3

$$dt' = \frac{\gamma}{\beta}(1-\beta^2)\frac{dx}{c} = \frac{1}{\gamma\beta}\frac{dx}{c}$$

$$\gamma\beta = (\gamma^2-1)^{1/2}$$

$$\gamma = 1 + (\gamma_f - 1)\frac{x}{L}$$

$$dt' = \frac{1}{(\gamma^2-1)^{1/2}}\,\frac{L/c}{\gamma_f-1}\,d\gamma \qquad d\gamma = (dx/L)(\gamma_f-1)$$

$$t' = \int dt' = \frac{L/c}{\gamma_f-1}\int_1^{\gamma_f}\frac{d\gamma}{(\gamma^2-1)^{1/2}}$$

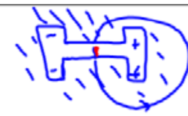
$$= \frac{L/c}{\gamma_f-1}\left(\cosh^{-1}\gamma_f - \underbrace{\cancel{\cosh^{-1} 1}}_{\to 0}\right)$$

$$\gamma_f = \frac{50\times10^3\,\text{MeV}}{m_e c^2 = 0.511\,\text{MeV}} = 9.78\times10^4 \qquad L/c = 10.75\,\mu s$$

$$\cosh^{-1} 9.78\times10^4 = 12.184$$

$$t' = \frac{12.184}{97800}\times 10.75\,\mu s = 1.339\ \text{ns}$$

(11) $\oint \vec{H}\cdot d\vec{\ell} = \int \vec{J}\cdot d\vec{s}$



$Hh = 2nI$

$\frac{Bh}{\mu_0} = 2nI \Rightarrow B = \frac{2\mu_0 n I}{h}$

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(12) $\vec{\nabla}\times\vec{B} = \underbrace{\mu_0\vec{J}}_{\approx 0} + \underbrace{\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}}_{=0}$

$0 = \ \Rightarrow \ \vec{B} = \vec{\nabla}\Phi_m$

$$\oint \vec{H}\cdot d\vec{\ell} = \int \vec{J}\cdot d\vec{s} = 2nI$$

$$= \int_{0}^{R} \frac{\vec{B}}{\mu_0}\cdot d\vec{\ell} + \int_{R}^{0} \frac{\vec{B}}{\mu_0}\cdot d\vec{\ell} = \frac{1}{\mu_0}\int \vec{\nabla}\Phi_m\cdot d\vec{\ell} + \frac{1}{\mu_0}\int_{R}^{0} \vec{\nabla}\Phi_m\cdot d\vec{\ell}$$

$$= \frac{1}{\mu_0}\left[\Phi_m(0) - \Phi_m(R_2) + \Phi_m(R_1) - \Phi_m(0)\right]$$

$$= \frac{1}{\mu_0}\left[\Phi_m(R_1) - \Phi_m(R_2)\right]$$

$$= 2nI$$

$\nabla^2\Phi_m = 0$ because $\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot\vec{\nabla}\Phi_m = \nabla^2\Phi_m = 0$

$$\Rightarrow \Phi_m = \sum_{n=1}^{\infty}\left(\Phi_{nc} r^n \cos n\theta + \Phi_{ns} r^n \sin n\theta\right)$$

Normal Quadrupole: $\Phi_m = c r^2 \sin 2\theta$

(a) $\Phi_m = c r^2\, 2\sin\theta\cos\theta = 2c(r\cos\theta)(r\sin\theta)$

$= 2cxy$

Boundary condition at pole-face $\vec{B} \perp$ face

$\vec{\nabla}\Phi_m \perp$ face $\Rightarrow$ Φ_m is constant on face.

