Assignments and Notes for Fundamental Accelerator Theory, Simulations and Measurements

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Accelerator Fundamentals
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Laboratory Assignment #1
Magnetic Measurement and Analysis of a Dipole Magnet

In this laboratory you will perform magnetic field measurements on a small dipole magnet. A power supply, two digital multimeters, cables and a Hall probe are supplied. You will need to connect these components together to be able to measure the total magnet coil current and voltage using the multimeters. Make sure you put the power supply in **current** control mode so the current does not change as the coils heat up (voltage knob turned up to maximum, then control current using the current knob). The magnet has two different types of return plates and pole pieces you will use to do the measurements. This experiment requires mechanical assembly and disassembly. Please be **VERY** careful when doing the assembly/dissassembly so that you do not damage the coils. They can easily be damaged and develop shorts across coil turns. Be careful not to over tighten the screws so the threads do not get stripped. Use MS Excel to plot and fit the data as well as calculate the field integrals.

1. Make a sketch of the mechanical dimensions of the magnet and all pole pieces/return plates for use later in POISSON magnetic field calculations (PC Assignment #1).

2. Sketch your electrical setup including magnet, coils, multimeter and power supply. Make sure you connect the coils in **series** with the power supply/multimeter circuit.

3. Assemble the dipole magnet with **thick** return plates and **wide** pole pieces.

   (a) Measure the magnetic field in the center of the magnet ($B_y(x = 0, y = 0, z = 0)$) and voltage across the coils as a function of current up to 0.5 amperes or so below the maximum current the supply can provide. Derive the power required at each current.

   (b) Plot the magnetic field in the center of the magnet ($B_y(x = 0, y = 0, z = 0)$) vs current and fit a line to the data. Derive the number of turns per coil from the slope of the fit.

   (c) Plot the power required vs magnetic field. What would the power be at each **magnetic** field point if the coils had twice the number of turns and half the copper cross sectional area? Assume the magnet is not saturated.

   (d) Measure the magnetic field distribution transversely across the gap (ie. $B_y(x, y = 0, z = 0)$)) at the highest current measured in part 3(a). What is the range in mm where the field is good (ie. varies by no more than 0.5%)? Fit the transverse field data to sixth order in transverse position as measured from the center of the magnet where $x = 0$ (ie. derive the fit coefficients a_n of the fit function

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$(B_y = \sum_{n=0}^{6} a_n x^n)$. Note: you should take more data points where the field starts to change so you get a good fit. Which fit coefficients dominate and why? To get a good fit you may want to only include field data that deviates by no more than 5 to 10 % from the central field.

(e) Measure the magnetic field distribution axially from far outside the magnet to the center of the magnet (ie. $B_y(x = 0, y = 0, z)$) highest current used in part 3(a). From this data, find the field integral for the magnet and the effective length for the magnet. You should take more data points where the field changes rapidly to accurately map out the field. Calculate the total deflection angle for a 30 MeV beam passing through this magnet. Make a sketch of the beam trajectory through the magnet showing entrance/exit angles etc. for your calculation.

4. Assemble the magnet with **thick** return plates and **narrow** pole pieces.

(a) Measure the magnetic field in the center of the magnet ($B_y(x = 0, y = 0, z = 0)$) as a function of current up to the highest current measured in part 3(a). Plot and fit the data as in part 3(b). Do you get the same slope as in part 3(b)? If so why if not why not?

(b) Measure the magnetic field distribution transversely across the gap (ie. $B_y(x, y = 0, z = 0)$) at the highest current used in part 3(a). What is the range in mm where the field is good (ie. varies by no more than 0.5%)? Is this range larger or smaller than that found in part 3(d)? What determines the good field range? Fit the transverse field data to sixth order in transverse position as measured from the center of the magnet where $x = 0$ (ie. derive the fit coefficients a_n of the fit function ($B_y = \sum_{n=0}^{6} a_n x^n$). Note: you should take more data points where the field starts to change so you get a good fit. Which fit coefficients dominate and why? To get a good fit you may want to only include field data that deviates by no more than 5 to 10 % from the central field. Compare the fit coeffients derived in this part with those derived in part 3(d).

5. Assemble the magnet with **thin** return plates **wide** pole pieces.

(a) Measure the magnetic field in the center of the magnet ($B_y(x = 0, y = 0, z = 0)$) as a function of current up to the highest current measured in part 3(a). Plot and fit magnetic field in the center of the magnet vs current as in part 3(b). Why is this data different than the data measured in part 3(b)? Using the number of turns/coil derived in part 3(b), derive a value for the part of the Ampere's Law contour integral outside the gap as a function of current. Explain why this part of the contour integral becomes larger with current.

(b) Measure the magnetic field distribution transversely across the gap (ie. $B_y(x, y = 0, z = 0)$) at the highest current measured in part 3(a). What is the range in mm where the field is good (ie. varies by no more than 0.5%)? Is this range larger or smaller than that in part 3(d)? Why?

(c) Measure the magnetic field distribution axially from far outside the magnet to the center of the magnet (ie. $B_y(x = 0, y = 0, z)$) at the highest current used in part 3(a). From this data, find the field integral for the complete magnet and the effective length for the magnet. Are the field integral and effective length larger or smaller that those found in part 3(e)? Why?

Laboratory Assignment #2
Magnetic Measurement and Analysis of a Quadrupole Magnet

In this assignment you will measure the magnetic field in a quadrupole magnet. A power supply, digital voltmeter, cables and a Hall probe mounted on x-z-movers are supplied. You will connect these components together to allow you to measure the magnet current with the multimeter. There are four coils, which must be connected in a certain way to make a quadrupole field. Make a sketch of the electrical wiring with power supply, multimeter to measure the current and coils.

1. Sketch your electrical setup including magnet, coils, multimeter and power supply. Pay particular attention to the polarity of each pole piece in your sketch. The mechanical dimensions of this magnet are supplied in a handout. You will need them for use in POISSON magnetic field calculations (PC Assignment #2).

2. With the quadrupole coils wired in series, measure the vertical magnetic field at $x = 1.0$ cm ($B_y(x = 1.0$ cm$, y = 0, z = 0)$) as a function of current up to 0.5 amperes or so below the maximum current the supply can provide. Using the slope of a linear fit to this data, derive the number of turns/coil.

3. Measure the vertical magnetic field ($B_y(x, y = 0, z = 0)$) as a function of transverse position x at the maximum current used part 2.

   (a) Fit the transverse field data to seventh order in transverse position as measured from the center of the magnet where $x = 0$ (ie. derive the fit coefficients a_n of the fit function ($B_y = \sum_{n=0}^{7} a_n x^n$). Note: you should take more data points where the field gradient ($g = a_1$) starts to change so you get a good fit. Which fit coefficients dominate and why? To get a good fit you may want to only include field data where the gradient deviates by no more than 5 to 10 % from the central gradient at $x = 0$ cm.

   (b) Determine the "good field region" from your fit in part 3(a) where the gradient does not vary by more than +/- 0.5%.

4. Measure the vertical magnetic field ($B_y(x = 1.0$ cm$, y = 0, z)$) as a function of axial position z for the maximum current used in part 2.

   (a) Determine the effective length of the quadrupole from the data.

   (b) Compare the effective length calculated in part 4(a) with the sum of the physical length of the quadrupole and the radius from the center of the quadrupole to the pole tip. How close are these two measurements? (ie. 1%, 5% etc).

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5. Do you have a horizontally focussing quadrupole for an electron beam? If not, how do you make it horizontally focussing?

6. Calculate the focal length of this quadrupole for an excitation current of 5 A and an electron beam with energy of 30 MeV.

Laboratory Assignment #3
Magnetic Measurement and Analysis of a Wiggler Magnet

In this laboratory you will perform magnetic field measurements on a small wiggler magnet. The wiggler magnet is made up of permanent magnet pieces. A Hall probe is supplied.

1. Set the wiggler gap height to 4 cm.

   (a) Measure the magnetic field in the center of the gap axially (z axis $B_y(x = 0, y = 0, z)$). Use small enough position steps to determine the field accurately. Note: there is a broken magnet on one side of the wiggler so the field will not be good there.

   (b) Plot the data and determine the magnetic field period length λ_p.

