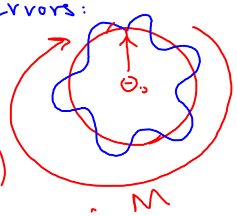
Orbit Changing Errors:

$$M\begin{pmatrix}x\\x_0\end{pmatrix}+\begin{pmatrix}0\\\Theta_0\end{pmatrix}=\begin{pmatrix}x\\x_0\end{pmatrix}$$

$$(I-M)\begin{pmatrix}x\\x_0\end{pmatrix}=\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$



$$e^{\mu J}=I\cos\mu+J\sin\mu$$

$$e^{(\mu_1+\mu_2)J}=e^{\mu_1 J}e^{\mu_2 J}$$

$$e^{-\mu J}e^{\mu J}=e^{(-\mu+\mu)J}=e^{0J}=I\cos 0+J\sin 0$$

So
$$\Big(\underbrace{e^{\mu\frac{J}{2}}e^{-\mu\frac{J}{2}}}_{I}-\underbrace{e^{\mu\frac{J}{2}}e^{\mu\frac{J}{2}}}_{M}\Big)\begin{pmatrix}x\\x'\end{pmatrix}_0=\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$
$=I$

$$e^{\mu J/2}\left(e^{-\mu J/2}-e^{\mu J/2}\right)\begin{pmatrix}x\\x'\end{pmatrix}_0=\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

$$\left(I\cos\frac{\mu}{2}-J\sin\frac{\mu}{2}-I\cos\frac{\mu}{2}-J\sin\frac{\mu}{2}\right)\begin{pmatrix}x\\x'\end{pmatrix}_0=e^{-\mu J/2}\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

$$-2J\sin\frac{\mu}{2}\begin{pmatrix}x\\x'\end{pmatrix}_0=e^{-\mu J/2}\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

$-J^{-1}=J$

$$\begin{pmatrix}x\\x'\end{pmatrix}_0=\frac{-J^{-1}}{2\sin\mu/2}e^{-\mu J/2}\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

$$=\frac{J}{2\sin\mu/2}\left(I\cos\frac{\mu}{2}-J\sin\frac{\mu}{2}\right)\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

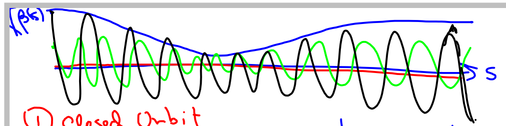
$$=\frac{1}{2\sin\mu/2}\left(I\sin\frac{\mu}{2}+J\cos\frac{\mu}{2}\right)\begin{pmatrix}0\\\Theta_0\end{pmatrix}$$

$$\begin{pmatrix}x\\x'\end{pmatrix}_0=\frac{1}{2\sin\frac{\mu}{2}}\begin{bmatrix}\sin\mu/2+\alpha_0\cos\mu/2 & \beta_0\cos\frac{\mu}{2}\\-\gamma_0\cos\frac{\mu}{2} & \sin\mu/2-\alpha_0\cos\frac{\mu}{2}\end{bmatrix}\begin{bmatrix}0\\\Theta_0\end{bmatrix}$$

$$\begin{bmatrix}x\\x'\end{bmatrix}_0=\frac{\Theta_0}{2\sin\frac{\mu}{2}}\begin{bmatrix}\beta_0\cos\mu/2\\\sin\frac{\mu}{2}-\alpha_0\cos\frac{\mu}{2}\end{bmatrix}$$
Closed Orbit

