New and Old Homework:

(19) $\cos\mu = \frac{1}{2}\mathrm{Tr}\,M = \pm\frac{1}{8}$

$\sin\mu = \sqrt{1-\cos^2\mu} = \left(1-\frac{1}{64}\right)^{1/2}$

$= \frac{\sqrt{63}}{8} = \frac{\sqrt{7}\,3}{8}$

$\beta = \frac{\beta_0}{\sin\mu} = \sqrt{\frac{64}{63}}\,\beta_0$

(20) $p = 10\ \mathrm{GeV}/c$

$pc = 10\ \mathrm{GeV}$

$p = \gamma m v = \gamma m \beta c = \gamma\beta m c$

$pc = \gamma\beta m c^2 = \gamma\beta \cdot \text{Rest mass}$

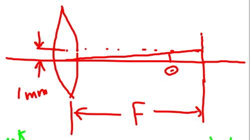
$\gamma\beta = pc/\text{Rest mass} = \frac{10\ \mathrm{GeV}}{0.93827\ \mathrm{GeV}}$

$\simeq 11$

(22) $\delta = 1\ \mathrm{mm}$

$\tan\theta = \frac{\delta}{F}$

$\approx \theta$



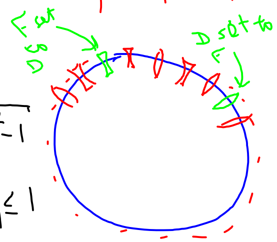
(25)

Stability:

$\lambda = \frac{\mathrm{Tr}\,M}{2} \pm \sqrt{\left(\frac{\mathrm{Tr}\,M}{2}\right)^2 - 1}$

stable $\Longleftrightarrow \left|\frac{\mathrm{Tr}\,M}{2}\right| \leq 1$

$\lambda_+\lambda_- = 1$



Assume n FODOs in Ring

Assume one messed up (backwards) magnet:

(1) Backward F magnet

$$\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix} =$$

$$= \begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{2}{F} & 1\end{pmatrix}$$

$$= M_{\text{FODO focus}} \times \begin{pmatrix}1 & 0\\ \frac{2}{F} & 1\end{pmatrix}$$

Defocusing Backwards:

$$M_{FODO\ \text{defocusing}} \begin{pmatrix} 1 & 0 \\ -\frac{2}{F} & 1 \end{pmatrix}$$

Focusing case: $M_{BAD} = M_{FODO} \times \begin{pmatrix} 1 & 0 \\ \frac{2}{F} & 1 \end{pmatrix}$

$$M_{RING} = M_{FODO}^{n-1} M_{BAD}$$

$$= M_{FODO}^{n} \begin{pmatrix} 1 & 0 \\ \frac{2}{F} & 1 \end{pmatrix}$$

$$M_{FODO} = e^{\mu J}$$

μ = phase advance for FODO (usually 60°–90°)

$$J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$$

$$M_{FODO}^{n} = e^{\mu n J}$$

$$= I \cos n\mu + J \sin n\mu$$

$$M_{RING} = \begin{bmatrix} \cos n\mu + \alpha \sin n\mu & \beta \sin n\mu \\ -\gamma \sin n\mu & \cos n\mu - \alpha \sin n\mu \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{F} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos n\mu + \alpha \sin n\mu + \frac{2}{F}\beta \sin n\mu & \beta \sin n\mu \\ -\gamma \sin n\mu + \frac{2}{F}\cos n\mu - \frac{2}{F}\alpha \sin n\mu & \cos n\mu - \alpha \sin n\mu \end{bmatrix}$$

$$\frac{1}{2} \operatorname{Tr} M_{ring} = \cos n\mu + \frac{\beta}{F} \sin n\mu$$

$$= \cos n\mu + 2\left(\frac{1+\sin\frac{\mu}{2}}{1-\sin\frac{\mu}{2}}\right)^{1/2} \sin n\mu$$

