1 Overview of the power system module

The THYME model is typical of models used for electro-mechanical transients. A brief overview of the model is given here. For a more detailed treatment, see the references listed at the end of this document. The model has two parts: differential equations for the generators and algebraic equations for the loads and transmission system. This note describes first the equations for the generators, second the equations for the loads and transmission systems and the inclusion of the generators within this model, and third the loads. Specific information on the models and simulation algorithms can be found in [2] and [5] respectively.

2 Generators

Each generator has the following state variables.

1. The frequency ω in the "per unit" system. Frequency is zero during normal operations: positive and negative values indicate excursions away from normal.

2. The mechanical power P_m that drives the generator's turbine. This also is in the "per unit" system.

3. The excitation voltage amplitude E, also in the "per unit" system. This value is adjusted dynamically to maintain the voltage at the terminals of the generator.

4. The phase angle θ of the voltage at the terminals of the generator.

5. A control signal c that regulates the mechanical power P_m for the purposes of i) keeping ω near zero and ii) keeping P_e at its reference level (i.e., at the desired output for the generator).

Each generator has the following parameters.

1. The "inertia" M of the turbine in the "per unit" system.

2. The synchronous reactance X of the turbine motor.

3. The droop setting R of the frequency regulator.

4. The reference point P_s for the mechanical power.

5. The area control gain A.

6. The reference voltage V_s for the terminals of the generator.

7. The time constants T_1, T_2, and T_e of the control circuits.

8. A resistance D to operation away from nominal frequency.

Each generator acts on the voltage phasor $V\angle\phi$ at terminals. The quantity $V\angle\phi$ is an input that originates with the loads, transmission network, and interaction (via the transmission network) with other generators. The voltage controller in the generator acts on V. The speed of the generator reacts to changes in P_e, which is

$$P_e = \mathrm{Re}\left\{ \left(V\angle\phi\right)\left(\frac{E\angle\theta - V\angle\phi}{X}\right)^* \right\} \tag{1}$$

The generator output is a current phasor I that it injects into the transmission network. This current is

$$I = \frac{E\angle\theta}{X} \tag{2}$$

The behavior of each generator is dictated by the following equations:

$$\dot{\omega} = \frac{P_m - P_e - D\omega}{M} \tag{3}$$

$$\dot{\theta} = \omega \tag{4}$$

$$\dot{c} = T_1(P_s - \omega/R - c) \tag{5}$$

$$\dot{P}_m = T_2(c_k - P_m) \tag{6}$$

$$\dot{E} = T_e(V_s - V) \tag{7}$$

The power set point P_s is adjusted whenever a signal arrives from the area generation controller. At these times $A\omega$ is subtracted from P_s. Over and under frequency protection devices are built into the generator models. These open when ω reaches its upper or lower threshold.

3 Transmission, loads, and generators in the network

Every load and generator is modeled by a Norton equivalent circuit: a current source and complex impedance in parallel and connected to ground at one end and its network terminal at the other. For generators, the impedance is its synchronous reactance X and the current source is I. For loads the current source is usually negative (i.e., it draws current from the network) or zero (this case creating a constant impedance load). Transmission lines are modeled by complex impedances that connect network nodes. The resulting network equations are in the form

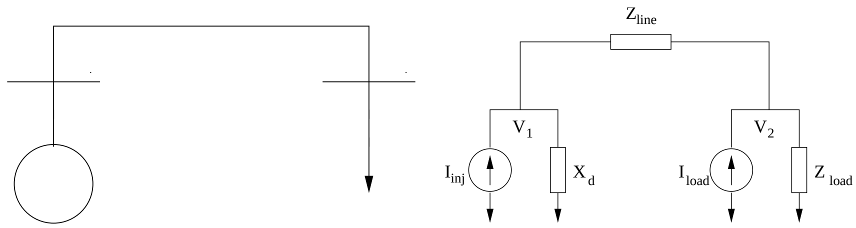
$$\mathbf{I} = \mathbf{Y}\mathbf{V} \tag{8}$$

The current vector $\mathbf{I}$ contain the current sources provided as output from the generators and any currents injected (or drawn) by the loads. The voltages $\mathbf{V}_t$ are the voltages seen at the network's terminals and contribute the V input to each generator. An example of a transmission network is shown in Fig. 1. Using $X = j$, $y_{line} = 1/Z_{line} = 100.0$, $y_{load} = 1/Z_{load} = 1$, and $I_{load} = 0$ we obtain the admittance matrix

$$\mathbf{Y} = \begin{bmatrix} 1/X + y_{line} & -y_{line} \\ -y_{line} & y_{load} + y_{line} \end{bmatrix} = \begin{bmatrix} j + 100 & -100 \\ -100 & 101 \end{bmatrix} \tag{9}$$

and voltage and current vectors

$$\mathbf{I} = \begin{bmatrix} I_{genr} \\ 0 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{10}$$



(a) One line drawing (b) Equivalent circuit

Figure 1: An example of the transmission model.

