$E_{cm} = 2\gamma m$

1000

$(ct, \vec{x})$

$(E, \vec{p}c)$

(1) $\overset{\beta c}{m} \longrightarrow \quad \longleftarrow \overset{\beta c}{m}$ (2)

$\xrightarrow[LT]{} \beta$

$E_1 = \gamma m c^2 \qquad E_2 = \gamma m c^2$

$p_1 = \gamma\beta m c \qquad p_2 = -\gamma\beta m c$

$E_1' = \gamma E_1 + \gamma\beta p_1 c = \gamma^2 m c^2 + \gamma^2\beta^2 m c^2$

$E_2' = \gamma E_2 + \gamma\beta p_2 c = \gamma^2 m c^2 - \gamma^2\beta^2 m c^2$

$E_1' = \gamma^2(1+\beta^2) m c^2 = \gamma^2\left(1+1-\frac{1}{\gamma^2}\right) m c^2 = \gamma^2\left(2-\frac{1}{\gamma^2}\right) m c^2$

$E_2' = \gamma^2(1-\beta^2) m c^2 = m c^2$

2×10^6

(3) L = length

$$E = mc^2 + (E_f - mc^2)\,x/L$$

$$\gamma = 1 + (\gamma_f - 1)\,x/L \qquad d\gamma = \frac{(\gamma_f - 1)}{L}\,dx$$

$$dt' = \gamma\,dt - \left(\frac{\gamma\beta}{c}\right)dx$$

$$= \left(\gamma\frac{dt}{dx} - \frac{\gamma\beta}{c}\right)dx = \left(\frac{\gamma}{\beta c} - \frac{\gamma\beta}{c}\right)\frac{L}{(\gamma_f - 1)}\,d\gamma$$

$$\gamma\beta = (\gamma^2 - 1)^{1/2}$$

(4) $$\vec F = \frac{d\vec p}{dt} = \frac{d}{dt}(\gamma m \vec v)$$

$$dE = \vec F \cdot d\vec s = \frac{d}{dt}(\gamma m \vec v) \cdot \frac{d\vec s}{dt}\, dt$$

$$= \vec v \cdot \frac{d}{dt}(\gamma m \vec v)\, dt = \vec v \cdot d(\gamma m \vec v)$$

$$= m v^2\, d\gamma + \gamma m \underbrace{\vec v \cdot d\vec v}_{=\frac{1}{2} dv^2}$$

$$= mc^2 \beta^2\, d\gamma + \frac{\gamma m c^2}{2}\, d\beta^2$$

$$= mc^2 \beta^2\, d\gamma + \frac{\gamma m c^2}{2}\, d\left(1 - 1/\gamma^2\right)$$

$$= mc^2 \beta^2\, d\gamma + mc^2 \frac{\gamma}{2} \frac{2}{\gamma^3}\, d\gamma$$

$$= mc^2\left(1 - \frac{1}{\gamma^2}\right) d\gamma + mc^2 \frac{1}{\gamma^2}\, d\gamma = mc^2\, d\gamma$$

(5) $p^2c^2 + m^2c^4 = (\gamma m v)^2 c^2 + m^2c^4$

$= \gamma^2 m^2 \beta^2 c^4 + m^2 c^4$

$= m^2c^4(\gamma^2\beta^2 + 1)$ $\quad \left(\gamma\beta = \sqrt{\gamma^2 - 1} \right.$

$= m^2c^4\gamma^2$

$= (mc^2\gamma)^2$

$= E^2$

8

$$B_{earth} = 1\,g = 10^{-4}\,T$$

$$g = 9.8\ m/s^2$$

$$mg = qvB$$

$m = 1.67 \times 10^{-27}$ kg

$q = 1.6 \times 10^{-19}$

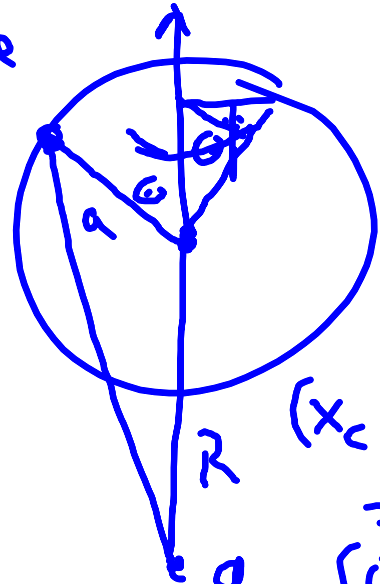
(9) solid Angle:

$d\Omega = \sin\theta\, d\theta\, d\phi$

$$\langle \Phi \rangle = \frac{1}{4\pi} \int_{\text{surface}} \Phi \, d\Omega$$

sphere

no charge inside



$(x_c, y_c, z_c) = (0, 0, R)$

$x = R + a\sin\theta\cos\phi$

$y = R + a\sin\theta\sin\phi$

$z = R + a\cos\theta$

(13) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \cancel{\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}}$

$\cong 0$

$\vec{B} = \vec{\nabla} \Phi_m$

$\vec{\nabla} \cdot \vec{B} = 0 = \nabla^2 \Phi_m$

$\vec{\nabla} \times \vec{H} = \vec{J}_{ext}$

$\vec{B} = \mu \vec{H}$

$\int \vec{\nabla} \times H \cdot d\vec{A} = \oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{A} = 2NI$

$$\nabla^2 \Phi_m = 0$$

Cylindrical coords:

$$\frac{1}{r}\partial_r\left(r\,\partial_r \Phi_m\right) + \frac{1}{r^2}\partial_\theta^2 \Phi_m = 0$$

$$\Phi_m = \sum_{m=0}^{\infty}\left[\Phi_{mc}(r)\cos m\theta + \Phi_{ms}(r)\sin m\theta\right]$$

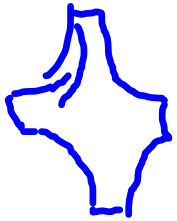
$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\Phi_{m\,c/s}\begin{Bmatrix}\cos m\theta\\ \sin m\theta\end{Bmatrix} - \frac{m^2}{r^2}\Phi_{m\,c/s}\begin{Bmatrix}\cos m\theta\\ \sin m\theta\end{Bmatrix} = 0$$

Guess $\Phi_m = a r^p$

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}(ar^p) - \frac{m^2}{r^2}ar^p = 0 \qquad p^2 - m^2 = 0$$

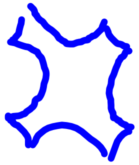
$$p^2 a r^{p-2} - m^2 a r^{p-2} = 0 \implies p = \pm m$$

$$\Phi_m = \phi_0 + \sum_{m=1}^{\infty} \left(a_{mc} r^m \cos m\theta + a_{ms} r^m \sin m\theta \right)$$



$$\frac{1}{r}\frac{d}{dr} r \frac{d\Phi_0}{dr} = 0$$

$$\Phi_0 = a_0 + \cancel{a_{log}}^{\,0} \ln r$$



$$\Phi_m = \sum_{m=1}^{\infty} \left(a_{mc} \cos m\theta + a_{ms} \sin m\theta \right) r^m$$

$r^1 \Rightarrow$ dipole

$r^2 \Rightarrow$ quadrupole

$r^3 \rightarrow$ sextupole

$r^4 \rightarrow$ octupole

$r^{2l} \rightarrow 2l$ pole