Adiabatic Damping:

Suppose acceleration

$$x'' + \underbrace{\frac{(\gamma\beta)'}{\gamma\beta} x'}_{\text{small}} + K(s)x = 0$$

Adiabatic $\Rightarrow$ $(\gamma\beta)' L \ll \gamma\beta$

Let $x = uv$

$$x' = u'v + uv'$$

$$x'' = u''v + 2u'v' + uv''$$

$$v u'' + \underbrace{\left(2v' + \frac{(\gamma\beta)'}{\gamma\beta} v\right)}_{\Rightarrow 0} u' + \left(v'' + \frac{(\gamma\beta)'}{\gamma\beta} v' + K v\right) u = 0$$

$$2v' + \frac{(\gamma\beta)'}{\gamma\beta} v = 0 \qquad \frac{2v'}{v} + \frac{(\gamma\beta)'}{\gamma\beta} = 0 \qquad \frac{v'}{v} = \frac{-(\gamma\beta)'}{2\gamma\beta} \ll 1/L$$

$$2\ln v + \ln(\gamma\beta) = \text{constant}$$

$$v = \frac{(\gamma_0\beta_0)^{1/2}}{(\gamma\beta)^{1/2}}$$

$$u'' + \Big(\underbrace{\frac{v''}{v}}_{\ll \frac{1}{L^2}} + \underbrace{\frac{(\gamma\beta)'}{\gamma\beta}\frac{v'}{v}}_{\ll \frac{1}{L^2}} + K\Big) u = 0$$

$$u'' + Ku = 0$$

$$x = \underbrace{\left(\frac{\gamma_0\beta_0}{\gamma\beta}\right)^{1/2}}_{v} \underbrace{\left(\frac{\varepsilon\beta(s)}{\pi}\right)^{1/2} \cos(\psi+\delta)}_{u}$$

$$x' = \underbrace{v'}_{\text{small}} u + v u'$$

$$\simeq v \left(\left(\frac{\varepsilon\beta(s)}{\pi}\right)^{1/2} \underbrace{\frac{1}{2\beta(s)}\beta'(s)}_{-\alpha/\beta} \cos(\psi+\delta) - \left(\frac{\varepsilon\beta(s)}{\pi}\right)^{1/2} \psi' \sin(\psi+\delta) \right) \qquad \psi' \to 1/\beta(s)$$

$$= v\left(-\frac{\alpha}{\beta} u - \frac{1}{\beta}\left(\frac{\varepsilon\beta(s)}{\pi}\right)^{1/2} \sin(\psi+\delta) \right)$$

$$v\left(\frac{\varepsilon\beta(s)}{\pi}\right)^{1/2} \sin(\psi+\delta) = -\alpha u v - \beta x'$$

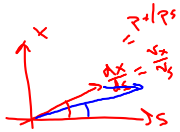
$$= -\alpha x - \beta x'$$

$$v^2 \frac{\varepsilon}{\pi} = \frac{x^2 + (\alpha x + \beta x')^2}{\beta(s)}$$

$$\varepsilon/\pi = \frac{\gamma\beta}{\gamma_0\beta_0} \frac{x^2 + (\alpha x + \beta x')^2}{\beta(s)} = \frac{\gamma\beta}{(\gamma\beta)_0} \frac{\varepsilon_{old}}{\pi}$$

$$\varepsilon_N = \gamma \beta \varepsilon_{old}$$

$$x' = \frac{dx}{ds} = \frac{p_x}{p}$$



Dispersion: $\delta \equiv dp/p_0$

$$x'' + K(s)\, x = \frac{\delta}{R}$$

Dispersion term

Consider solving:

$$x'' + Kx = f(s)$$

Assume solutions to $x''+Kx=0$

$C(s), S(s)$: $C(s_0)=1 \quad C'(s_0)=0$

$S(s_0)=0 \quad S'(s_0)=1$

$$\begin{vmatrix} C & S \\ C' & S' \end{vmatrix} = 1$$

Define: $G(s,s') \equiv S(s)\,C(s') - C(s)\,S(s')$

$$\frac{dG(s,s')}{ds} = G'(s,s') = S'(s)\,C(s') - C'(s)\,S(s')$$

$$\begin{aligned} G''(s,s') &= S''(s)C(s') - C''(s)S(s') \\ &= -K\big(S(s)\,C(s') - C(s)S(s')\big) \\ &= -KG(s,s') \end{aligned}$$

$s'=s$ ↘

$G(s,s) = 0 \qquad G'(s,s) = 1$

Want to solve $x'' + Kx = f(s)$

$$\mathcal{D}(s) \equiv \int_{s_0}^{s} G(s,s')\, f(s')\, ds'$$

$$\mathcal{D}(s) = \int_{s_0}^{s} G(s,s')\, f(s')\, ds'$$

$$\mathcal{D}'(s) = \underbrace{G(s,s) f(s)}_{0} + \int_{s_0}^{s} G'(s,s')\, f(s')\, ds'$$

$$\mathcal{D}''(s) = \underbrace{G'(s,s)}_{1} f(s) + \int_{s_0}^{s} G''(s,s')\, f(s')\, ds'$$

$$= f(s) - K \underbrace{\int_{s_0}^{s} G(s,s')\, f(s')\, ds'}_{\mathcal{D}(s)}$$

$$\boxed{\mathcal{D}''(s) + K(s)\,\mathcal{D}(s) = f(s)}$$

$$\mathcal{D}(s) = \int_{0}^{s} G(s,s')\, f(s')\, ds'$$

is a particular solution of

Note that $\mathcal{D}(0) = \mathcal{D}'(0) = 0$