   (c) Determine the field integral over the data measured in part 1(a) $\int B_y(x = 0, y = 0, z)dz$. Is this field integral equal to zero? Explain how there will be a small angular deviation of the wiggler exit trajectory relative to the entrance trajectory when the field integral is not zero. For your explanation make a sketch of the electron trajectory using a wiggler with only one complete period.

   (d) using the field integral for this wiggler, estimate the angular deviation of the exit trajectory from the entrance trajectory for a gap of 4.0 cm gap and an electron energy of 7 GeV. This is one source of dipole errors in a storage ring light source.

2. With the wiggler gap height at 4 cm, position the Hall probe 1 cm above the midplane (ie. $B_y(x = 0, y = 1$ cm$, z)$).

   (a) Measure the magnetic field 1 cm above the gap midplane axially (z axis) $B_y(x = 0, y = 1$ cm$, z)$). Use small enough position steps to determine the field accurately.

   (b) Plot the data and determine the magnetic field period length λ_p for the field 1 cm above the gap midplane.

   (c) Plot the field measured in part 1(a) and 2(a) on the same graph. Explain why the field distributions are different.

   (d) Determine the field integral for the data measured in part 2(a) $\int B_y(x = 0, y = 0, z)dz$). Is this field integral equal to zero?

   (e) using the field integral determined in part 2(d), estimate the angular deviation of the exit trajectory from the entrance trajectory for a gap of 4.0 cm gap and an electron energy of 7 GeV. Compare this result with that found in part 1(d).

3. Position the Hall probe in the center of the gap on the midplane $B_y(x = 0, y = 0, z)$.

   (a) Repeat the measurement in part 1(a) for 3 or 4 different gap apertures greater than 4 cm.

   (b) For each gap aperture, determine the period length λ_p and the maximum magnetic field.

   (c) Plot the maximum magnetic field as a function of "normalized" gap aperture $x = g/\lambda_p$. Fit this data with the fit function $B_y = B_o e^{-x(b-cx)}$, $x = g/\lambda_p$, where and g is the gap height and λ_p is the period length. Compare the fit coefficients B_o, b and c with the typical undulator values of $B_o = 3.2$ T, $b = 5.28$ and $c = 1.67$.

4. What phase relationship does the electron trajectory have relative to the magnetic field in the midplane? Assume the electron moves in the midplane (make a sketch of the particle trajectory and magnetic field on the same graph).

Laboratory Assignment #4
Measurements and Analysis of simple Microwave Cavities

In this experiment you will perform measurements on two rf-cavities. The first one is a pill-box cavity (some might call it a cookie box). The second cavity is a coaxial cavity with a tuning stub. Properties of a pill-box cavity can be calculated easily with the attached formulas. You can also simulate these cavities with the program URMEL or SuperFish (maybe you did already). Properties of the coaxial cavity are similar to those of a pill-box cavity.

1. Measure mechanical dimensions of the large pill-box cavity for computer simulations if you did not do this already. A small pill-box cavity is provided for comparison but measurements in this assignment should be done using the large cavity. Estimate the lowest order resonant frequency (TM_{010} mode) from the dimensions and the formulas given. Sketch the electrical and magnetic field configuration of this mode.

2. Open the pill box and observe the installed antennas. One is a loop and the other a straight antenna. What fields do you excite with each of the antennas.

3. Connect the pill box cavity to the network analyzer and find the exact TM_{010} mode frequency.

4. Display the resonant curve and measure the full width half maximum, FWHM and determine the quality factor Q.

5. Detune the cavity,

   (a) lightly squeeze the cylindrical part of the cavity and observe the movement of the TM_{010} mode frequency. Does the frequency get lower or higher? Why?

   (b) squeeze the end plates and observe movement of the TM_{010} mode resonant frequency. Which "squeeze" is more effective and why?

6. Cavities can resonate not only in the TM_{010} mode, but also in TM modes which are characterized by different field patterns. From the dimensions and the formula for the frequency of TM modes (or the attached "mode graph") identify four or five other TM modes, make a rough sketch of their field pattern, and try to find their resonant curve on the network analyzer. We ignore TE modes, because we need longitudinal, not transverse, electric fields for acceleration.

7. Now put two Ferrite pieces ($\mu_r \gg 1$) flat on the bottom around the axis of the cavity. Measure the shift in resonant frequency for the TM_{010} mode and the TM_{011} mode. Do the same with four Ferrite pieces arranged in a circle closer to the edge of the pill box. Explain the magnitude of the frequency shift in either case in terms of the change in electric or magnetic field energy (Hint: think about the relative strength of electric and magnetic fields at different radial positions in the cavity).

8. A dielectric also changes the resonant frequency. Put a small vial of water, piece of plastic or pencil eraser in the center of the cavity and observe the shift in frequency of the TM_{010} and TM_{011} modes. Make sure the dielectric does not distort the mode shape as measured on the network analyzer (if it does distort the mode shape the dielectric is no longer a perturbation to the fields). Measure the frequency shift as a function of radial position of the vial and plot frequency shift versus radial position of the dielectric. This is a crude electric field measurement. The frequency shift is proportional to the square of the electric field at the location of the dielectric.

9. The second cavity is a coaxial cavity. The cavity dimensions are: outer tube length is 20.00 cm, inner tube length is 19.00 cm, inner tube outer diameter is 2.16 cm and outer tube inner diameter is 7.30 cm. Check as many of these dimensions as possible with a ruler. The walls are probably around 0.125" thick. There is a tuning stub to tune the resonant frequency of the cavity to an exact frequency.

   (a) Calibrate the tuning stub by measuring and plotting the frequency as a function of the tuning stub position.

   (b) Adjust the tuning stub to a frequency of 358.0 MHz. Identify 8 to 10 higher frequency TM modes using the network analyzer. Which of the frequencies are modes and which are harmonics? Hint: Different harmonics of the same mode have different values of the integer p which specifies the axial variation of the fields but the same m and n. Modes are specified by m and n which define the azimuthal and radial variation of the fields. You can assume the frequency of modes in the coaxial cavity has the same functional form as the pillbox cavity.

Laboratory Assignment #5
Beam Current and Position Monitor Measurements and Analysis

To measure a pulsed beam current we use a transformer type **current monitor**. It consists of a vacuum tube with an insulating ceramic piece, a ferrite ring and two coils wound around the ferrite ring. The beam functions as the primary coil of the transformer. Sending a pulsed signal from a signal generator through the current monitor (coax cable where the beam would be) generates a signal in the secondary coils which is observed with on an oscilloscope.

1. Open the brass shield of the current monitor and inspect the inside of the monitor. Note all relevant details then close the brass shield. To get clean signals it is important to close the brass shield. You can do a measurment while the shield is still open and then close it for comparison. The shield is there to guide the image currents back to the signal generator or in case of a real beam to guide the image currents from one side of the ceramic break to the other.

2. Derive a formula for the beam current as a function of the voltage measured on the scope. See the following page for hints on deriving your formula. Should you use the 1-turn or 8-turn secondary coil?

3. From the signal generator send various amplitude (square) current pulses through the monitor and measure the signal from each secondary coil on the scope. Derive the calibration constant k in Amperes/Volt (or mA/mV) for each secondary coil (make sure you terminate the unused secondary coil in 50 Ohms).

4. Measure the decay time constant on the scope for each secondary coil. Derive the inductance of each secondary coil from the measured time constant. Are the inductances in the ratio 8:1? Are the measured time constants in the ratio 8:1?

5. In a real current monitor in an accelerator you could not send a pulse from a signal generator through the aperture of the current monitor. To calibrate the monitor anyway, you use the 1-turn coil on the ferrite to simulate the beam current and measure the signal on the 8-turn coil. Send various amplitude current pulses through the 1-turn coil and measure the voltage induced on the 8-turn coil. Do you get the same calibration constant k in Volts/Ampere as in part 3? Is this measurement setup equivalent with the measurment in part 3?