26

$$\delta\nu = \frac{1}{4\pi} \sum_i \beta_i \, \Delta\left(\frac{1}{f}\right)_i$$

$$M = \begin{pmatrix} \cos\sqrt{K+\Delta K}\, L & \frac{1}{\sqrt{K+\Delta K}} \sin\sqrt{K+\Delta K}\, L \\ -\sqrt{K+\Delta K} \sin\sqrt{K+\Delta K}\, L & \cos\sqrt{K+\Delta K}\, L \end{pmatrix}$$

$$\left.\begin{matrix} L \to 0 \\ K \to \infty \end{matrix}\right\} LK \text{ finite}, \ L\sqrt{K} \to 0$$

$$M = \begin{pmatrix} 1 & 0 \\ \underbrace{-(K+\Delta K)L}_{-\left(\frac{1}{f} + \Delta\frac{1}{f}\right)} & 1 \end{pmatrix} \Rightarrow \Delta\frac{1}{f} = \Delta K L$$

$$K = \frac{B_N'}{RB}$$

$$\Delta K = \frac{\Delta B_N'}{RB}$$

$$\delta\nu = \frac{1}{4\pi} \sum_i \beta_i \Delta\left(\frac{1}{f_i}\right) = \frac{1}{4\pi} \sum_i \beta_i \Delta K_i L_i$$

$$= \frac{1}{4\pi} \oint \beta \Delta K \, ds$$

$$\delta\nu = \frac{1}{4\pi}\sum_i \beta_i \, \Delta\left(\frac{1}{f}\right)_i$$

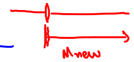
$$\delta\nu^2 = \frac{1}{(4\pi)^2}\sum_i \beta_i \Delta\left(\frac{1}{f}\right)_i \sum_j \beta_j \Delta\left(\frac{1}{f}\right)_j$$

$$= \frac{1}{(4\pi)^2}\left\{ \overbrace{\sum_i}^{N\,\text{errors}} \beta_i^2 \left(\Delta\left(\frac{1}{f}\right)_i\right)^2 + \underbrace{\sum_{i \neq j} \beta_i \beta_j \Delta\left(\frac{1}{f}\right)_i \Delta\left(\frac{1}{f}\right)_j}_{0} \right\}$$

$$\approx \frac{1}{(4\pi)^2} N \bar{\beta}^2 \left\langle \Delta\left(\frac{1}{f}\right)^2 \right\rangle$$

$$\delta\nu = \frac{\sqrt{N}}{4\pi}\bar{\beta}\left\langle \Delta\left(\frac{1}{f}\right)^2 \right\rangle^{1/2}$$

27 $\quad \Delta\left(\frac{1}{f}\right) = q = \frac{\Delta B_N'}{\rho B}$


Mnew

$$M_{old} = \begin{bmatrix} \left(\frac{\beta_s}{\beta_1}\right)^{1/2}(\cos\psi_0 + \alpha_1 \sin\psi_0) & (\beta_1\beta_s)^{1/2}\sin\psi_0 \\ m_{21} & m_{22} \end{bmatrix}$$

$$M_{New} = \begin{bmatrix} \left(\frac{\beta_s^N}{\beta_1}\right)^{1/2}(\cos\psi_N + \alpha_1 \sin\psi_N) & (\beta_1\beta_s^N)^{1/2}\sin\psi_N \\ m_{21}^N & m_{22}^N \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{\beta_s}{\beta_1}\right)^{1/2}(\cos\psi_0 + \alpha_1 \sin\psi_0) & (\beta_1\beta_s)^{1/2}\sin\psi_0 \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ q & 1 \end{bmatrix}$$

$$11 \Rightarrow \left(\frac{\beta_s^N}{\beta_1}\right)^{1/2}(\cos\psi_N + \alpha_1\sin\psi_N) = \left(\frac{\beta_s}{\beta_1}\right)^{1/2}(\cos\psi_0 + \alpha_1\sin\psi_0) + q(\beta_1\beta_s)^{1/2}\sin\psi_0$$

$$12 \Rightarrow (\beta_s^N \beta_1)^{1/2}\sin\psi_N = (\beta_1\beta_s)^{1/2}\sin\psi_0$$

$$\boxed{(\beta_s^N)^{1/2}\sin\psi_N = (\beta_s)^{1/2}\sin\psi_0}$$

$$11 \rightarrow (\beta_s^N)^{1/2}\underbrace{\cos\psi_N}_{\sqrt{1-\sin^2\psi_N}} + \alpha_1(\beta_s^N)^{1/2}\sin\psi_N = \beta_s^{1/2}\cos\psi_0 + \alpha_1\beta_s^{1/2}\sin\psi_0 + q\beta_1\beta_s^{1/2}\sin\psi_0$$

$$(\beta_s^N - \underbrace{\beta_s^N\sin^2\psi_N}_{\beta_s\sin^2\psi_0})^{1/2} + \alpha_1\beta_s^{1/2}\sin\psi_0$$

$$= \beta_s^{1/2}\cos\psi_0 + \alpha_1\beta_s^{1/2}\sin\psi_0 + q\beta_1\beta_s^{1/2}\sin\psi_0$$