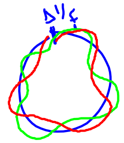
# Focusing Errors

$\begin{pmatrix} 1 & 0 \\ \Delta\frac{1}{f} & 1 \end{pmatrix}$




$M_0$ = desired error-free matrix

Actual matrix $M = M_0 \begin{pmatrix} 1 & 0 \\ -\Delta\frac{1}{f} & 1 \end{pmatrix}$

$\mu = 2\pi\nu \qquad \mu_0 = 2\pi\nu_0$

$$M = \begin{pmatrix} \cos 2\pi\nu_0 + \alpha\sin 2\pi\nu_0 & \beta\sin 2\pi\nu_0 \\ -\gamma\sin 2\pi\nu_0 & \cos 2\pi\nu_0 - \alpha\sin 2\pi\nu_0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\Delta\frac{1}{f} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\pi\nu_0 + \left(\alpha - \Delta\frac{1}{f}\beta\right)\sin 2\pi\nu_0 & \beta\sin 2\pi\nu_0 \\ \left(\Delta\frac{1}{f}\alpha - \gamma\right)\sin 2\pi\nu_0 - \Delta\frac{1}{f}\cos 2\pi\nu_0 & \cos 2\pi\nu_0 - \alpha\sin 2\pi\nu_0 \end{pmatrix}$$

$$\cos 2\pi\nu = \frac{1}{2}\operatorname{Tr} M$$

$$= \cos 2\pi\nu_0 - \Delta\frac{1}{f}\,\frac{\beta}{2}\sin 2\pi\nu_0$$

$$\nu = \nu_0 + \Delta\nu$$

$$\cos(2\pi\nu_0 + 2\pi\Delta\nu) = \cos 2\pi\nu_0 \cos 2\pi\Delta\nu - \sin 2\pi\nu_0 \sin 2\pi\Delta\nu$$

$$= \cos 2\pi\nu_0 - \frac{\beta}{2}\Delta\left(\frac{1}{f}\right)\sin 2\pi\nu_0$$

$$\cos 2\pi\Delta\nu = 1 - \frac{1}{2}(2\pi\Delta\nu)^2 + \frac{1}{4!}(2\pi\Delta\nu)^4 - \ldots$$

$$\approx 1$$

$$\sin 2\pi\Delta\nu = 2\pi\Delta\nu - \frac{1}{3!}(2\pi\Delta\nu)^3 + \ldots$$

$$\simeq 2\pi\Delta\nu$$

$$\cos 2\pi\nu_0 \times 1 - \sin 2\pi\nu_0 \times 2\pi\Delta\nu = \cos 2\pi\nu_0 - \frac{\beta}{2}\Delta\left(\frac{1}{f}\right)\sin 2\pi\nu_0$$

$$\boxed{\Delta\nu = \frac{\beta}{4\pi}\Delta\left(\frac{1}{f}\right)}$$

$$\boxed{\Delta\nu = \frac{\beta}{4\pi}\,\Delta\!\left(\frac{1}{f}\right)}$$

Quad Matrix
Focusing const $K$ : $\begin{bmatrix} \cos\sqrt{K}\,L & \frac{1}{\sqrt{K}}\sin\sqrt{K}\,L \\ -\sqrt{K}\sin\sqrt{K}\,L & \cos\sqrt{K}\,L \end{bmatrix}$
Length $L$

Consider error in $K \to K+\Delta K$

$$\begin{pmatrix} \cos\sqrt{K+\Delta K}\,L & \frac{1}{\sqrt{K+\Delta K}}\sin\sqrt{K+\Delta K}\,L \\ -\sqrt{K+\Delta K}\sin\sqrt{K+\Delta K}\,L & \cos\sqrt{K+\Delta K}\,L \end{pmatrix}$$

$$\left.\begin{array}{l} \text{Let } L \to \text{small} \\ K \to \text{big} \\ KL \to \text{finite} \\ \sqrt{K}L \to \text{small} \end{array}\right\} \Rightarrow \begin{pmatrix} 1 & 0 \\ (K+\Delta K)L & 1 \end{pmatrix}$$

$$(K+\Delta K)L = \frac{1}{f} + \Delta\!\left(\frac{1}{f}\right)$$

$$\Delta\nu = \frac{\beta}{4\pi}\,\Delta\!\left(\frac{1}{f}\right) = \frac{\beta L}{4\pi}\,\Delta K$$

$$K = \frac{B_N'}{RB_0} \qquad \Delta K = \frac{\Delta B_N'}{RB_0}$$

$$RB_0 \equiv P_0/q$$

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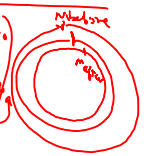
$$M_{Before} = \begin{pmatrix} \cos\mu_0 + \alpha_0\sin\mu_0 & \beta_0\sin\mu_0 \\ -\gamma_0\sin\mu_0 & \cos\mu_0 - \alpha_0\sin\mu_0 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 \\ -\Delta\left(\frac{1}{f}\right) & 1 \end{pmatrix}$$




$$M_{after} = \begin{pmatrix} 1 & 0 \\ -\Delta\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} \cos\mu_0 + \alpha_0\sin\mu_0 & \beta_0\sin\mu_0 \\ -\gamma_0\sin\mu_0 & \cos\mu_0 - \alpha_0\sin\mu_0 \end{pmatrix}$$

$$M_{before} = \begin{pmatrix} \cos\mu_0 + \alpha_0\sin\mu_0 - \beta_0\Delta\left(\frac{1}{f}\right) & \beta_0\sin\mu_0 \\ -\gamma_{before}\sin\mu & \cos\mu_0 - \alpha_0\sin\mu_0 \end{pmatrix}$$

$$M_{after} = \begin{pmatrix} \cos\mu_0 + \alpha_0\sin\mu_0 & \beta_0\sin\mu_0 \\ -\gamma_{after}\sin\mu & \cos\mu_0 - \alpha_0\sin\mu_0 - \beta\Delta\left(\frac{1}{f}\right)\sin\mu_0 \end{pmatrix}$$

$$\alpha_{\text{before}} \sin\mu = \frac{M_{\text{before }11} - M_{\text{before }22}}{2}$$

$$= \alpha_0 \sin\mu_0 - \frac{1}{2}\beta_0 \Delta\left(\frac{1}{f}\right) \sin\mu_0$$

$$\alpha_{\text{after}} \sin\mu = \frac{M_{\text{after }11} - M_{\text{after }22}}{2}$$

$$= \alpha_0 \sin\mu_0 + \frac{1}{2}\beta_0 \Delta\left(\frac{1}{f}\right) \sin\mu_0$$

$$\Delta\alpha = \alpha_{\text{after}} - \alpha_{\text{before}} = -\beta_0 \Delta\left(\frac{1}{f}\right) \frac{\sin\mu_0}{\sin\mu}$$

$$\Delta \approx -\beta_0 \Delta\left(\frac{1}{f}\right)$$