A **beam position monitor (BPM)** is based on the signal a passing beam induces on an electrode. The amplitude of this signal, S, depends on the distance to the beam and scales

like $S \propto 1/r$ (for r not too small). There are two pairs of electrodes in the position monitor. We may use two opposite electrodes and measure the difference of the electrode signal $\Delta S \propto 1/r_{\text{left}} - 1/r_{\text{right}} = 1/(R-\delta) - 1/(R+\delta) \approx \frac{2\delta}{R}$, where R is the monitor aperture radius and δ the distance of the beam center from the monitor center. To normalize the signal one measures the beam position P by evaluating $P \equiv \frac{\Delta S}{\Sigma S} = A\frac{1/r_{\text{left}} - 1/r_{\text{right}}}{1/r_{\text{left}} + 1/r_{\text{right}}} = A\frac{2\delta/R}{2/R} = A\delta$, where A is the calibration factor.

1. Take the position monitor apart and note the interior components. No measurements need to be taken. The monitor interior is just a coaxial line matched to 50 Ohm to allow the transmission of a fast signal pulse through the monitor. Put the monitor together again.

2. Calibrate the position monitor by observing and measuring the signals on the scope as a function of the position of the central conductor. This is difficult to do accurately but you should be able to determine the calibration factor A by measuring the difference over sum signal P for three or four values of δ.

Current monitor circuit

Here are some useful hints for writing a circuit equation for the current monitor.

Figure 1 shows the cross section of the current monitor assembly. The return current flows the opposite way along the outside surface. The surface current K on the outside is approximately

$$K = \frac{i_b}{2\pi r}, \tag{1}$$

giving an azimuthal magnetic field H in the cavity of about

$$H = \frac{4\pi}{c}K = \frac{2i_b}{cr}. \tag{2}$$

or

$$H = \mu_0 K = \frac{\mu_0 i_b}{2\pi r} \tag{3}$$

in MKS units. The exact H will vary with time and position in the cavity.

The magnetic field in the ferrite follows $B = \mu H$ assuming that both H and B are azimuthal.

The toroid behaves as an infinite solenoid. The field produced by a coil around the toroid (of radius r) is

$$B = \frac{\mu\mu_0 NI}{2\pi r}, \tag{4}$$

where N is the number of turns, I is the current, and μ is the susceptibility of the toroid material at the particular H field present.
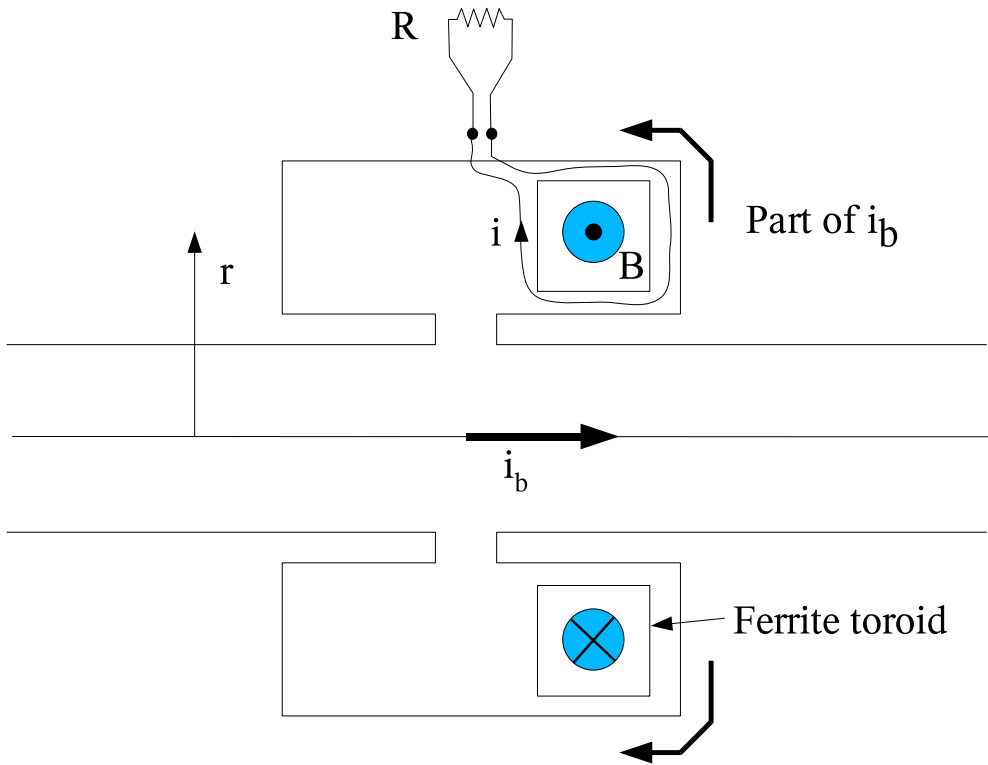
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Figure 1: Schematic cross section of the current monitor.

In Figure 1, i is the reaction current driven by the voltage generated from the changing flux. The direction of i is drawn so that i is positive for a positive i_b. Express B in terms of i_b and i. Write equation for the i current loop with the derivative of the magnetic flux, assuming a toroid crosssection area A. Find the expression for the inductance of the toroid signal coil.

You should finally get a first order differential equation for i with i_b driving terms. Solve for a square pulse of i_b.

PC-Assignment #1
Magnetic Field Calculations for a Dipole Magnet Using POISSON

A PC computer program (POISSON)) is available to simulate magnetic fields. The program is largely self-guiding, but we've included some tips in the section "Using POISSON." Start with an input file for a magnet of the same type and modify profiles/coils as desired.

If you have not yet done so, go to the dipole measurement location and measure relevant geometrical dimensions of the dipole. Do the geometry measurements together with the group that is doing dipole magnetic measurements. This laboratory is meant to be done along with Laboratory Assignment #1 so the student can compare the field calculations with actual measurements.

1. Calculate the dipole magnetic field at a coil current equal to one used (or will use) in Laboratory Assignment #1 (Dipole magnet measurements...read this assignment if you have not already done so). Use a current above 5 Amperes. Do this for **thin/wide, thick/narrow, thick/wide** return flux plates and pole width combinations respectively.

2. Print out the results from part 1 and compare the central field, (B_y at $x = y = z = 0$ cm) field distributions (B_y vs. x) with measurement data from Laboratory Assignment #1.

3. use a **thick** and then the **thin** return yoke with the **wide** poles. Run the program in both cases for increasing currents ($0 < I < 10$ A) and determine the excitation curve and saturation. For this, you increase the current for each run and note down only the resulting field in the center of the magnet $[B_y(x = 0, y = 0, z = 0)]$. As you increase the current, note the fields in the iron at the locations where the field contours are the most dense. These are regions where saturation will occur first. (Saturation means the field no longer increases linearly with current.) If saturation becomes too large, the program may not converge anymore. Where in the magnet does the most severe saturation occur? Why? Do you have any suggestion to improve the magnet design if you had to build it again?

4. Plot both excitation curves $[B_y(x = 0, z = 0)$ vs $I_{coil}]$ on the same graph along with the measurement data from Laboratory Assignment #1.

5. Can you figure out a way to model the longitudinal field distribution (B_y vs. z) ? (Hint: look at the magnet at 90 deg with respect to the beam axis and put the return yoke at, say 10×gap apertures to the left from the magnet.)

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PC-Assignment #2
Magnetic Measurements for a Quadrupole Magnet Using POISSON

A PC computer program (POISSON) is available to simulate magnetic fields. The program is largely self-guiding, but we've included some tips in the section "Using POISSON." Start with an input file for a magnet of the same type and modify profiles/coils as desired.

This laboratory is meant to be done along with Laboratory Assignment #2 so the student can compare the field calculations with actual measurements. The geometry of the quadrupole for Laboratory Assignment #2 is given in a handout. You can verify the geometry by direct measurement too.

1. Calculate the quadrupole magnetic field at a current equal to the one you used (or will use) in Laboratory Assignment #2 (Quadrupole magnet measurements...read this assignment if you have not already done so). If you have not done the measurement yet, use a power supply current of $5A$ and 100 turns/coil. Print out the results and compare the vertical field $B_y(x)$ on the midplane with measurements. If you did quadrupole measurements already, compare the measured field gradient with the simulated one. Do you have any suggestion to explain the difference?

2. Run the program for different currents and determine the excitation curve and saturation (if any). For this, you increase the current for each run and note down only the resulting field $(B_y(x = 1.0 \text{ cm}, y = 0, z = 0))$ at $x = 1$ cm. As you increase the current note down the magnetic fields where the field lines are most dense. At what point does the gradient fail to follow the current linearly? Where in the magnet does the most severe saturation occur? Do you have any suggestion to improve the quadrupole magnet design if you had to build one?

