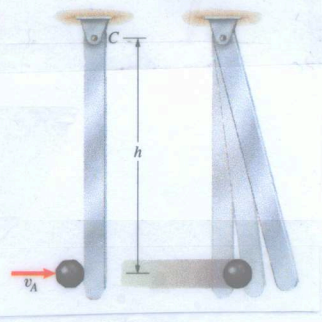
EX 19.70

Problem 19.70 In Active Example 19.7, suppose that the ball A weighs 2 lb, the bar B weighs 6 lb, and the length of the bar is 3 ft. The ball is translating at $v_A =$ 10 ft/s before the impact and strikes the bar at $h = 2$ ft. What is the angular velocity of the bar after the impact if the ball adheres to the bar?



Solution

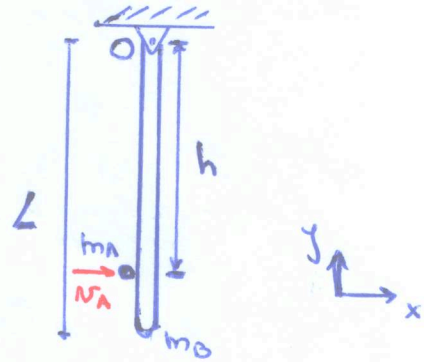
Data:

$W_A = 2$ lb $\qquad v_A = 10$ ft/s

$W_b = 6$ lb $\qquad h = 2$ ft

$L = 3$ ft

Sphere adheres to bar after impact

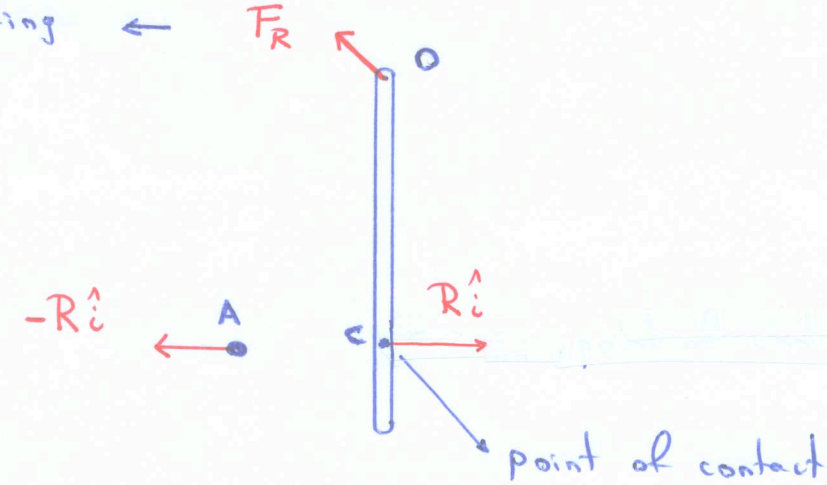


Concepts:

(1) We assume the sphere is a point mass

(2) Total angular momentum about O is conserved

Force acting at pin joint $\leftarrow$ $\vec{F}_R$



Angular momentum about O is conserved:

- $\vec{F}_R$ acts at O $\Rightarrow$ produces no angular momentum change
- Angular momentum contribution of contact forces $\vec{R}$ and $-\vec{R}$ vanishes.

Total Angular Momentum about O:

$$\vec{H}_O = \vec{H}_{O,\text{sphere}} + \vec{H}_{O,\text{bar}}$$

$$= \underbrace{\vec{r}_{A/O} \times m_A \vec{v}_A}_{\text{point mass}} + \underbrace{I_O \omega \hat{k}}_{\substack{\text{rigid body} \\ \text{rotating about fixed point O.}}}$$

$$\vec{r}_{A/O} = -h\hat{j} \qquad \vec{v}_A = v_A \hat{i}$$

$$\Rightarrow \vec{H}_O = m_A v_A h \hat{k} + I_O \omega \hat{k}$$

Let ω = angular velocity of bar just before impact

ω' = " " " " " after "

$\vec{v}_A$ = velocity of sphere just before impact

$\vec{v}_A'$ = " " " " after "

By conservation of total angular momentum

$$m_A v_A h + I_O \underset{\substack{\parallel \\ 0}}{\omega} = m_A v_A' h + I_O \omega' \qquad (*)$$

bar is at rest

By kinematics:

Since sphere adheres to bar after impact

$\Rightarrow$ sphere is in circular motion about O after impact

$$\vec{V}_A' = \vec{\omega}' \times \vec{r}_{A/O} \Rightarrow \vec{V}_A' = \omega' h \hat{i} \Rightarrow v_A' = \omega' h$$

$$(*) \Rightarrow \quad m_A v_A h = m_A h^2 \omega' + I_O \omega' = \left(m_A h^2 + \frac{1}{3} m_B L^2\right) \omega'$$

$$I_O = \frac{1}{3} m_B L^2$$

$$\boxed{\omega' = \frac{m_A v_A h}{m_A h^2 + \frac{1}{3} m_B L^2}}$$

$$\boxed{\omega' = 1.54 \text{ rad/s}}$$