We can always write

$$x=A\sqrt{\beta}\cos(\psi-\delta)$$



① Closed Orbit
② Small Oscillation
③ Large Oscillation

$$\alpha = -\frac{\beta'}{2}$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Closed Orbit: $\begin{pmatrix} x \\ x' \end{pmatrix}_0 = \frac{\theta_0}{2\sin\mu/2}\begin{bmatrix} \beta_0 \cos\frac{\mu}{2} \\ \sin\frac{\mu}{2} - \alpha_0\cos\frac{\mu}{2} \end{bmatrix}$

$$x = A\sqrt{\beta}\cos(\psi-\delta)$$

$$x' = -A\sqrt{\beta}\,\frac{1}{\beta}\sin(\psi-\delta) + \frac{A\sqrt{\beta}}{2\beta}\beta'\cos(\psi-\delta)$$

$$= -A\sqrt{\beta}\,\frac{1}{\beta}\sin(\psi-\delta) - A\sqrt{\beta}\,\frac{\alpha}{\beta}\cos(\psi-\delta)$$

$$x_0 = A\sqrt{\beta_0}\cos(-\delta) = A\sqrt{\beta_0}\cos\delta$$

$$x_0' = -A\sqrt{\beta_0}\,\frac{1}{\beta_0}\sin(-\delta) - A\sqrt{\beta_0}\,\frac{\alpha_0}{\beta_0}\cos(-\delta)$$

$$= A\sqrt{\beta_0}\,\frac{1}{\beta_0}\sin\delta - A\sqrt{\beta_0}\,\frac{\alpha_0}{\beta_0}\cos\delta$$

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① $A\sqrt{\beta_0}\cos\delta = \frac{\theta_0}{2\sin\mu/2}\beta_0\cos\mu/2$

② $A\sqrt{\beta_0}\,\frac{1}{\beta_0}\sin\delta - A\sqrt{\beta_0}\,\frac{\alpha_0}{\beta_0}\cos\delta = \frac{\theta_0}{2\sin\mu/2}\times$

$\times\left[\sin\frac{\mu}{2} - \alpha_0\cos\frac{\mu}{2}\right]$

Try $\delta = \mu/2$, Then ① $A = \frac{\theta_0}{2\sin\mu/2}\sqrt{\beta_0}$

Substitute into ② $\frac{\theta_0}{2\sin\mu/2}\sin\mu/2 - \frac{\theta_0}{2\sin\mu/2}\alpha_0\cos\mu/2$ ✓

Finally

$$x = \frac{\theta_0}{2\sin\mu/2}\sqrt{\beta_0\beta}\cos(\psi - \mu/2)$$

$\mu \equiv 2\pi\nu$ where ν = Tune

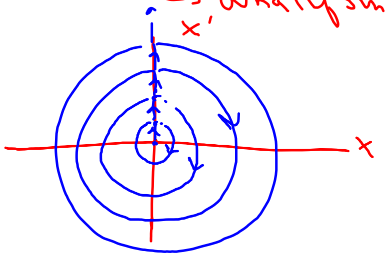
$$x = \frac{\theta_0}{2\sin\pi\nu}\sqrt{\beta_0\beta}\cos(\psi - \pi\nu)$$

Closed Orbit with error θ_0 at 0.

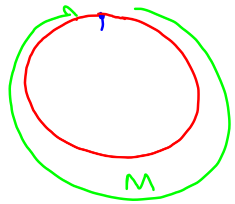
Closed Orbit with Error

$$x = \frac{\theta_0}{\underbrace{2\sin\pi\nu}}\sqrt{\beta\beta_0}\,\cos(\psi - \pi\nu)$$

→ What if $\sin\pi\nu = 0$?



Focusing Errors :



$\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$ Quad → desired

$\begin{pmatrix} 1 & 0 \\ \frac{1}{f}+\Delta\frac{1}{f} & 1 \end{pmatrix}$ Quad What you actually have.

$$= \underbrace{\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}}_{\text{Already in period so}}\underbrace{\begin{pmatrix} 1 & 0 \\ \Delta 1/f & 1 \end{pmatrix}}_{\text{Represents error.}} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f}+\Delta\frac{1}{f} & 1 \end{pmatrix}$$

So focusing error represents as thin lens : $\begin{pmatrix} 1 & 0 \\ \Delta\frac{1}{f} & 1 \end{pmatrix}$

and periodic matrix

$$M_0 \rightarrow M = M_0\begin{pmatrix} 1 & 0 \\ \Delta 1/f & 1 \end{pmatrix}$$