3. Plot the excitation curve $[B_y(x = 1, y = 0) \text{ vs } I_{\text{coil}}]$ and compare with measurements in Laboratory Assignment #2.

4. What are the higher harmonics up to 5th order from the POISSON output? How do these compare with coefficients to the fit to the transverse field data in part 3(a) in Laboratory Assignment #2?

PC-Assignment #3
Cavity simulation with URMEL

The program URMEL calculates fields in 2D-cavity (rotationally symmetric) structures. Once a cross section of the cavity is defined, the program calculates field patterns, the field along the axis and many more technical cavity parameters. Study the examples in `c:\student[1-30]` where the [1-30] is the number that you have been assigned for this class. Backup copies can be found at `c:\USPASExamples\Urmel`. Warning: do not overwrite the examples in `c:\USPASExamples\Urmel`. This laboratory is meant to be done along with Laboratory Assignment #4 so the student can compare the frequencies of TM modes measured with calculations.

To run URMEL double click on the runUrmel.tcl script. It will ask for an URMEL input file (`<rootname>.urmi`). Output will be plotted on the screen. Various output files will also be created. While URMEL is running these output files will have predefined names which can cause old output files to be overwritten. The runUrmel.tcl script waits for the plot output from URMEL to be closed and then it moves these output files to `<rootname>.*`. This is done to avoid overwritting the output when using different URMEL input files. This program can also be run from the DOS prompt by typing "`tclsh runUrmel.tcl <rootname>`".

1. Find those TM modes and frequencies found in the measurements in Laboratory Assignment #4 using URMEL for the pill-box cavity. You will need to know the pill-box cavity dimensions. Print out field configurations.

2. The coaxial cavity outer tube is 20.00 cm long, the inner tube length is 19.00 cm, the outer diameter of the inner conductor is 2.16 cm and the outer diameter of the outer conductor is 7.94 cm. (The inner diameter of the outer conductor is likely to be 7.30 cm, assuming 0.125" thickness. Check the dimensions of the actual cavity anyway.) Find the lowest frequency TM mode field pattern. Does it match the frequency measured in Laboratory Assignment #4?

3. The coaxial cavity has harmonics and modes. Find the 3rd and 5th harmonic fields by the frequency obtained in the URMELOUT listing. Plot and discuss the difference between harmonics and modes (See Laboratory Assignment #4 for a discussion of modes and harmonics).

PC-Assignment #3
RF Cavity TM Mode Calculations with SUPERFISH

The program SUPERFISH calculates fields in 2D-cavity structures. Once a cross section of the cavity is defined, the program calculates field patterns, the field along the axis and many more technical cavity parameters. Study the examples in `c:\USPASExamples\Superfish\` `FrequencyScan\CoaxialCavity`. The last section of the assignement explains how SUPERFISH can scan the frequency to get the frequencies of different harmonics and calculate the fields in one mode. This laboratory is meant to be done along with Laboratory Assignment #4 so the student can compare the frequencies of TM modes measured with calculations.

1. Measure the dimensions of the pill-box cavity and determine the TM-modes and estimate their frequencies using the formula or mode graph (See Laboratory Assignment #4).

   Note: `c:\USPASExamples\Superfish\FrequencyScan\pillCavity.am` is an example of calculating one mode, you need create a scan-frequency file similar to `coaxialCavity_scan.am` but with pillbox cavity structure to get the harmonics (follow the steps at the end of the assignment and then create the files for each mode.

   Warning: do not overwrite these examples. Copy them to your own directory and modify them for your assignments. The examples are all automesh input files, if you change the extension `.am` to `.af` (autofish file), double-clicking the `.af` file will run superfish automatically. The contents of `.am` and `.af` are exactly the same.

2. Find those TM modes and frequencies found in the measurements in Laboratory Assignment #4 using SUPERFISH for the pill-box cavity. You will need to know the pill-box cavity dimensions. Print out field configurations.

3. The coaxial cavity outer tube is 20.00 cm long, the inner tube length is 19.00 cm, the outer diameter of the inner conductor is 2.16 cm and the outer diameter of the outer conductor is 7.94 cm. (The inner diameter of the outer conductor is likely to be 7.30 cm, assuming 0.125" thickness. Check the dimensions of the actual cavity anyway.) Find the lowest frequency TM mode field pattern. Does it match the frequency measured in Laboratory Assignment #4?

   Note: you may modify `c:\USPASExamples\Superfish\FrequencyScan\` `CoaxialCavity\coaxialCavity_scan.am` file with the above coaxial cavity structure and run it to find out the lowest harmonics frequency and create `mode010.am` file. Run `mode010.am` file in two steps as described in frequencyscannotes.txt to get the accurate frequency and fields.

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4. The coaxial cavity has harmonics and modes. Find the 3$^{\text{rd}}$ and 5$^{\text{th}}$ harmonic fields from the frequency obtained in FishScan.TBL. Plot and discuss the difference between harmonics and modes.

Steps for running a frequency scan in SUPERFISH.

1. Double-click `coaxiaCavity_scan.am` to run `automesh` and `coaxialCavity_scan.T35` will be created after `automesh` is completed.

2. Double-click`coaxialCavity_scan.T35` to see the structure of half of the cavity.

3. Right-click`coaxialCavity_scan.T35`, select "run fish" till it completes. `Fishscan.TBL` will be generated.

4. Double-click `Fishscan.TBL`, a plot of D(k2) versus frequency. You can find out the fundmental and high harmonics frequencies.

5. Create `mode010.am` file with start frequency close to the first harmonics and `mode050.am` file with start frequency close to the fifth harmonics.

6. Double click `mode010.am` (or `mode050.am`) to run automesh

7. Right-click on `mode010.T35` (or `mode050.T35`) and select "run fish".

8. Double-click on `mode010.T35` (or `mode050.T35`), the frequency and field will be shown for the corresponding harmonics.

PC Assignment #4
Design quadrupole doublet focusing

Use PC Program: Beam Optics

1. Design a quadrupole focusing doublet such that the focal point in both the horizontal and vertical plane is 3 m from the end of the second quadrupole. Choose your own quadrupole length.

   (a) What are the focal length for both quadrupoles?

   (b) What is the focal length, of the doublet in both the horizontal and vertical plane. Use thin lens approximation.

   (c) Make 2 graphs with plots of two trajectories each. The starting conditions are:
   (1) $[x_0 = 1 \text{ mm}, x'_0 = 0.0 \text{ mrad}]$ and
   (2) $[x_0 = 0 \text{ mm}, x'_0 = 1.0 \text{ mrad}]$ in one plot and
   (3) $[y_0 = 1 \text{ mm}, y'_0 = 0.0 \text{ mrad}]$ and
   (4) $[y_0 = 0 \text{ mm}, y'_0 = 1.0 \text{ mrad}]$ in the other.
   Having a sharp focal point makes the doublet a close 90-degree transformer, where the sine-like function determines the beam size at the focal point. Show and verify this.

2. Vary the distance between both quadrupoles and find the focal point at a distance of 3 m from the end of the second quadrupole. Use three or more values for the quadrupole distance.

   (a) How do the individual quadrupole focal lengths scale with quadrupole distance? Are they longer or shorter?

3. Use a parallel beam (beam size option) with a starting beam size of say 1cm and plot the evolution of the beam through the doublet to the focal point.

4. Choose one of your solutions and calculate the required field gradients for a 1 GeV electron beam, a 1TeV proton beam and a 10 keV A^+-ion beam. What would the electric field have to be if you use an electrostatic quazdrupoles for the 10 keV A^+-ion beam?

PC-Assignment #5
FODO Cell Optics

In this computer laboratory you will view machine functions and trajectories in the simple periodic FODO cell. You will see demonstrated matched phase ellipses, mismatched phase ellipses, and unstable lattices. While there aren't many numerical results generated in this lab, the student is encouraged in exploring FODO cells as much as desired.

The `FODOcells` GUI application runs the optics code `elegant` internally with a user-defined FODO cell beamline. This application calculates and plots the beta functions and trajectories through these FODO cells.

