Synchrotron Tune Shift:

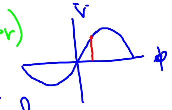
$$F_s^{sc} = qE_s \underset{+q(\vec{v}\times\vec{B})_s}{} = -\frac{q^2\lambda'}{2\pi\varepsilon_0\gamma^2}\begin{cases}\ln(b/r), & r>a\\ \ln(b/a)+\frac{1}{2}\left(1-\frac{r^2}{a^2}\right), & r<a\end{cases}$$

Longitudinal Motion:

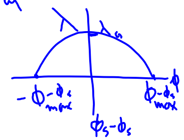
$$\frac{d}{dn}\Delta\phi = \frac{\eta\,\omega_{rf}\tilde{T}_s}{\beta_s^2 E_s}\Delta E$$

$$\frac{d}{dn}\Delta E = e\bar{V}(\sin\phi - \sin\phi_s) + L F_s^{sc}$$

$$q(\vec{v}\times\vec{B})_s = q(v_r B_\theta - v_\theta B_r)$$



Parabolic Longitudinal Profile:



$$\lambda = \lambda_0\left(1 - \frac{(\phi-\phi_s)^2}{\phi_{max}^2}\right)$$

$$\lambda' = \frac{d\lambda}{ds} = \frac{-2\lambda_0}{\phi_{max}^2}(\phi-\phi_s)\frac{d\phi}{ds}$$

$$\frac{d\phi}{ds} = \frac{-2\pi h}{L}$$ where h = rf harmonic #

$$\lambda' = \frac{4\pi h\lambda_0}{L\phi_{max}^2}\Delta\phi$$

$$e\bar{V}(\sin\phi - \sin\phi_s) \simeq e\bar{V}\cos\phi_s\,\Delta\phi$$

$$\frac{d}{dn}\Delta E = e\bar{V}\cos\phi_s\,\Delta\phi - \frac{Lq^2}{2\pi\varepsilon_0\gamma^2}\lambda'\,\text{Radial}$$

$$= \left(e\bar{V}\cos\phi_s - \frac{Lq^2}{2\pi\varepsilon_0\gamma^2}\text{Radial}\frac{4\pi h\lambda_0}{L\phi_{max}^2}\right)\Delta\phi$$

$$\frac{d\Delta\phi}{dn} = \frac{\eta\,\omega_{rf}\tilde{T}_s}{\beta_s^2 E_s}\Delta E \qquad \omega_{rf}\tilde{T}_s = 2\pi h$$

$$\frac{d^2\Delta\phi}{dn^2} = \frac{\eta\,2\pi h}{\beta_s^2 E_s}\frac{d\Delta E}{dn} = \frac{\eta\,2\pi h}{\beta^2 E_s}e\bar{V}\cos\phi_s \times$$

$$\times\left(1 - \text{Radial}\frac{8\pi h\lambda_0}{\gamma^2\phi_{max}^2}\frac{q^2}{4\pi\varepsilon_0}\frac{1}{e\bar{V}\cos\phi_s}\right)\Delta\phi$$

$$\frac{d^2\Delta\phi}{dn^2} = \frac{\eta 2\pi h}{\beta_s^2 E_s}\frac{d\Delta E}{dn} = \frac{\eta 2\pi h}{\beta^2 E_s} e\bar{V}\cos\phi_s \times$$

$$\times \left(1 - \text{Radial}\,\frac{8\pi h\lambda_0}{\gamma^2\phi_{max}^2}\,\frac{q^2}{4\pi\varepsilon_0}\,\frac{1}{e\bar{V}\cos\phi_s}\right)\Delta\phi$$

$$\frac{d^2\Delta\phi}{dn^2} = -(2\pi\nu_s)^2\Delta\phi$$

$$= -\left(2\pi\left(\nu_s^0+\delta\nu_{sc}\right)\right)^2\Delta\phi$$

small

$$= -(2\pi)^2\left(\nu_s^{0\,2}+2\nu_s^0\,\delta\nu_{sc}+\cancel{(\delta\nu_{sc})^2}\right)\Delta\phi$$

$$= -(2\pi\nu_s^0)^2\left(1+2\frac{\delta\nu_{sc}}{\nu_s^0}\right)\Delta\phi$$

$$= \frac{\eta 2\pi h}{\beta_s^2 E_s} e\bar{V}\cos\phi_s\left[1-\frac{\text{Radial}\;8\pi h\lambda_0 r_0 mc^2}{\gamma^2\phi_{max}^2\;\; e\bar{V}\cos\phi_s}\right]\Delta\phi$$

$$(2\pi\nu_s^0)^2 = -\frac{\eta 2\pi h}{\beta_s^2 E_s}e\bar{V}\cos\phi_s$$

$$\frac{\delta\nu_{sc}}{\nu_s^0} = -\text{Radial}\;\frac{4\pi h\lambda_0 r_0}{\gamma^2\phi_{max}^2}\;\frac{mc^2}{e\bar{V}\cos\phi_s}$$

SNS: Ring

$$\eta = -\frac{1}{\gamma^2}+\frac{1}{\gamma_T^2} \qquad h=1$$

$$\simeq -\frac{1}{7^2}+\frac{1}{5^2} = 0.04-0.25$$

$$\approx 0.2$$

$$\beta_s^2 = 0.75 \qquad E_s = 2\,\text{GeV} = 2\times10^6\,\text{keV}$$

$$\cos\phi_s = 1 \qquad (\phi_s = 0)$$

$$e\bar{V} = 40\,\text{kV}$$

$$(2\pi\nu_s^0)^2 = \frac{0.2\times2\pi}{0.75}\;\frac{40}{2\times10^6}$$

$$= 3.4\times10^{-5}$$

$$\boxed{\nu_s^0 \approx 10^{-3}}\ \text{SNS}$$

$$\text{Radial} \simeq 1$$

$$r_0 = 1.535\times10^{-18}\,\text{m}$$

$$\lambda_0 = \frac{2\times1.5\times10^{14}}{248}\,\text{m}^{-1}$$

$$= 1.21\times10^{12}\,\text{m}^{-1}$$

$$\phi_{max} = 120^\circ = \frac{2\pi}{3}$$

$$\gamma \approx 4 \qquad mc^2 = 1\,\text{GeV} \approx 10^6\,\text{keV}$$

$$\frac{\delta\nu_{sc}}{\nu_s^0} = \frac{-4\pi\times1.535\times10^{-18}\times1.21\times10^{12}\times.93827\times10^6}{4.267\times\left(\frac{2\pi}{3}\right)^2 40}$$

$$= -0.029$$

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