EX 19.83

Solution

Concepts:

(1) The weight is not an impulsive force, so it does not introduce a change in the angular momentum:

By the Moment - Angular Momentum Principle

$$\sum \vec{M}_o = \frac{d\vec{H}_o}{dt}$$

angular impulse $\leftarrow$ $$\int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \vec{H}_o(t_2) - \vec{H}_o(t_1)$$

Assume $t_2 = t_1 + \Delta t$ and $\Delta t \to 0$, then $\vec{r}_{G/O} = \text{constant}$

$$\Longrightarrow \int_{t_1}^{t_2} \vec{r}_{G/O} \times mg\hat{j} \, dt \approx (\vec{r}_{G/O} \times mg\hat{j})\Delta t \xrightarrow[\Delta t \to 0]{} 0$$

$$\Longrightarrow \boxed{\vec{H}_o(t_2) = \vec{H}_o(t_1)}$$

$\Longrightarrow$ We have "Conservation of Angular Momentum"

The angular momentum about O is

$$\vec{H}_o = \vec{H} + \vec{r}_{G/O} \times m\vec{V}_G$$

$$\underset{\substack{\uparrow \\ \text{rigid body}}}{=} I\omega\hat{k} + \vec{r}_{G/O} \times m\vec{V}_G$$

$I \rightarrow$ moment of inertia about center of mass

(2) By the Principle of Linear Impulse and Momentum

$$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_G(t_2) - m\vec{v}_G(t_1) \longrightarrow \text{vector equation}$$

Then,

$$\int_{t_1}^{t_2} \sum F_x\, dt = m\left(\vec{v}_G(t_2)\right)_x - m\left(\vec{v}_G(t_1)\right)_x .$$

All forces act on x-direction

$$\Longrightarrow \boxed{\left(\vec{v}_G(t_2)\right)_x = \left(\vec{v}_G(t_1)\right)_x = 0}$$

There is no change in the velocity in x-direction.

<u>Solve</u>:

Let

$\vec{v}_G = \vec{v}_G(t_1) = -v_G\hat{j}$: velocity of center of mass just before impact

$\vec{v}_G' = \vec{v}_G(t_2) = -v_G'\hat{j}$: velocity of center of mass just after impact

ω = angular velocity just befor impact

ω' = " " " after impact

$$\vec{H}_o = I\omega\hat{k} + \left(-\frac{L}{2}\cos\sigma\,\hat{i} + \frac{L}{2}\sin\sigma\,\hat{j}\right) \times m(-v_G)\hat{j}$$

$$= \left(I\omega + m v_G \frac{L}{2}\cos\sigma\right)\hat{k}$$

(1) By conservation of angular momentum (assum σ = constant)

$$I\omega + m v_G \frac{L}{2}\cos\sigma = \underset{\substack{\parallel \\ 0}}{I\omega'} + m v_G' \frac{L}{2}\cos\sigma$$

$$I = \frac{1}{12} m L^2$$

$$\Longrightarrow \quad \boxed{\frac{1}{12} m L^2 \omega + m v_G \frac{L}{2} \cos\theta = m v_G' \frac{L}{2} \cos\theta} \quad (*)$$

(2) By the coefficient of restitution

$$e = \frac{(\vec{v}_{bar\,o}')_y - \cancel{(\vec{v}_{floor\,o}')_y}}{\cancel{(\vec{v}_{floor\,o})_y} - (\vec{v}_{bar\,o})_y}$$

negligible

0

$$\Longrightarrow \quad \boxed{-(\vec{v}_{bar\,o})_y \, e = (\vec{v}_{bar\,o}')_y} \quad (**)$$

(3) By kinematics

$$\vec{v}_{bar\,o} = \vec{v}_G + \vec{\omega} \times \vec{r}_{o/G} = -v_G \hat{j} + \omega \hat{k} \times \left(\frac{L}{2} \cos\theta \hat{i} - \frac{L}{2} \sin\theta \hat{j} \right)$$

$$= \left(-v_G + \omega \frac{L}{2} \cos\theta \right) \hat{j} + \omega \frac{L}{2} \sin\theta \, \hat{i}$$

$$\vec{v}_{bar\,o}' = \vec{v}_G' + \vec{\omega}' \times \vec{r}_{o/G} = -v_G' \hat{j}$$

$\omega' = 0$

$$\begin{cases} (\vec{v}_{bar\,o})_y = -v_G + \omega \frac{L}{2} \cos\theta \\ (\vec{v}_{bar\,o}')_y = -v_G' \end{cases}$$

$$(**) \Longrightarrow \quad \boxed{e \left(v_G - \frac{\omega L}{2} \cos\theta \right) = -v_G'}$$

Use this expression in (*)

$$\frac{1}{12} m L^2 \omega + m v_G \frac{L}{2} \cos\sigma = - m e \left(v_G - \omega \frac{L}{2} \cos\sigma \right) \frac{L}{2} \cos\sigma$$

$$= - m v_G \frac{L}{2} e \cos\sigma + m \omega \frac{L^2}{4} \cos^2\sigma \, e$$

$$\cancel{m} v_G \frac{\cancel{L}}{2} \cos\sigma \, (1+e) = \cancel{m} \omega \frac{L^{\cancel{2}}}{4} \left(\cos^2\sigma \, e - \frac{1}{3} \right)$$

$$\boxed{\omega = \frac{2 v_G \cos\sigma (1+e)}{e \cos^2\sigma - \frac{1}{3}}}$$

$$\boxed{\omega = -24.0 \text{ rad/s}}$$