4 Loads

Loads are modeled as recommended in [1]. A reduced form of this model and parameters for it can be found in [3]. The power consumed by the load is modulated by changing the current injected by that load into the network. The load admittance remains constant. The equation for calculating injected load is derived from the load's Norton equivalent circuit with injected current I_{inj}, load admittance y, terminal voltage V, and base demand $S_0 = P_0 + jQ_0$ for the circuit. These give the injected current as

$$I_{inj} = Vy - \left(\frac{\alpha_r P_0 + \alpha_i j Q_0}{V} \right)^*$$

where $S_0 = V_0 y$ and V_0 is the initial voltage at the load and the model is always parameterized so that $I_{inj} = 0$ at $V = V_0$. The terms α_r and α_i modulate I_{inj}. These are

$$\begin{aligned} \alpha_r &= K_{pz} \left(\frac{|V|}{|V_0|} \right)^2 + K_{pi} \left(\frac{|V|}{|V_0|} \right) + K_{pc} \\ &+ K_{p1} \left(\frac{|V|}{|V_0|} \right)^{n_{pv1}} (1 + n_{pf1} f) + K_{p2} \left(\frac{|V|}{|V_0|} \right)^{n_{pv2}} (1 + n_{pf2} f) \\ \alpha_i &= K_{qz} \left(\frac{|V|}{|V_0|} \right)^2 + K_{qi} \left(\frac{|V|}{|V_0|} \right) + K_{qc} \\ &+ K_{q1} \left(\frac{|V|}{|V_0|} \right)^{n_{qv1}} (1 + n_{qf1} f) + K_{q2} \left(\frac{|V|}{|V_0|} \right)^{n_{qv2}} (1 + n_{qf2} f) \end{aligned}$$

A description of the parameters can be found in [1]. The voltages V are calculated from the network equations and frequency f at the load using the method described in [4]. Note that this requires the solution to a fixed point problem in the form $\bar{I}_{inj} = f(x, \bar{I}_{inj})$ where x is a vector with the state variables of the generators and $\bar{I}_{inj}$ is the vector of currents injected by the loads.

5 Initializing from power flow data

The electrical model can be initialized from solved power flow data in the IEEE Common Data Format or the PTI format. These are described at `http://www.ee.washington.edu/research/pstca/` and several sample data files are also provided at that location. Parameters assigned to the generators and loads are described in the source code in these locations: see the class AutoInitilizingData for generator parameters and ElectricalModelEqns for load parameters.

6 Full equations from a one load, one generator system

To better illustrate the numerical issues posed by the above equations, a complete model consisting of a generator and load connected by a transmission line is constructed in this section. The following set of equations is derived from Fig. 1, the above discussion and the material presented in [4]. Note that voltages, currents, and impedances shown in Fig. 1 are complex quantities.

1. Transmission equations.

$$\mathbf{Y} = \begin{bmatrix} 1/X + y_{line} & -y_{line} \\ -y_{line} & y_{load} + y_{line} \end{bmatrix}$$

$$\begin{bmatrix} I_{genr} \\ I_{load} \end{bmatrix} = \mathbf{Y} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The transmission equations are used to solve for the voltages V_1 and V_2 given $\mathbf{Y}$, which is a determined by the structure of the electrical network, and the currents I_{genr} and I_{load}, which are determined by the generator and load equations.

