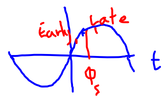
L = distance between cavities



$$L = v_s \tau_s$$

$$\log L = \log v_s + \log \tau_s$$

$$\frac{\Delta L}{L} = \frac{\Delta v_s}{v_s} + \frac{\Delta \tau_s}{\tau_s}$$

$\frac{\Delta v}{v_s}$:

$$p = \gamma m v$$

$$\Delta p = \Delta \gamma\, m v + \gamma m \Delta v$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\Delta \gamma = -\frac{1}{2} \frac{-\frac{2v}{c^2} \Delta v}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} = \frac{v}{c} \frac{\Delta v}{c} \gamma^3$$

$$\Delta p = m v \gamma^3 \frac{v}{c} \frac{\Delta v}{c} + m \gamma \Delta v$$

$$= \gamma m \Delta v \left(1 + \gamma^2 \beta^2\right)$$

$$= \gamma^3 m \Delta v \qquad \left(1 + \gamma^2\left(1 - \frac{1}{\gamma^2}\right)\right) = \gamma^2$$

$$\frac{\Delta p}{p_s} = \frac{\gamma m \Delta v\, \gamma^2}{\gamma m v_s} = \frac{\gamma^2 \Delta v}{v_s}$$

$$\frac{\Delta v}{v_s} = \frac{1}{\gamma^2} \frac{\Delta p}{p_s}$$

$$\frac{\Delta L}{L} = \frac{\Delta p}{\gamma^2 p_s} + \frac{\Delta \tau}{\tau_s}$$

Definition of γ_T

$$\frac{\Delta L}{L} \equiv \frac{1}{\gamma_T^2} \frac{\Delta p}{p_s} \Rightarrow$$

$$\frac{\Delta \tau}{\tau_s} = \left(\frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}\right) \frac{\Delta p}{p_s}$$

$$\equiv \eta \frac{\Delta p}{p_s}$$

$\eta \equiv$ Phase slip factor

Linac: $\gamma_T = \infty$ No path length change with Δp.

Fixed B:

$$wp = qwrB$$

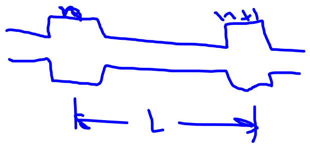
$$p + \Delta p = qB(r + \Delta r)$$

$$p\left(1 + \frac{\Delta p}{p}\right) = qBr\left(1 + \frac{\Delta r}{r}\right) \Rightarrow 1 + \frac{\Delta p}{p} = 1 + \frac{\Delta r}{r}$$

$$\frac{\Delta p}{p} = \frac{\Delta r}{r} = \frac{\Delta L}{L}$$

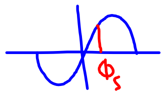
$\gamma_T = 1$

$$\frac{\Delta \tilde{T}}{\tilde{T}} = \eta \frac{\Delta P}{P}$$



$$\phi^s_{n+1} = \phi^s_n = \phi^s$$

$$\phi_{n+1} = \phi_n + \omega_{rf} \Delta \tilde{T}$$



$$= \phi_n + \omega_{rf} \tilde{T}_s \frac{\Delta \tilde{T}}{\tilde{T}_s}$$

$$= \phi_n + \omega_{rf} \tilde{T}_s \eta \frac{\Delta P}{P_s}$$

$$\frac{\Delta P}{P_s} = \frac{1}{\beta^2} \frac{\Delta E}{E_s}$$

$$= \phi_n + \frac{\omega_{rf} \tilde{T}_s \eta}{\beta^2 E_s} \Delta E$$

$$\Delta\phi_{n+1} = \Delta\phi_n + \frac{\omega_{rf} \tilde{T}_s \eta}{\beta^2 E_s} \Delta E$$

$$\boxed{\frac{d}{dn}(\Delta\phi) = \frac{\omega_{rf} \tilde{T}_s \eta}{\beta^2 E_s} \Delta E}$$

$$E^s_{n+1} = E^s_n + e\bar{V} \sin\phi_s$$

$$E_{n+1} = E_n + e\bar{V} \sin\phi_{n+1}$$

$$E_{n+1} - E^s_{n+1} = \Delta E_{n+1}$$

$$\Delta E_{n+1} = \Delta E_n + e\bar{V}(\sin\phi_{n+1} - \sin\phi_s)$$

$$\boxed{\frac{d}{dn}(\Delta E) = e\bar{V}(\sin\phi - \sin\phi_s)}$$

$$\boxed{\begin{aligned} \phi &\equiv \phi_s + \Delta\phi \\ E &\equiv E_s + \Delta E \end{aligned}}$$

$$\frac{d}{dn}(\Delta\phi) = \frac{\eta\, \omega_{rf} T_s}{\beta_s^2 E_s} \Delta E$$

$$\frac{d}{dn}(\Delta E) = e\hat{V}(\sin\phi - \sin\phi_s)$$

Small $\Delta\phi$, ΔE:

$$\sin\phi - \sin\phi_s = \sin(\phi_s + \Delta\phi) - \sin\phi_s$$

$$= \sin\phi_s \cos\Delta\phi + \cos\phi_s \sin\Delta\phi - \sin\phi_s$$

$$= \sin\phi_s(\cos\Delta\phi - 1) + \cos\phi_s \sin\Delta\phi$$

$\Delta\phi$ small:

$$\cos\Delta\phi - 1 = -1 + 1 - \frac{1}{2}\Delta\phi^2 + \frac{1}{4!}\Delta\phi^4 + \dots$$

$$= -\tfrac{1}{2}\Delta\phi^2 + \dots$$

Ignore!

