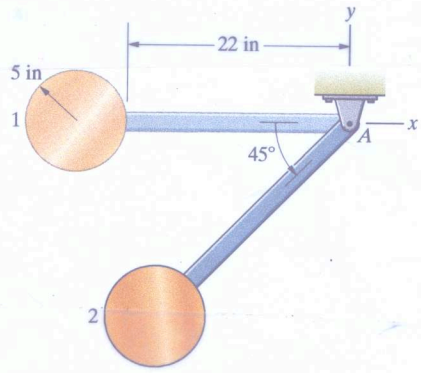
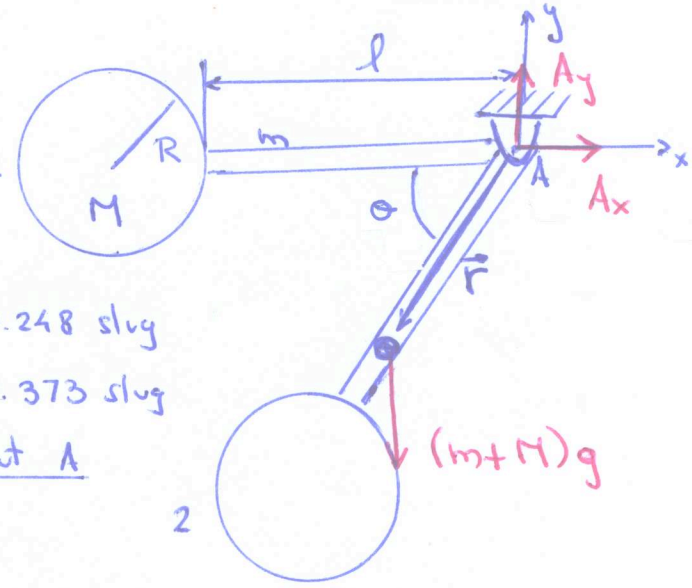
EX 19.11

Problem 19.11 The object consists of an 8-lb slender bar welded to a 12-lb circular disk. The object is released from rest in position 1. Determine the x and y components of force exerted on the object by the pin support when it is in position 2.



Solution



Data: $R = 5\text{ in} = 0.417\text{ ft}$

$l = 22\text{ in} = 1.83\text{ ft}$

$\theta = 45\text{ in}$

$w = 8\text{ lb} \longrightarrow m = \frac{w}{g} = 0.248\text{ slug}$

$W = 12\text{ lb} \longrightarrow M = \frac{W}{g} = 0.373\text{ slug}$

(1) Find moment of inertia about A

The moments of inertia about centers of mass are

$$I_{bar} = \frac{1}{12} m l^2 \; ; \quad I_{disk} = \frac{1}{2} M R^2$$

$$I_A = (I_{bar})_A + (I_{disk})_A = I_{bar} + \left(\frac{l}{2}\right)^2 m + I_{disk} + (l+R)^2 M$$

$$= \frac{1}{12} m l^2 + m \frac{l^2}{4} + \frac{1}{2} M R^2 + M (l+R)^2$$

$$\boxed{I_A = \frac{1}{3} m l^2 + M\left(l^2 + 2lR + \frac{3}{2} R^2\right)}$$

$$I_A = 2.193\text{ slug ft}^2 = 2.193\ \frac{\text{lb s}^2\ \text{ft}^{\cancel{2}}}{\cancel{\text{ft}}} \Rightarrow \boxed{I_A = 2.193\text{ lb-s}^2\text{-ft}}$$

Find distance from A to center of mass

$$\boxed{d = \frac{m\,\ell/2 + M(\ell + R)}{m + M}} = 20.6\,\text{in} = 1.72\,\text{ft}$$

(2) <u>Equations of motion at position 2</u>

(i) Motion of center of mass

$$\sum F_x = A_x = (m+M)\,a_x$$

$$\sum F_y = A_y - (m+M)g = (m+M)\,a_y$$

(ii) Angular motion

$$\sum M_A = I_A\,\alpha$$

(3) <u>Kinematic constraints</u>

$$\vec{r} = -d\cos\sigma\,\hat{i} - d\sin\sigma\,\hat{j}$$

$$\vec{a} = \underset{\overset{\shortparallel}{\vec{0}}}{\vec{a}_A} + \vec{\alpha}\times\vec{r} - \omega^2\,\vec{r}$$

$$= \alpha\hat{k}\times(-d\cos\sigma\,\hat{i} - d\sin\sigma\,\hat{j}) - \omega^2(-d\cos\sigma\,\hat{i} - d\sin\sigma\,\hat{j})$$

$$= (\alpha\,d\sin\sigma + \omega^2 d\cos\sigma)\hat{i} + (-\alpha\,d\cos\sigma + \omega^2 d\sin\sigma\,\hat{j})$$

$$\sum \vec{M}_A = \vec{r}\times(-(m+M)g\,\hat{j}) = (m+M)\,g\,d\cos\sigma\,\hat{k}$$

$\Longrightarrow$ (i) $A_x = (m+M)(\alpha d \sin\theta + \omega^2 d \cos\theta)$

(ii) $A_y = (m+M)(g - \alpha d \cos\theta + \omega^2 d \sin\theta)$

(iii) $(m+M) g d \cos\theta = I_A \alpha$

Unknowns: A_x, A_y, ω, α

Need 1 more equation

(4). <u>Work energy equation</u>

$$U_{12} = T_2 - \underset{0}{\underset{\shortparallel}{T_1}}$$

$\longrightarrow$ released from rest

$\Longrightarrow \omega = 0$

Fixed axis rotation

$$T_2 = \frac{1}{2} I_A \omega^2$$

$$U_{12} = \int_1^2 \vec{F} \cdot d\vec{r} = \int_0^{-d\sin\theta} -(m+M) g \, dy = (m+M) g d \sin\theta$$

Force $\vec{A}$ is applied at a fixed point for which $d\vec{r} = \vec{0}$, only the weight do work

$$\Longrightarrow (m+M) g d \sin\theta = \frac{1}{2} I_A \omega^2 \quad \text{(iv)}$$

Solve:

(iv) $\Rightarrow$ $2\frac{(m+M)}{I_A} g\, d\sin\sigma = \omega^2$ $\Rightarrow$ $\boxed{\omega = 4.70 \text{ rad/s}}$

(iii) $\Rightarrow$ $\frac{(m+M)}{I_A} g\, d\cos\sigma = \alpha$ $\Rightarrow$ $\boxed{\alpha = 11.0 \text{ rad/s}^2}$

(i) $\Rightarrow$ $A_x = (m+M)\left(\frac{(m+M)}{I_A} g\, d\cos\sigma\, d\sin\sigma + 2\frac{(m+M)}{I_A} g\, d\sin\sigma\, d\cos\sigma\right)$

$$\boxed{A_x = 3\frac{(m+M)^2 d^2}{I_A} g \sin\sigma\cos\sigma}$$

$$\boxed{A_x = 25.0 \text{ lb}}$$

(ii) $\Rightarrow$ $A_y = (m+M)\left(g - \frac{(m+M)}{I_A} g\, d\cos\sigma\, d\cos\sigma + 2\frac{(m+M)}{I_A} g\, d\sin\sigma\, d\sin\sigma\right)$

$$= (m+M)\left(g - \frac{m+M}{I_A} g\, d^2(\cos^2\sigma - 2\sin^2\sigma)\right)$$

$$\boxed{A_y = (m+M)\, g\left(1 - \frac{(m+M)}{I_A} d^2(\cos^2\sigma - 2\sin^2\sigma)\right)}$$

$$\boxed{A_y = 28.3 \text{ lb}}$$