2. Generator equations.

$$
\begin{aligned}
\dot{\omega} &= \frac{P_m - P_e - D\omega}{M} \\
\dot{\theta} &= \omega \\
\dot{c} &= T_1(P_s - \omega/R - c) \\
\dot{P}_m &= T_2(c_k - P_m) \\
\dot{E} &= T_e(V_s - |V_1|) \\
P_e &= \Re\{V_1 I_{out}^*\} \\
I_{out} &= I_{genr} - V_1/X \\
I_{genr} &= \frac{E\cos\theta + jE\sin\theta}{X}
\end{aligned}
$$

Note that the generator equations are coupled to the load equations via the transmission equations. Specifically, the voltage V_1 is a function of the load current I_{load}. Because the load equations (see below) comprise a set of non-linear, algebraic equations, the entirety of the model constitutes a semi-explicit differential-algebraic model.

3. Load equations.

$$
\begin{aligned}
I_{load} &= V_2 y_{load} - \left(\frac{\alpha_r P_0 + \alpha_i j Q_0}{V_2}\right)^* \\
\alpha_r &= K_{pz}\left(\frac{|V_2|}{|V_0|}\right)^2 + K_{pi}\left(\frac{|V_2|}{|V_0|}\right) + K_{pc} + K_{p1}\left(\frac{|V_2|}{|V_0|}\right)^{n_{pv1}}(1 + n_{pf1}f) + K_{p2}\left(\frac{|V_2|}{|V_0|}\right)^{n_{pv2}}(1 + n_{pf2}f) \\
\alpha_i &= K_{qz}\left(\frac{|V_2|}{|V_0|}\right)^2 + K_{qi}\left(\frac{|V_2|}{|V_0|}\right) + K_{qc} + K_{q1}\left(\frac{|V_2|}{|V_0|}\right)^{n_{qv1}}(1 + n_{qf1}f) + K_{q2}\left(\frac{|V_2|}{|V_0|}\right)^{n_{qv2}}(1 + n_{qf2}f)
\end{aligned}
$$

$$
\begin{bmatrix} \cos(\arg V_2) & -|V_2|\sin(\arg V_2) \\ \sin(\arg V_2) & |V_2|\cos(\arg V_2) \end{bmatrix} \begin{bmatrix} \dot{|V_2|} \\ f \end{bmatrix} = \mathbf{Y}^{-1} \begin{bmatrix} \frac{1}{X}E\omega(\sin\theta - j\cos\theta) \\ 0 \end{bmatrix}
$$

As before, the load equations are coupled to the generator equations. Specifically, the voltage V_2 is a function of the load generator I_{genr} and the frequency f is a function of the generator state variables E, ω, and θ. The entirety of the model therefore constitutes a semi-explicit differential-algebraic model. Numerically, it is the solution to the load equations that pose the greatest difficulty. There are two special case of practical interest, however, in which the non-linearities vanish and the complete model becomes a set of simpler, ordinary differential equations. The first case is when constant impedance loads are modeled by setting $K_{pz} = K_{qz} = 1$ and all other of the K parameters to zero. This gives $I_{load} = 0$. The second case is when $V \approx V_0$ and so we can take $I_{load} \approx 0$.

The general form of the system can be seen by taking the vector $\mathbf{x}$ to be the state variables in the generator model(s), and the vector $\mathbf{I} = [\mathbf{I}_{load}\ \mathbf{I}_{genr}]^T$ to be the currents injected by the generator(s) and load(s). The form of the equations is then

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{Y}^{-1}\mathbf{I})$$
$$\mathbf{I}_{load} = g(\mathbf{x}, \mathbf{Y}^{-1}\mathbf{I})$$

where $\mathbf{I}$ depends, of course, on both $\mathbf{x}$ and $\mathbf{I}_{load}$. In the special cases described for the load model, the term $\mathbf{I}_{load}$ becomes zero and this is reduced to the system of ordinary differential equations

$$\dot{\mathbf{x}} = f(\mathbf{x})$$

where the vector $\mathbf{I}$ appearing in the previous equations now depends solely on $\mathbf{x}$ via the generator equations.

References

[1] Standard load models for power flow and dynamic performance simulation. *IEEE Transactions on Power Systems*, 10(3):1302–1313, August 1995.

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[4] J. Nutaro and V. Protopopescu. Calculating frequency at loads in simulations of electro-mechanical transients. Authors are with the Computational Sciences and Engineering Division at Oak Ridge National Laboratory, Oak Ridge, TN (contact email: nutarojj@ornl.gov)., November 2010.

[5] James J. Nutaro. *Building Software for Simulation: Theory and Algorithms with Applications in C++*. John Wiley and Sons, Inc, Hoboken, New Jersey, 2011.