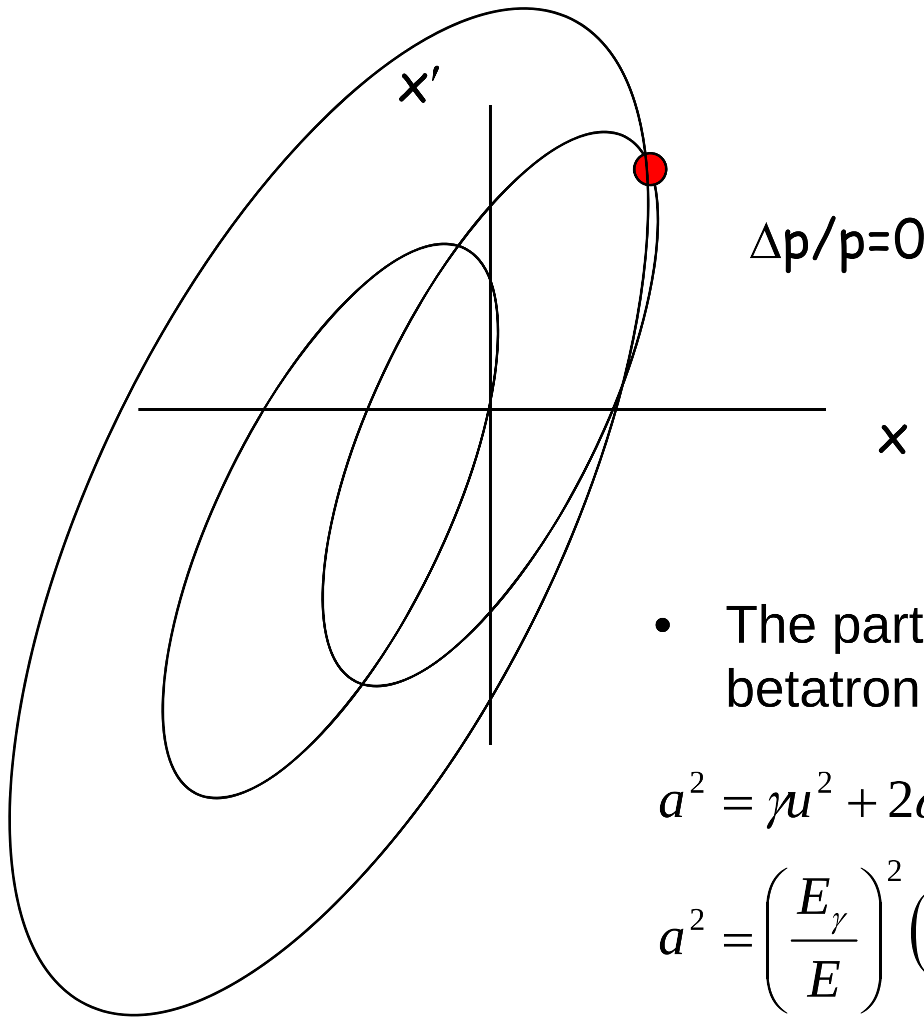
- Eventually, the individual beam particles become excited by the emission of synchrotron radiation, a process known as quantum excitation
- After emission of a SR photon, the particle finds itself displaced from its new closed orbit
- The particles position and angle in real-space do not change, but it acquires a betatron amplitude about a new closed orbit given by:

$$u_\beta = u_{\beta 0} + \eta \frac{E_\gamma}{E_0} \qquad u'_\beta = u'_{\beta 0} + \eta' \frac{E_\gamma}{E_0}$$



Quantum Excitation

- The particle oscillates at a larger betatron amplitude after emission of a SR photon



- The particle's new betatron amplitude is:

$$a^2 = \gamma u^2 + 2\alpha u u' + \beta u'^2$$

$$a^2 = \left(\frac{E_\gamma}{E}\right)^2 \left(\gamma \eta^2 + 2\alpha \eta \eta' + \beta \eta'^2\right)$$