$$\sin\Delta\phi \simeq \Delta\phi - \frac{1}{3!}\Delta\phi^3 + \dots \simeq \Delta\phi$$

$$\frac{d\Delta E}{dn} \simeq e\hat{V}\cos\phi_s\, \Delta\phi$$

$$\frac{d\Delta\phi}{dn} = \eta\frac{\omega_{rf} T_s}{\beta_s^2 E_s} \Delta E$$

$$\frac{d^2\Delta\phi}{dn^2} = \eta\frac{\omega_{rf} T_s}{\beta_s^2 E_s} \frac{d}{dn}\Delta E$$

$$= \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s} e\hat{V}\cos\phi_s\, \Delta\phi$$

$$\equiv -(2\pi\nu_s)^2\, \Delta\phi$$

$\nu_s \equiv$ Synchrotron tune

$$\Delta\phi \approx \cos(2\pi\nu_s n)$$

$$\frac{d}{dn}\Delta\phi = \frac{\eta\,\omega_{rf} T}{\beta_s^2 E_s}\Delta E$$

$$\frac{d}{dn}\Delta E = e\bar{V}\left(\sin\phi - \sin\phi_s\right)$$

$$= e\bar{V}\left(\sin(\phi_s+\Delta\phi) - \sin\phi_s\right)$$

$$\frac{d^2\Delta\phi}{dn^2} = \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\frac{d}{dn}\Delta E$$

$$= \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\left(\sin(\phi_s+\Delta\phi) - \sin\phi_s\right)$$

$$\frac{d\Delta\phi}{dn}\frac{d^2\Delta\phi}{dn^2} = \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\left(\sin(\phi_s+\Delta\phi) - \sin\phi_s\right) \times \frac{d\Delta\phi}{dn}$$

$$\frac{d}{dn}\,\frac{1}{2}\left(\frac{d}{dn}\Delta\phi\right)^2 = \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\,\frac{d}{dn}\left[-\cos(\phi_s+\Delta\phi) - \sin\phi_s\,\Delta\phi\right]$$

$$\frac{d}{dn}\left[\frac{1}{2}\left(\frac{d\Delta\phi}{dn}\right)^2 + \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\left(\cos(\phi_s+\Delta\phi) + \sin\phi_s\,\Delta\phi\right)\right] = 0$$

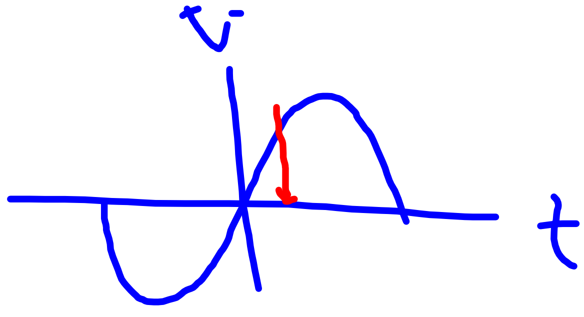
$$\frac{1}{2}\Big(\underbrace{\frac{d\Delta\phi}{dn}}_{\frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\Delta E}\Big)^2 + \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\left(\cos(\phi_s+\Delta\phi) + \Delta\phi\sin\phi_s\right) = \text{constant}$$

$$\frac{1}{2}\left(\frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\right)^2 \Delta E^2 + \frac{\eta\,\omega_{rf} T_s}{\beta_s^2 E_s}\, e\bar{V}\left(\cos(\phi_s+\Delta\phi) + \Delta\phi\sin\phi_s\right) = \text{const}$$

$$\boxed{\Delta E^2 + 2\,\frac{\beta_s^2 E_s}{\eta\,\omega_{rf} T_s}\, e\bar{V}\left(\cos(\phi_s+\Delta\phi) + \Delta\phi\sin\phi_s\right) = \text{const}}$$

$$\Delta E^2 + \frac{2\beta_s^2 E_s e \hat{V}}{\eta \omega_{rf} T_s}\left(\cos\phi + \phi \sin\phi_s\right) = \text{const}$$

$$\Delta E^2 + \frac{2\beta_s^2 E_s e \hat{V}}{\eta \omega_{rf} T_s}\cos\phi$$



$$= \text{constant}$$

$$\text{constant} = \frac{2\beta_s^2 E_s e \hat{V}}{\eta \omega_{rf} T_s}\left(\cos\phi\right)_{max}$$

$$\Delta E^2 = \frac{2\beta_s^2 E_s e \hat{V}}{\eta \omega_{rf} T_s}$$