The main parameter of the FODO cell are the inter-quadrupole distance L and the focal lengths of quadrupoles QF (focusing horizontally) and QD (defocusing vertically). A dipole is centered in the drift space between quadrupoles. The length of the magnets are entered as a fraction of L. Thus this GUI allows the scaling of distances and lengths with L.

Other inputs to the application are the number of cells and the number of trajectories to track. A large number of cells is necessary for observing the long-term behavior of the focusing channel, and a large number of trajectories is useful in displaying a beam envelope.

There are three calculations to run: twiss functions, trajectories for a matched beam ellipse, and trajectories for a beam ellipse matched to a different cell (i.e., mismatched condition).

For each calculation, there is an execution command, a plot command, and a printout command (in case values in the plots are needed).

To get help on any GUI widget, double click with the right mouse button over the widget and a help window with text will pop up.

Exercise 1. Beta functions. Using default FODO cell parameters press "Calculate twiss functions", then "Plot beta". Periodic beta functions are calculated and plotted.

The layout of outlined blocks at bottom of the beta function plot represent the magnet layout of the FODO cell along a beam axis: QF-dipole-QD-QD-dipole-QF. The first block is a QF quadrupole. The long and thin block is a dipole. Two consecutive blocks with no space in between are two QD quadrupoles meant to represent in reality a quadrupole magnet of twice the length of a QD. The QD blocks are pictured below the axes to show a negative strength.

The default focal length of the QF is set to 2 m. The default focal length of one QD is -2 m. The combination of the 2 QD will then have a focal length of -1 m. The default values for the quadrupole lengths are 0.25 L.

Because the quadrupoles in this calculation are thick lenses, the formulas for the thin-lens FODO cell, which can be evaluated by hand, will only approximate the optical properties of the FODO cell.

Using the thin-lens approximation formulas, calculate the β_x and β_y functions at the QF, QD, and the minimum and maximum dispersion for the default case ($L = 1$ m, $f = 2$ m, bend angle $= 0.01$ rad).

Read out the values of β_x, β_y and dispersion (η_x) from the printout file ("View Data" button) and compare with thin-lens calculation.

Note the phase advance output (phix, phiy) in the status output box at the top of the GUI. Compare with the calculation from the thin-lens approximation.

Repeat the calculation, printout and phase advance output with the quadrupole length equal to 0.01 times L ("fraction Q"). Note how closer the values of β_x, β_y, η_x, and phase advances are to the thin-lens values.

Multiply the values for L and for the two focal lengths by two. Note what happens to β_x, β_y, η_x, and phase advances.

Exercise 2. Trajectories. Return to the default values of the application and set the number of cells to 4.

Calculate the beta functions and plot them. Note the periodicity of the functions. This is expected since the quadrupole layout is periodic.

Press the "Calculate" and "Plot" in the "Matched Trajectories" frame. The "Calculate" button does two things: generates the initial coordinates for tracking, and tracks the particle trajectories through the FODO cells. Two plot windows appear. The first window gives the phase space plot of the initial coordinates of the trajectories in the x-plane and the y-plane. The y-plane data is on the second frame of the window (press n on the keyboard to advance to the next frame, press p to go back to the previous frame). In the default setting there are 4 initial trajectories arranged "90 degrees" apart in the x- and y-planes. The second plot window shows the trajecories along the beamline (as a function of longitudinal coordinate). The y-plane trajectories are shown in the second frame (again, press n to view the second frame).

Repeat the calculation and plot with the quadrupole length changed to 0.01 times L. Note the slight change in trajectories.

Estimate the phase advance per cell in both planes just using the shape of the trajectories.

The blue line in the trajectory plots is the envelope calculated from β.

Exercise 3. More trajectories Starting from conditions in Exercise 2 (default values + 4 cells), change the number of trajectories to 40.

The plots of initial coordinates will now show a collection of points that form an ellipse.

The plot of trajectories fill out the envelope (blue line). Zoom in on the graph at the

corners of the envelope and observe that the individual trajectories fill in the space inside the envelope.

An ellipse of particles is said to be matched to the beamline if the resulting trajectories fit inside an envelope given by the periodic β functions.

Repeat above with quadrupole length equal to 0.01 times L.

Exercise 4. Mismatched ellipse. A mismatched input ellipse is expected to produce trajectories that do not fill out a symmetric envelope throughout the beamline. The initial coordinates are generated for a FODO cell that is different from the FODO cell of the tracked beamline. One has to run a combination of command from 'Matched Trajectories" and "Mismatched Trajectories", as acheived in the following example.

Set the number of trajectories to 40, and number of cells to 10. Set the focal lengths of QF and QD to 2.5 m, and -2 m. Press the "Calculate" button from "Matched Trajectories". (This action internally generates the initial coordinates.) Then change the focal length of QF back to 2 m. Press the "Calculate" button from "Mismatched Trajectories". Then press "Plot."

Observe the modulation on the envelope formed by the trajectories. Check both planes.

Exercise 5. Unstable cells. Change the focal length of QF to 3.0 m and press the "Calculate" and "Plot" buttons from "Mismatched Trajectories".

The decrease in horizontal focusing will cause a divergence in the x-plane trajectories. Check the vertical plane. Effectively there is not enough focusing in the horizontal plane to counteract the defocusing effect of the QDs. Vary the focal length of QF by steps of $\pm$ 0.1 m (up and down) and observe the changes in the trajectory envelope.

In an unstable FODO cell the trace of the 2x2 transfer matrix is outside the stable range of -2 and 2. Since the trace of the transfer matrix for one cell is $2\cos\mu$, with μ the phase advance per cell, one could solve for the imaginary part of the phase advance μ and interpret this as an exponential growth factor per cell seen in trajectories. Note that only one plane needs to be unstable for the cell to be considered unstable.

In the thin-lens approximation, a divergence occurs in one of the planes when the difference in the absolute values of the focal lengths of the two quadrupole is equal or exceeds the FODO cell parameter L, i.e. $L < |f_{QF}| - |f_{QD}|$ produces horizontal instability, while $L < |f_{QD}| - |f_{QF}|$ produces vertical instability. In our case the QF stability limit is 3 m because $L = 1$ m and $|f_{QD}| = 2$ m. Scan the QF focal length below 2 m until you reach an instability in the y-plane.

Start application `NecktieDiagrams`. This application produces 2D contour plots that display the limits of stability for FODO cells with QD and QF quadrupoles as variables. There are only few options to this application. Explore all options (i.e. press the plot button for all possible selections of radio buttons.)

PC-Assignment #6
Design a Storage Ring

General guide lines for the storage ring design: Design a storage ring and some of its components. Do not assume that you end up with an optimized design. In reality, to design and optimize a storage ring takes a long time and many iterations to accommodate goals as well as physical, technical and financial constraints. The exercise during these two weeks is therefore only a start, but you will go through many basic steps of optimization.

Start with a group discussion on your goals for the storage ring. Should it be a synchrotron light source or a high energy physics machine. However, as a first step, just consider the ring to be composed of FODO cells. For more complicated lattices we need to use more powerful computer programs. Decide on beam energy, beam current, number/length of insertion straight sections, rough circumference of the ring. At this point you may ask yourself, how do I make the right decision with my limited experience? Most probably, you will make the "wrong" decision or "not so right" decision. That is fine, nobody makes the right decision at the beginning of a design effort. That's the point where the iterations start. You start with some parameters, use them for your designs and when you get into some problems (physical, technical or financial) you discuss which and how to alter basic design parameters to get to realistic and affordable solutions. In this school, never mind "affordable" since we do not perform a cost estimate, but you may make here and there certain choices purely on an economic basis if there are no physical or technical reasons to go one or another way. An example for this is the choice of the rf-frequency. Basically, any frequency could be used, but it is prudent to choose a frequency of say 500 to 1000MHz because this is the TV frequency range and there are many sources for competitively priced rf-power sources (klystrons).

