(1) $\left|\frac{dp}{dt}\right| = \omega p = qvB = q\omega r B$

$$B = \frac{p}{qr} = \frac{\gamma\beta m c}{qr}$$

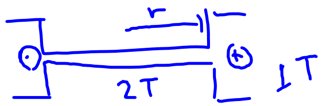
$$\gamma = \frac{E}{mc^2} = \frac{300 \text{ MeV}}{0.511 \text{ MeV}}, \quad \beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} = 1$$

$r = 1$ $\quad q = 1.602 \times 10^{-19}$ Coul.

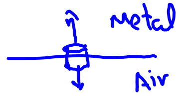
$c = 2.9979 \times 10^8$ m/s

Plug in $\Rightarrow B = 1$ T

$$\Phi = 2\pi r^2 B \Rightarrow \Phi = 2\pi r^2 B = 2\pi \text{ T-m}^2$$



$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \int_V \vec{\nabla} \cdot \vec{B} \, dV$



$$= \oint_S \vec{B} \cdot d\vec{S} = 0$$

$$= A\left(B_\perp^{metal} - B_\perp^{air}\right)$$

What if want 10 GeV Betatron?

$$r \sim \frac{\gamma\beta m c}{qB} \qquad \frac{r_{10\,GeV}}{r_{300\,MeV}} = \frac{\gamma_{10\,GeV}}{\gamma_{300\,MeV}}$$

$$= 33\tfrac{1}{3} \text{ m} \qquad = \frac{10 \text{ GeV}}{300 \text{ MeV}} = \frac{10}{.3} = 33\tfrac{1}{3}$$

② ρ resistivity

$$\delta = \left(2\rho / \mu_0 \omega\right)^{1/2}$$

$$\rho_s = \rho/\delta = \rho\left(\frac{\mu_0 \omega}{2\rho}\right)^{1/2}$$

$$= \left(\frac{\rho \mu_0 \omega}{2}\right)^{1/2}$$

$$\rho_{copper} = 1.7 \times 10^{-8}\ \Omega\text{-m}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\omega = 400\ \text{MHz} = 2\pi \times 4 \times 10^{8}\ \text{sec}^{-1}$$

Plug in $\Rightarrow$ $\rho_s = 5.2 \times 10^{-3}\ \Omega$

③ $R_s = \frac{Z_0^2}{\pi \rho_s} \frac{L}{R} \frac{T^2}{(1+R/L)\, J_1^2(2.405)}$

$T = \sin(\omega L/2v) \,/\, (\omega L/2v)$

$\frac{\omega R}{c} = 2.405 \Rightarrow$

$\frac{\omega L}{2v} = \frac{2.405\, c}{R} \frac{L}{2v} = \frac{2.405}{2\beta} \frac{L}{R}$

Define $x \equiv L/R$

$R_s = \underbrace{\frac{Z_0^2}{\pi \rho_s J_1^2(2.405)}}_{\alpha} \frac{x}{1+\frac{1}{x}} \frac{\sin^2\left(\frac{2.405}{2\beta} x\right)}{\left(\frac{2.405}{2\beta}\right)^2 x^2}$

$= \alpha \frac{1}{x+1} \sin^2(Kx)$ $\quad K = \frac{2.405}{2\beta}$

maximum is when $\frac{dR_s}{dx} = 0$

Equivalently when $\underbrace{\frac{d}{dx}(\log R_s)}_{\frac{1}{R_s}\frac{dR_s}{dx}} = 0$

$\log R_s = \log \alpha - \log(1+x) + 2 \log(\sin Kx)$

$\frac{d}{dx} \log R_s = \frac{-1}{1+x} + \frac{2K \cos Kx}{\sin Kx} = 0 \Rightarrow$

$\tan Kx = 2K(1+x)$

$r_s = \frac{R_s}{L} = \alpha \frac{1}{L} \frac{1}{L/R+1} \sin^2 \frac{KL}{R}$

$\log r_s = \log \alpha - \log L - \log(L/R+1) + 2 \log\left(\sin \frac{KL}{R}\right)$

$\frac{\partial \log r_s}{\partial L} = -1/L - \frac{1}{L/R+1} \frac{1}{R} + 2 \frac{K \cos \frac{KL}{R}}{R \sin \frac{KL}{R}} = 0$

$2K \frac{\cos KL/R}{\sin KL/R} = \frac{R}{L} + \frac{1}{L/R+1} = \frac{R}{L}\left(1 + \frac{L/R}{L/R+1}\right)$

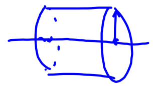
$\tan(Kx) = \frac{2Kx}{1 + \frac{x}{x+1}}$

(4) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} 2.405 \frac{c}{R}$

$= 1.64 E_k^2 e^{-8.5/E_k}$

Energy gain = $E L T \cos\phi$

$= 1.7 E_k L T$



$f = 400$ MHz

$\Rightarrow R = 28.7$ cm $= \frac{2.405 c}{\omega}$

Assume $v \approx c \Rightarrow \beta = 1$

$T: L/R = 2/3 \Rightarrow \frac{\omega L}{2v} = \frac{2.405c}{R}\frac{L}{2c}$

$= 0.8 = u$

$T = \frac{\sin u}{u} = 0.9$

$L = 2/3 R = 19.1$ cm

$400 = 1.64 E_k^2 e^{-8.5/E_k}$

$\Rightarrow E_k = 19.4 \frac{MV}{m}$

Plug in $\Rightarrow$ Gain = 5.67 MV

1 Cavity

Power Dissipation:

$P = \frac{1}{2} \rho_s \frac{\bar{E}_0^2}{Z_0^2} 2\pi R L \left(1 + \frac{R}{L}\right) J_1^2(2.405)$

$J_1(2.405) = 0.52$

$\rho_s = 0.518 \times 10^{-3}\,\Omega$ (problem 2, copper)

$E_0 = 33$ MV/m $= 1.7 E_k$

$Z_0 = 377\,\Omega$

Plug in $\Rightarrow P = 308$ kW

Title: Sep 14 - 3:21 PM (4 of 4)