$xy =$ const is hyperbola $y \sim a/x$

(b) $\Phi(R_1) - \Phi(R_2) = 2\mu_0 n I$

$= c r^2 \sin 2\theta$

$\theta_1 = \pi/4 \qquad \theta_2 = 3\pi/4$

$\sin 2\theta_1 = 1 \qquad \sin(2\theta_2) = -1$

$cR^2 + cR^2 = 2\mu_0 n I$

$c = \frac{2\mu_0 n I}{R^2}$

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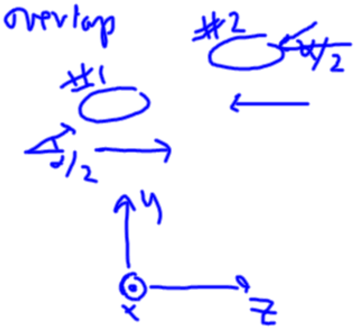
Assume @ $t=0$ complete overlap
$\alpha \ll 1$



Bunch 1: $x(t) = x$

$$y(t) = y + \beta c t \sin\frac{\alpha}{2} \simeq y + \frac{\alpha}{2}\beta c t$$

$$z(t) = z + \beta c t \cos\frac{\alpha}{2} \simeq z + \beta c t$$

Bunch 2: $x(t) = x$

$$y(t) = y - \frac{\alpha}{2}\beta c t$$

$$z(t) = z - \beta c t$$

$$n_{\substack{1\\2}}(x,y,z,t) = \frac{N}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} e^{-\frac{x^2}{2\sigma_x^2}} \cdot e^{-\frac{(y\mp\frac{\alpha}{2}\beta c t)^2}{2\sigma_y^2}} \cdot e^{-\frac{(z\mp\beta c t)^2}{2\sigma_z^2}}$$

Note: Gaussian in 1-D $= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2}$

Trick: $\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} = \left\{ \int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2} dy \right\}^{1/2}$

$x^2 + y^2 = r^2$

$dx\,dy = dA = 2\pi r\,dr$

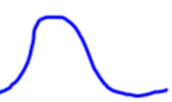
$$= \left\{ \int\!\!\int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2\sigma^2}} dx\,dy \right\}^{1/2} = \left\{ \int_0^{\infty} e^{-r^2/2\sigma^2} 2\pi r\,dr \right\}^{1/2}$$

$$= \left\{ \pi \int_0^{\infty} e^{-r^2/2\sigma^2} d(r^2) \right\}^{1/2} = \left\{ \pi \int_0^{\infty} e^{-\eta/2\sigma^2} d\eta \right\}^{1/2}$$

$$= \left\{ \pi\, 2\sigma^2 \left(-e^{-\eta/2\sigma^2}\right)\Big|_0^{\infty} \right\}^{1/2}$$

$$= \sqrt{2\pi}\,\sigma$$

dR = # collision in dt of 1 bunch crossings



$$= \int n_1(x,y,z,t)\, n_2(x,y,z,t)\; 2\beta c\, dt\; \sigma_{int}\; dx\,dy\,dz$$

$$= \int \frac{N^2}{(2\pi)^3 \sigma_x^2\sigma_y^2\sigma_z^2} e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{(y-\frac{\alpha}{2}\beta c t)^2 + (y+\frac{\alpha}{2}\beta c t)^2}{2\sigma_y^2}} \times e^{-\frac{(z-\beta c t)^2 + (z+\beta c t)^2}{2\sigma_x^2}}\; 2\beta c\, dt\; \sigma_{int}\; dx\,dy\,dz$$

(17) 10^{11} proton/sec

Liquid H, $L = 1\,m$, $\rho = 0.07\,g/cm^3$

$$R = \frac{\text{Protons}}{\text{time}} \times \frac{\text{Target particles}}{\text{area}} \times \sigma_{int}$$

$$n_{Target} = 0.07 \times N_A\,cm^{-3} = 0.07 \times 10^{6} \times N_A\,m^{-3}$$

$$\frac{\text{Target parts}}{\text{area}} = n_{Target} \times L$$

$$R = 10^{11} \times 0.07 \times 10^{6} \times 6.022 \times 10^{23} \times \sigma_{int}$$

$$L = R/\sigma_{int} = 0.422 \times 10^{40}\,m^{-2} = 0.422 \times 10^{12}/\text{Barn}$$