With these general guidelines start your first storage ring design effort. For a high energy physics machine you may go "overboard" with your design parameters, just check that the components you need can be built. To simulate somewhat economic constraints (e.g. for synchrotron light sources) the design should be within the following boundaries:

Beam energy between 1.3 and 3 GeV (choose energy to meet desired radiation characteristics)

Ring circumference no more than 150 m (VUV ring) or 300 m (x-ray ring)

(smaller is better, but do include the desired number of insertion straight sections and consider the desired small beam emittance)

The total length of bending magnets should be between 25 and 30% of circumference

Beam current no more than 500 mA

(try to meet your goals with the least current)

Maximum bending magnet field is 1.7T

Maximum pole tip field in quadrupoles is 1.0T

Maximum pole tip field in sextupoles is 0.5T

always keep at least 20 cm between magnets as space for magnet coils!

at least 4 straight sections (2 to 3 m long) available for insertion magnets

Maximum pumping speed per pump be 1000 lt/sec.

Place pumps only where there is at least 15 cm of magnetfree space available.

To obtain a high brightness synchrotron radiation source you want to design the ring for a small beam emittance. The emittance scales like E^2/ρ and θ^3, where θ is the deflection angle of each bending magnet. Therefore, many lattice cells with small deflection per unit results in a small emittance, but at the price of a large circumference.

Design goals for a storage ring may be:

Keep rf power below about 300 kW

Bending magnet radiation: critical energy 1 keV or more

Wiggler magnet radiation: critical energy 1 keV or more

Undulator radiation: should cover a photon energy spectrum

(use up to 7$^{\text{th}}$ order) up to about 2 keV.

You are very welcome to choose your own design goals different from the above, but give reason in report for your choice. You may, for example, study the design of a ring in your home institution. There are many input files for existing storage rings in the subdirectory of BeamOptics.

Design a storage ring with Program BeamOptics

Tasks:

1. Decide on a few basic storage ring parameters and goals: VUV, x-ray ring, HEP etc., energy, approximate circumference, number/length of unit cells and insertion straight sections.

2. Before you start working on the computer make a sketch of the ring cell (sequence of quadrupoles and bending magnets depending on your choice of lattice). Then also sketch the whole ring showing cells, insertion straight sections, number of superperiods.

3. Use PC program to design first one cell and then the whole ring composed of many such cells

(a) determine bending magnet parameters: length, bending radius, field (keep field less than 1.7 Tesla !), choose aperture (see 3d).

(b) determine quadrupole parameters: length, strengths, choose aperture radius (see 3d), max. pole tip field (keep pole tip field to less than 1.0 Tesla !).

(c) general ring parameters: circumference, number of superperiods, number of insertion straight sections, length of insertions.

(d) determine magnet apertures: assume you want the ring acceptances to be at least 50 mm×mrad horizontally and 20 mm×mrad vertically. Further assume an extra allowance of ±5mm for orbit distortion and ±5mm for the vacuum chamber thickness.

(e) beam optics parameters: plot/list betatron functions and dispersion function for one superperiod, ring tunes.

4. Start your report: import magnet list, betatron functions, plot of betatron function with magnet lattice (*.wmf file) into WORD. Add some text describing your goals and choices and what the numbers and plots mean.

   IMPORTANT: Don't forget to save your results!

5. we must correct for chromatic aberrations of the focusing structure. This is done by inserting sextupoles where the dispersion is nonzero. The degree of aberration in storage rings is measured in terms of natural chromaticity which is always negative. Insert sextupoles, SF's and SD's into the lattice to correct the natural chromaticity to +1.0 in both planes.

6. sextupoles are nonlinear elements causing potential problems on particle stabilities at large amplitudes. The range of stability is determined by particle tracking.

7. track individual particles starting with increasing horizontal amplitudes and track for 1000 turns. Watch the phase space trajectories evolve from a pure elliptical shape to a distorted shape and eventually to instability.

8. track particles with negative as well as positive amplitudes and record the limiting amplitudes.

   (a) do the same for vertical oscillation, but use only positive amplidutes. Motion and limits are symmetric in the vertical plane. Check it out!

   (b) finally start with a finite horizontal amplitude within the stability range and vary the vertical amplitude.

   (c) create a an $x - y$-plot showing the amplitudes at which the particle oscillation becomes unstable. The range of stability is called the dynamic aperture of the ring. Compare this stability range with the acceptance of the ring. The dynamic aperture should be larger than the physical aperture. Note: you cannot compare apertures (in mm) you must compare emittances/acceptances. The acceptance of a ring is equal to the largest emittance a beam may have to just fit through the vacuum chambers.

9. Choose RF-parameters, cavity voltage and power. You can have only 50 kW per cavity. if you need more power you need to increase number of cavities. Adjust rf-voltage until you get desired beam lifetime.

10. calculate beam lifetime (use default values for rf- and vacuum parameters). Your goal should be a total beam lifetime of at least 20 hours.

11. insert vacuum pumps into the ring such that the pressure distribution is minimum. Add pumping until you get the desired beam lifetime.

PC-Assignment #7
Synchrotron Radiation

In this assignment you will determine various synchrotron radiation parameters with the use of the PC program *SynRad.* (Reading: section 9.1; 9.3.1)

Task:

1. Specify radiation characteristics from bending magnets. From your ring design (Assignment on storage ring design or take a synchrotron light source of your choice) the beam energy, current and bending magnet field is known. Use these parameters to determine general characteristics of bending magnet radiation:

   (a) what is the total radiation power from each bending magnet and for the total ring?

   (b) calculate the radiation power density in units of (watt/mrad). Use the a vertical opening angle of $2/\gamma$ and calculate the power density in units of (Watt/mm^2) at an experimental station 10m away from the source.

2. Determine spectral radiation characteristics of bending magnet radiation:

   (a) what is the critical photon energy?

   (b) estimate at what photon energy (round off to an integer multiple of the critical photon energy) the spectral intensity has dropped down to about 1% of the maximum.

   (c) calculate the photon spectrum and specify the useful photon energy range, that is the spectral range for which the intensity is at least 1% of the maximum intensity.

   (d) suppose, for your experiment you are interested in 8 keV radiation within a bandwidth of $\Delta e_{\mathrm{ph}}/e_{\mathrm{ph}} = 10^{-4}$. The beam line has a horizontal acceptance of 5mrad. What is the photon flux at your experiment?

   (e) assume a 1:1 imaging from the source to the experiment. Calculate for a photon energy equal to the critical photon energy the photon flux density at the experiment. Choose the optimum source location within the bending magnet

3. Photon beam polarization

   (a) what is the plane of polarization for bending magnet radiation?

(b) for a photon energy of $0.1\,e_{\text{crit}}$ and $0.5\,e_{\text{crit}}$ determine the photon flux for π-mode radiation as a function of vertical observation angle.

4. Specify radiation characteristics from an undulator magnet. Use the parameters of your storage ring to determine the characteristics of undulator radiation:

   (a) given the length of a straight section in your ring and the vertical aperture required by beam and vacuum chamber specify an undulator magnet for a strength parameter of at least $K = 2.0$. Determine length and number of periods, field and gap.

   (b) for which value (within 10%) of the undulator strength parameter K is the intensity of the fundamental radiation a maximum.

   (c) calculate the maximum transverse oscillation amplitude of the beam while passing through the undulator magnet.

   (d) what is the horizontal and vertical opening angle of the radiation?

5. Determine spectral radiation characteristics of undulator radiation:

   (a) determine the fundamental wavelength of undulator radiation as a function of gap aperture. How far can you vary the wavelength by varying the undulator gap?

   (b) plot the line spectrum for $K = 0.5, 1.0$ and 2.0.

   (c) considering all harmonics up to 7th order, plot the photon beam brightness as a function of undulator gap. Does the undulator cover a continuous spectrum at these harmonics?

Using POISSON for Magnet Simulation
Michael Borland, ANL

The PC computer program (POISSON) is available to simulate magnetic fields. The program is largely self-guiding, but we've included some tips in this section.

POISSON is actually part of a suite of programs that includes programs for magnetic, electrostatic, and rf field calculations. The suite is maintained by the Los Alamos Accelerator Code Group and available (for Windows-based PCs only) from their web site.

The programs that you'll use in this part are

1. AUTOMESH: the purpose of automesh is to take input giving the profiles of the iron, air, and coil regions of the problem, and generate a mesh that matches the problem. AUTOMESH uses an adaptive triangular mesh that is well-suited to magnets with curved boundaries.

2. POISSON: this program performs the actual magnetic field computations, using data generated by AUTOMESH.

3. WSFPLOT: this program plots problem geometry from AUTOMESH, along with field lines and arrows from POISSON.

4. SF7: this program extracts data from the field map generated by POISSON and saves it in a form that can be easily plotted with TBLPLOT.

