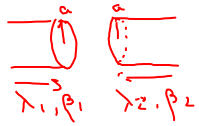
More Space Charge:

Colliding Beams:




Consider 2 uniform beams of particle densities $\lambda_1, \lambda_2$, velocities $\beta_1, \beta_2$, and radii $a$.

Find force on 1 due to 2:

E field: $E_r = \dfrac{q\lambda_2}{2\pi\varepsilon_0}\dfrac{r}{a^2}$ (inside $r \le a$ from Gauss' Law)

B field: $B_\theta = \dfrac{q\lambda_2}{2\pi\varepsilon_0}\dfrac{r}{a^2}\dfrac{\beta_2}{c}$ (inside $r < a$ from Ampere's Law)

$$= \frac{\beta_2}{c} E_r$$

$$\vec{F}_{2\to 1} = q\left(\vec{E} + \vec{v}_1 \times \vec{B}\right) = \frac{q^2\lambda_2}{2\pi\varepsilon_0}\frac{r}{a^2}\left(\hat{r} + \beta_1 c\,\hat{s} \times \frac{\beta_2}{c}\hat{\theta}\right)$$

$$= \frac{2\lambda_2 q^2}{4\pi\varepsilon_0}\frac{r}{a^2}\hat{r}\left(1 - \beta_1\beta_2\right)$$

Fraction of time in collision

$$x'' + \left(\frac{2\pi\nu_0}{L}\right)^2 x = \frac{F_x}{\gamma_1\beta_1^2 mc^2} f$$

$$x'' + \left(\frac{2\pi\nu_0}{L}\right)^2 x = \frac{2\lambda_2 f}{\gamma_1\beta_1^2}\overbrace{\left(\frac{q^2}{4\pi\varepsilon_0 mc^2}\right)}^{r_0}\frac{x}{a^2}\left(1 - \beta_1\beta_2\right)$$

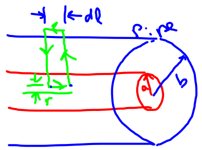
$$= \frac{2\lambda_2 r_0 f}{a^2}\frac{1-\beta_1\beta_2}{\gamma_1\beta_1^2}\,x$$

$$x'' + \left(\frac{2\pi}{L}\right)^2\left(\nu_0^2 - \left(\frac{L}{2\pi}\right)^2\frac{2\lambda_2 r_0 f}{a^2}\frac{1-\beta_1\beta_2}{\gamma_1\beta_1^2}\right)x$$

$$= 0 = x'' + \left(\frac{2\pi}{L}\right)^2\left(\nu_0 + \Delta\nu\right)^2 x$$

# Longitudinal Space Charge:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$




$$\oint_{\partial A} \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_A \vec{B}\cdot d\vec{s}$$

$$E_s(r)\,dl + \int_r^b E_r(r, s+dl)\,dr + 0\ (\text{from outside piece})$$

$$+\int_b^r E_r(r,s)\,dr = \oint \vec{E}\cdot\vec{dl}$$

Assume uniform in transverse, but $\lambda = \lambda(s)$

$$E_s\,dl + \int_r^b \left(E_r(s+dl) - E_r(s)\right) dr = \oint \vec{E}\cdot\vec{dl}$$

$$\oint \vec{E}\,\vec{dl} = E_s\cdot dl + \int_r^a \frac{\lambda(s+dl)-\lambda(s)}{2\pi\varepsilon_0}\frac{r}{a^2}\,dr$$

$$+\int_a^b \frac{\lambda(s+dl)-\lambda(s)}{2\pi\varepsilon_0}\frac{1}{r}\,dr$$

$$= E_s\,dl + \frac{\left(\lambda(s+dl)-\lambda(s)\right)}{2\pi\varepsilon_0}\begin{bmatrix} \left.\frac{r^2}{2a^2}\right|_r^a \\ + \left.\ln r\right|_a^b \end{bmatrix}$$

$$\oint E\cdot dl = \boxed{E_s\,dl + \frac{\lambda'\,dl}{2\pi\varepsilon_0}\begin{bmatrix} \frac{1}{2} - \frac{r^2}{2a^2} + \ln\frac{b}{a} & r<a \\ \ln b/r & r>a \end{bmatrix}}$$

$$= -\frac{d}{dt}\oint \vec{B}\cdot\vec{ds} = -\frac{d}{dt}\int_r^b dl\,dr\,B_\theta$$

$$= \beta c \frac{d}{ds}\int_r^b B_\theta\,dr\ dl \qquad B_\theta = \frac{\beta}{c}E_r$$

$$= dl\,\beta^2\frac{d}{ds}\int_r^b E_r\,dr = \boxed{dl\,\beta^2\frac{d}{ds}\frac{\lambda}{2\pi\varepsilon_0}\begin{bmatrix} \frac{1}{2} - \frac{r^2}{2a^2} + \ln\frac{b}{a} & r<a \\ \ln b/r & r>a \end{bmatrix}}$$

$$E_s = -\frac{\lambda'}{2\pi\varepsilon_0}\begin{bmatrix} \frac{1}{2} - \frac{r^2}{2a^2} + \ln\frac{b}{a} & r<a \\ \ln b/r & r>a \end{bmatrix}\left[1-\beta^2\right]$$

$$\left[1-\beta^2\right] \rightarrow 1/\gamma^2$$

$$E_s = -\frac{\lambda'}{2\pi\varepsilon_0\gamma^2}\begin{bmatrix} \frac{1}{2} - \frac{r^2}{2a^2} + \ln b/a & r<a \\ \ln b/r & r>a \end{bmatrix}$$