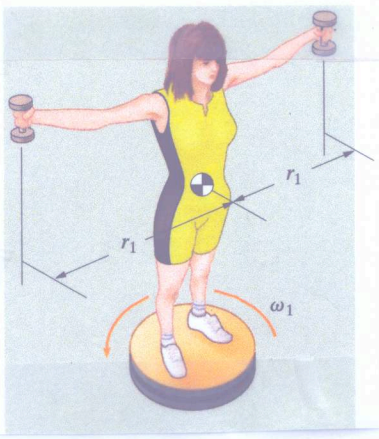
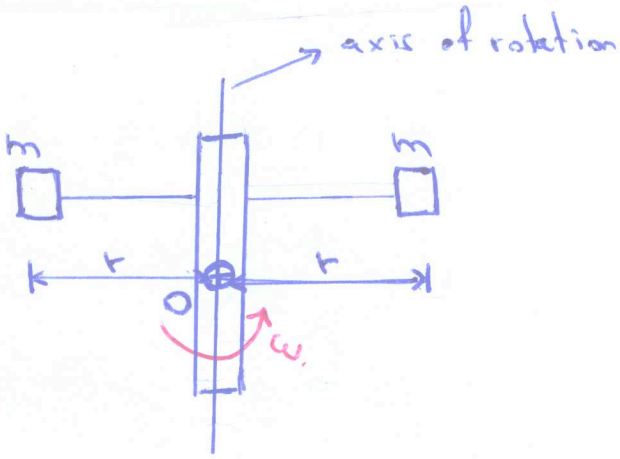
EX 19.58

Problem 19.58 In Example 19.6, we neglected the moments of inertia of the two masses m about the axes through their centers of mass in calculating the total angular momentum of the person, platform, and masses. Suppose that the moment of inertia of each mass about the vertical axis through its center of mass is $I_M = 0.001$ kg-m^2. If the person's angular velocity with her arms extended to $r_1 = 0.6$ m is $\omega_1 = 1$ revolution per second, what is her angular velocity ω_2 when she pulls the masses inward to $r_2 = 0.2$ m? Compare your result to the answer obtained in Example 19.6.



Solution



Data: $I_p = 0.4$ kgm^2

$m = 4$ kg

$I_m = 0.001$ kgm^2

$r_1 = 0.6$ m $\qquad r_2 = 0.2$ m

$\omega_1 = 1$ rev/s

(1) We denote by O the center of mass of the combined person and platform.

Using the equation

$$H_O = (\vec{r} \times m\vec{v}) \cdot \hat{k} + I\omega$$

we compute the total angular momentum of the system as

$$(H_O)_{Total} = \underset{\substack{\uparrow \\ \text{person + platform}}}{I_p \omega} + 2(r m v + I_m \omega)$$

(2) By kinematics

$$v = r\omega$$

$$\Rightarrow \quad (H_o)_{Total} = \left[I_p + 2(mr^2 + I_m)\right]\omega$$

(3) Neglecting friction in the rotating platform, the total angular momentum is conserved

$$\Rightarrow \quad \left[I_p + 2(mr_1^2 + I_m)\right]\omega_1 = \left[I_p + 2(mr_2^2 + I_m)\right]\omega_2$$

$$\boxed{\omega_2 = \left[\frac{I_p + 2(mr_1^2 + I_m)}{I_p + 2(mr_2^2 + I_m)}\right]\omega_1}$$

$$\boxed{\omega_2 = 4.55 \text{ rev/s}}$$