5. TBLPLOT: this program makes line plots of data extracted from the POISSON output by SFO. It also allows saving the data in ASCII form, which can be imported into a EXCEL.

Here are step-by-step instructions for using these programs:

1. Find an input file for a magnet of the same type (e.g., quadrupole or dipole) that you want to simulate. You will find a selection of input files in subdirectories of `c:\LANL\Examples\Magnetostatic`. Do not modify the files here! Rather, copy them to your own area before you begin work. The files you start with are "Automesh Input" files, with file extension .AM. To be concrete, lets assume this file is called INPUT.AM.

   Note: We have also made several additional examples of our own, namely, HMagnet-Full.am and Quadrupole.am. You may wish to start with these examples as they are simpler and closer to the problems you are doing. These are in the `c:USPASExamples\POISSON` directory.

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2. Before setting up your own problem, you should run the input file without modification to get some experience. To start, simply double-click on the file to run AUTOMESH. A window will pop up that will show the status of AUTOMESH as it runs.

3. Once AUTOMESH completes (the window disappears), you'll find a new file called INPUT.T35. Double-click on this file to run WSFPLOT and show the magnet profile. You'll also see the mesh. When you are finished looking at this, close the window.

4. Right-click on INPUT.T35 and select "Run Poisson." Again, a window will pop up showing the progress of the program. Watch the convergence numbers. They'll (generally) decrease monotonically until they are below 10^{-6}, at which point POISSON terminates.

5. Once POISSON completes, double-click on INPUT.T35 (which now contains the POISSON solution). WSFPLOT will again pop up a plot of the magnet profile, along with field contours and the mesh. You can turn on and off the various elements of the plot using the "View" menu. Moving the mouse around on the plot allows you to determine the field at any point in the magnet.

6. To see a text printout of the POISSON results, look for the file OUTPOI.TXT. To read it, right-click and select Edit. In the "old days," this was all you'd get from POISSON. These days, you don't need it for much except harmonic analysis. The harmonic analysis tables appear at the very end of the file.

7. To plot field data vs x or y, you need to extract data from the INPUT.T35 file another form. Start by right-clicking on INPUT.T35 and selecting "Interpolate." A dialog will pop up. Note the four entry boxes in the center. You can use these to enter the x and y coordinates of the start and end of a line along which you want the fields. (Warning: the default values give a line extending from the lower-left and to the upper-right corner of the problem, which is not really very useful. Usually, you can get what you want by just setting both y coordinates to zero.)

8. After completing this dialog, another file with a name like INPUT01.TBL will appear. Double-click on this file to run TBLPLOT, which will plot the selected data. You can use the "Select X and Y" submenu under the "Data" menu to change which data is plotted. You can also use the "Save Data" submenu of the "File" menu to write the data to an ASCII file; the name of this file is chosen automatically, and is typically something like INPUT0100.TXT.

Now that you are familiar with POISSON, you should copy INPUT.AM and modify it to reflect your problem. To do this, copy the file and right-click on the copy. Choose "Edit" to edit the file, then go through the steps above. Note that units for x and y are centimeters, while units for current are ampere-turns.

Some common problems using POISSON:

1. *AUTOMESH doesn't complete successfully.* Usually this means that you have an error in your input. Occassionally, it means that your problem is too complex and AUTOMESH is having trouble. Check for things like:

    (a) Two points in a row with the same coordinates.

    (b) Mesh spacing (DX and DY) too large.

    (c) Region boundary not closed. The first and last points of a region should be identical.

    (d) Line regions (XREG1, YREG1, etc) that are close to but not exactly corresponding to a problem feature. Line regions (which determine where AUTOMESH changes the mesh spacing) should either coincide exactly with problem features (e.g., a line or corner) or else be in a location with no problem features.

2. *POISSON runs for a long time and doesn't converge.* The magnet is probably very saturated. You may have entered an unreasonably high current.

3. *The field doesn't agree with what's expected.* Did you enter the source current instead of the total current in ampere-turns?

4. *There is documentation available on the computer.* To find it, go to LANL/Docs and double-click on SFCODES.DOC.

Microwave cavities

We consider only cylindrical microwave cavities with its axis in the z-direction and resonating in a TM-mode. Cavities can resonate in a variety of modeseach with it's own frequency and field pattern. We are interested only in such modes which give a finite electrical field component in the z-direction. That excludes transverse electrical (TE-) modes and leaves transverse magnetic (TM-) modes. The expressions for thew fields are:

$$
\begin{aligned}
E_r &= -\mathrm{i}\frac{\beta}{k_c}E_0 J'_n(k_c r)\, \exp\left[\mathrm{i}(\omega t - n\theta)\right] \sin\beta z, \\
E_\theta &= -\frac{\beta n}{k_c^2}E_0\frac{1}{r}J_n(k_c r)\, \exp\mathrm{i}(\omega t - n\theta) \sin\beta z, \\
E_z &= E_0 J_n(k_c r)\, \exp\mathrm{i}(\omega t - n\theta) \sin\beta z, \\
H_r &= -\frac{\omega\epsilon n}{k_c^2}E_0\frac{1}{r}J_n(k_c r)\, \exp\mathrm{i}(\omega t - n\theta) \sin\beta z, \\
H_\theta &= -\mathrm{i}\frac{\omega\epsilon}{k_c}E_0\frac{1}{r}J'_n(k_c r)\, \exp\mathrm{i}(\omega t - n\theta) \sin\beta z, \\
H_z &= 0,
\end{aligned}
$$

where $\beta^2 = k^2 - k_c^2$, $k^2 = \omega^2\epsilon\mu$, $J_n(k_c a) = 0$ and a the radius of the cavity. The quantities ϵ and μ are the relative dielectric constant and permeability of the medium inside the cavity respectively. Different TM-modes are distinguished by indices TM_{nml}, where n is the number of azimuthal nodes, m is the number of radial modes as determined by the mth root of $J_n(k_c a) = 0$, and l the number of longitudinal modes.

The following table displays zero's of Bessel's Functions $J_n(x)$.

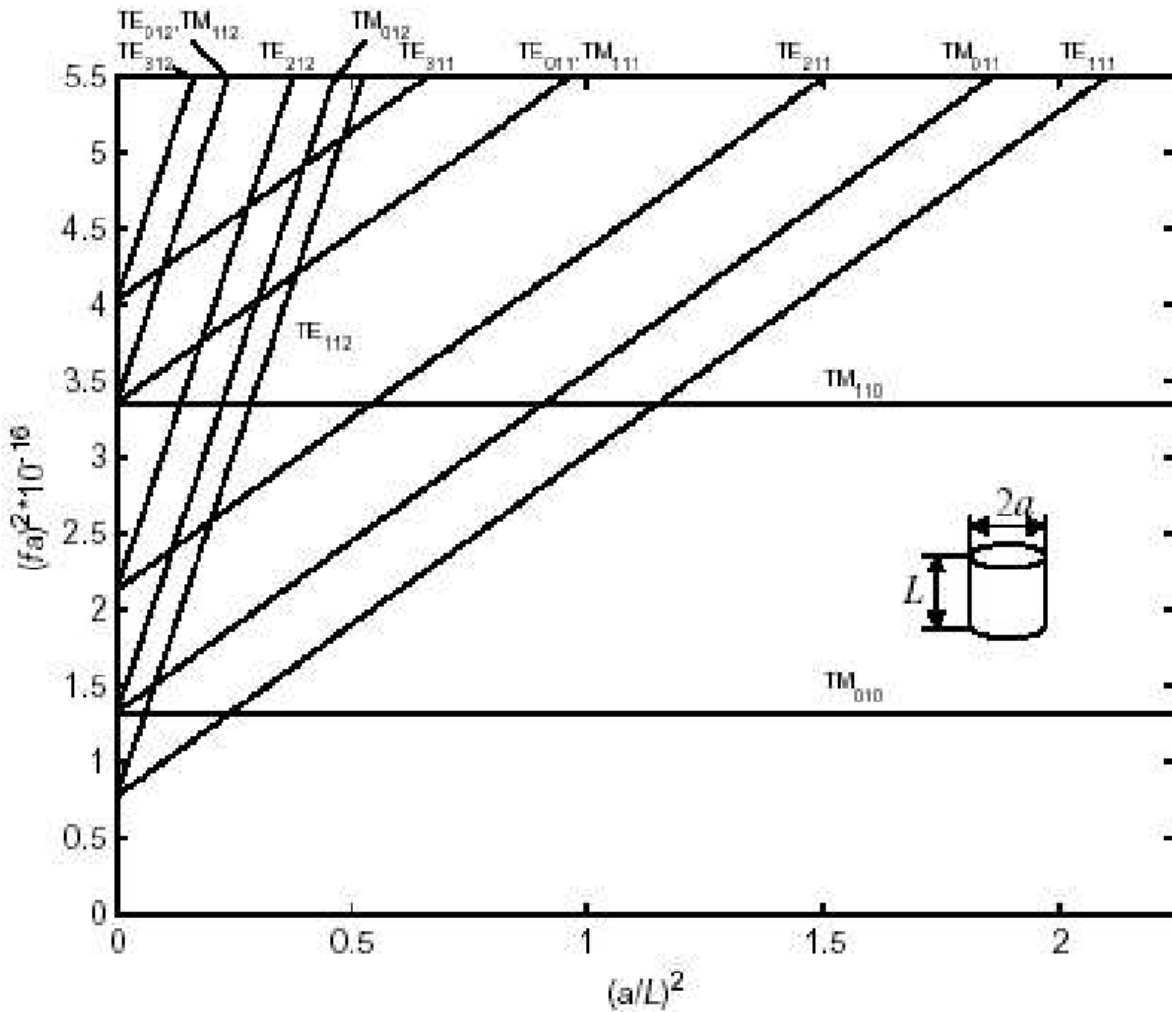| Mode	m^{th} zero	of	at $x =$
TM_{01}	1	$J_0(x)$	2.40
TM_{11}	1	$J_1(x)$	3.83
TM_{21}	1	$J_2(x)$	5.14
TM_{02}	2	$J_0(x)$	5.52
TM_{12}	2	$J_1(x)$	7.02

