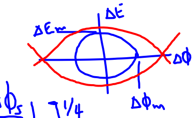
# Homework:

**2-10** Bucket + Bunch $\Delta\phi_m$, $\Delta E_m$




want $\left(\frac{\Delta p}{p_s}\right)_m = \frac{1}{\beta^2}\frac{\Delta E_m}{E_s}$

$$= \frac{1}{\beta^2}\frac{1}{E_s}\left[\left|\frac{\beta^2 E_s e\bar{V}\cos\phi_s}{\eta\,\omega_{rf}\,T^3}\right|\right]^{1/4}$$

$$\omega_{rf}T = h2\pi$$
$$\frac{1}{T} = \frac{\beta c}{L}$$

$$\left(\frac{\Delta p}{p_s}\right)_m = \frac{1}{\beta^2}\frac{1}{E_s}\left[\left|\frac{\beta^2 E_s}{\eta}\beta^2\right|\right]^{1/4}\underbrace{\left[\left|\frac{e\bar{V}\cos\phi_s \times c^2}{2\pi h L^2}\right|\right]^{1/4}}_{\text{constant}}$$

$$\propto \frac{1}{\beta}\left[\frac{1}{|\eta| E_s^3}\right]^{1/4}$$

**a** $\gamma << \gamma_T$ $\quad |\eta| = \left|\frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}\right| \approx \frac{1}{\gamma^2}$

$\propto 1/E_s^2$

$$\boxed{\frac{\Delta p}{p_s} \propto \frac{1}{\beta} E_s^{-1/4}}$$

$$\beta^2 = 1 - 1/\gamma^2 = \frac{\gamma^2 - 1}{\gamma^2} = \frac{(\gamma-1)(\gamma+1)}{\gamma^2}$$

$\gamma \approx 1$ @ low energy $\quad \approx 2(\gamma - 1)$

$$\approx \frac{2(E_s - mc^2)}{E_s}$$

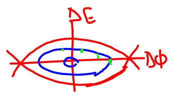
**b** $\gamma >> \gamma_T \Rightarrow \eta = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2} \approx \frac{1}{\gamma_T^2} = \text{const}$

$\beta = 1$

$$\left|\frac{\Delta p}{p_s}\right|_m \propto \frac{1}{\beta}\left|\frac{1}{|\eta| E_s^3}\right|$$

$$\propto E_s^{-3/4}$$

**11** Stationary Buckets $\phi_s = 0$.




$$\frac{d^2\phi}{dn^2} + \underbrace{\left[2\pi\,\nu_s(0)\right]^2}\sin\phi = 0$$

$$(2\pi\nu_s(0))^2 = \left(\frac{\eta\,\omega_{rf}\,T e\bar{V}}{\beta^2 E_s}\right.$$

Stable Fixed Pt

Unstable F.P.

Small oscillations:

$\sin\phi \approx \phi$

$$\frac{d^2\phi}{dn^2} + (2\pi\nu_s(0))^2\phi = 0$$

solution: $\phi = \hat{\phi}_m \sin\left[2\pi\nu_s(0)n\right]$

$$\left\{\frac{d^2\phi}{dn^2} + (2\pi\nu_s(0))^2 \sin\phi = 0\right\} \times 2\frac{d\phi}{dn}$$

$$\frac{d}{dn}\left(\frac{d\phi}{dn}\right)^2 + 2(2\pi\nu_s(0))^2 \frac{d}{dn}(-\cos\phi) = 0$$

$$\frac{d}{dn}\left[\left(\frac{d\phi}{dn}\right)^2 - 2(2\pi\nu_s(0))^2\cos\phi\right] = 0$$

$$\left(\frac{d\phi}{dn}\right)^2 - 2(2\pi\nu_s(0))^2\cos\phi = \text{constant}$$

$$= -(2\pi\nu_s(0))^2\cos\phi_m$$

$$\frac{d\phi}{dn} = \sqrt{2}(2\pi\nu_s(0))\left(\cos\phi - \cos\phi_m\right)^{1/2}$$

$$\sqrt{2}(2\pi\nu_s(0))\int_0^n dn = \oint \frac{d\phi}{\sqrt{\cos\phi - \cos\phi_m}}$$

If $\nu_s$ = tune. Phase advance $= 2\pi\nu_s$ in 1 Turn $\Rightarrow$ Phase advance $= 2\pi$ in $1/\nu_s$ turns. If phase advance $= 2\pi$ then $\int_0^n dn = n = 1/\nu_s$

$$\frac{\sqrt{2}\ 2\pi\ \nu_s(0)}{\nu_s(\phi_m)} = 4\int_0^{\phi_m}\frac{d\phi}{\sqrt{2\left(\sin^2\frac{\phi_m}{2} - \sin^2\frac{\phi}{2}\right)}}$$

$$\cos\phi - \cos\phi_m = 1 - 2\sin^2\frac{\phi}{2} - 1 + 2\sin^2\frac{\phi_m}{2}$$

Define $\sin x \equiv \dfrac{\sin\phi/2}{\sin\phi_m/2}$

$$\cos x = \sqrt{1 - \sin^2\phi/2 \,/\, \sin^2\phi_m/2}$$

$$\cos x\, dx = \frac{1}{\sin\phi_m/2}\,\frac{1}{2}\cos\frac{\phi}{2}\,d\phi$$

$$\sqrt{2}\ 2\pi\ \frac{\nu_s(0)}{\nu_s(\phi_m)} = \frac{4}{\sqrt{2}}\int_0^{\pi/2}\frac{2\cos x\,dx\,\sin\frac{\phi_m}{2}}{\left(\sqrt{\sin^2\frac{\phi_m}{2}(1-\sin^2 x)}\right)\times(\cos\phi/2)}$$

$$\cos\phi/2 = \sqrt{1 - \sin^2\phi/2} = \sqrt{1 - \sin^2\frac{\phi_m}{2}\sin^2 x}$$

$$\frac{\nu_s(0)}{\nu_s(\phi_m)} = \frac{2}{\pi}\int_0^{\pi/2}\frac{\cos x\,dx}{\cos x\sqrt{1 - \sin^2\phi_m/2\,\sin^2 x}}$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\frac{dx}{\sqrt{1 - \sin^2\phi_m/2\,\sin^2 x}} = \frac{2}{\pi}K\left(\sin^2\frac{\phi_m}{2}\right)$$