Equilibrium Beam Parameters

- The beamsize in an accelerator where synchrotron radiation is important eventually reaches emittance values in all three planes that are *an equilibrium between radiation damping and quantum excitation*
- The equilibrium beam energy spread in an electron storage ring depends only on the beam energy and bending radius

$$\frac{\sigma_\varepsilon^2}{E^2} = C_q \frac{\gamma^2}{J_z} \frac{\langle 1/\rho^3 \rangle}{\langle 1/\rho^2 \rangle} \qquad C_q = 3.84 \times 10^{-13} \mathrm{m}$$

- The transverse beamsizes are given by

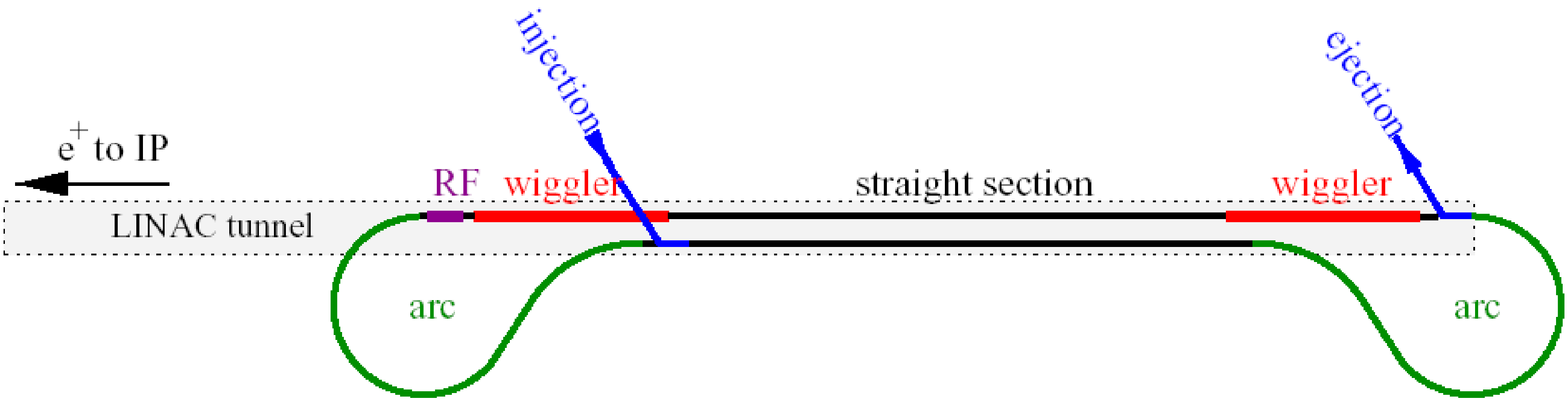
$$\varepsilon_u = \frac{\sigma_u^2}{\beta_u} = C_q \frac{\gamma^2}{J_u} \frac{\langle \mathcal{H} / \rho^3 \rangle}{\langle 1/\rho^2 \rangle}$$

$$\mathcal{H}(s) = \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2$$

- For the vertical plane, dispersion and therefore H are zero. Does the vertical emittance shrink to zero?
- No: the vertical beamsize is theoretically limited by $1/\gamma$ angular emission of synchrotron radiation. In practice it is limited by more mundane issues like orbit errors

Damping Ring

- A Damping Ring has parameters tuned to minimize quantum excitation while providing damping, so that the equilibrium emittance can be reduced.
- This can be accomplished by producing more synchrotron radiation with strong bending fields (wiggler magnets) placed in dispersion-free straight sections



International Linear Collider Damping Ring

Colliders and Luminosity

- Two beams of opposite charge counter-rotating in a storage ring follow the same trajectories and have the same focusing
- The beams collide and produce particle reactions with a rate given by

$$R = \sigma_{physics}\mathcal{L}$$

- where

$$\mathcal{L} = f_{rev}\frac{N_1 N_2}{Area} = f_{rev}\frac{N_1 N_2}{4\pi\sigma_x\sigma_y}$$

- Beamsizes are reduced by special quadrupole configurations “low-beta” to reduce the beamsizes at the collision points