Figure 2: Mode chart for a cylindrical cavity with diameter d and length l.

Maxwell Equations

$$\begin{aligned}
\boldsymbol{\nabla} \cdot \mathbf{D} &= \boldsymbol{\rho} \\
\boldsymbol{\nabla} \cdot \mathbf{B} &= 0 \\
\boldsymbol{\nabla} \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\boldsymbol{\nabla} \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned}$$

- $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}$ are vector fields

    - Each has three components
    - Values of each component vary over space

- Fields vanish inside of perfect metals

- $\mathbf{E}$ field lines start and stop on charges $\Rightarrow$ $\mathbf{E}$ is always perpendicular to a metallic wall

- $\mathbf{H}$ field lines never start or stop (they are loops) $\Rightarrow$ $\mathbf{H}$ is always tangent to a metallic wall

- For constant fields, $\mathbf{E}$ and $\mathbf{H}$ are decoupled

- For time-varying fields, $\mathbf{E}$ and $\mathbf{H}$ couple to form a wave equation

Wave Equations

- Field configuration travels with the wave velocity

- Waves satisfying the wave equation can travel either forward or backward

- For electromagnetic waves with a fixed wavelength and frequency, the wave equation relates these quantities by

$$\mathbf{k}^2 = \frac{\omega^2}{c^2} \tag{5}$$

  where $\mathbf{k}$ is the vector wavenumber

$$k_{x,y,z} = \frac{2\pi}{\lambda_{x,y,z}} \tag{6}$$

  and ω is the radian frequency $\omega = 2\pi f$.

- Can represent a sinusoidal wave traveling in the $\mathbf{k}$ direction as

$$e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{7}$$

  where i$= \sqrt{-1}$ and we have used Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{8}$$

Fields in Waveguides

- Waves traveling in the forward **z** direction have the (z, t) dependence

$$e^{i(k_z z - \omega t)} \qquad (9)$$

  and those in the backward direction have

$$e^{i(k_z z + \omega t)} \qquad (10)$$

- Transverse dependence of fields, in the (x, y) directions, depends on the cross-sectional geometry of the waveguide

    - **E** fields must be perpendicular to the walls
    - **H** fields must be tangential to the walls

- Find k_z by

    - Pick the frequency, ω, of interest
    - Calculate the transverse wavenumber from the geometry of the guide
    - Solve

$$k_z = \sqrt{(\omega/c)^2 - k_\perp^2} \qquad (11)$$

- Many waveguides have a minimum $k_\perp$. For such guides there is a minimum frequency which will propagate

$$\omega_{\text{cutoff}} = c k_\perp \qquad (12)$$

Fields in Cavities

- Choose a convenient $\mathbf{z}$ axis

- Solve for the transverse wavenumber

- Impose the Maxwell equations boundary conditions

    - $\mathbf{E}$ fields must be perpendicular to the walls
    - $\mathbf{H}$ fields must be tangential to the walls

- Only certain frequencies can satisfy all of the boundary conditions. These frequencies, ω_{mnl}, are called the cavity modes

- This often leads to a cosine

$$\cos k_z z = \frac{1}{2}\left(e^{ik_z z} + e^{-ik_z z}\right) \tag{13}$$

  or a sine

$$\sin k_z z = \frac{1}{2i}\left(e^{ik_z z} - e^{-ik_z z}\right) \tag{14}$$

  dependence of the z component.

- Traveling waves turn into standing waves

Quality Factor

- Perfect metals (superconductors) never lose any energy; once excited, the mode lasts forever

- Real metals dissipate the field energy

- A quality factor, Q, is defined to characterize how good the cavity is

$$Q_{mnl} = \omega_{mnl} \frac{\text{Stored Energy}}{\text{Power Loss per Cycle}} \tag{15}$$

- A high Q means an efficient cavity

Coupling to a Cavity

- The modes that can exist in a cavity are determined solely by the cavity geometry

- In order to excite those modes, one needs to have a source that can couple to that mode

- Probes can couple either to $\mathbf{E}$ or $\mathbf{H}$, depending on their shape

  - $\mathbf{E}$ probes point in the direction of the field
  - $\mathbf{H}$ probes are loops that enclose magnetic field lines

Cavity Perturbations

- Small changes in cavity shape or composition (dielectric constant or magnetic permittivity) from a uniform cavity shift the cavity resonant frequency

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta U_H - \Delta U_E}{U} \tag{16}$$

where ΔU_H is the magnetic energy removed, ΔU_E is the electric energy removed, and U is the total stored energy.

Electromagnetic Fields from Relativistic Particles

- Stationary particles, in free space, create an isotropic electric field and no magnetic field

- Moving particles are a current and therefore also generate a magnetic field

- Moving particles also compress the electric field in the transverse direction, with the compression proportional to the relativistic

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \tag{17}$$

- Relativistic particles, for which $\gamma \gg 1$, have their electromagnetic fields compressed almost completely into the transverse plane

- Fields generated by a relativistic bunch of particles, while the particles are present, look like that generated from a pulse along a coaxial line

Cavity Measurements: Scattering Matrix

- Devices that measure cavities send waves along a coaxial transmission line into the cavity

- Cavity parameters are measured by looking at the reflected and transmitted waves

- Typical devices can launch waves into two ports and measure the output of those same two ports

- The matrix that relates the input waves to the output waves is the scattering matrix

$$
\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix} \tag{18}
$$

  where $V_{1,2}^+$ are the incident waves and $V_{1,2}^-$ are the output waves.

- Many measurements involve S_{11}, the reflected wave on port 1, or S_{21}, the transmitted wave from port 1 to port 2

Other Notes

Match impedances when possible to maximize power transfer

- 50Ω is the common termination for all radio frequency work

- For low frequency applications, for most signals less than ~ 100 kHz, a high impedance input is usually acceptable

Use logarithmic scales when working with large magnitudes

- RF signals can be measured over a very large range of amplitudes

- Logarithmic scales allow more of the range to be seen

- The common unit for logarithmic ratios is the deciBel

$$\frac{V_1}{V_2}_{dB} = 20 \log_{10} \left(\frac{V_1}{V_2} \right) \tag{19}$$

- A unit of power using the logarithmic scale is the dBm. One dBm is one milliWatt of power into a 50Ω load.