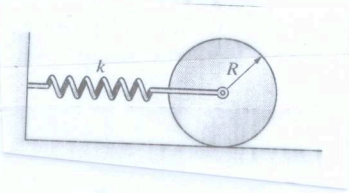
EX 21.15

Problem 21.15 The 20-lb disk rolls on the horizontal surface. Its radius is $R = 6$ in. The spring constant is $k = 15$ lb/ft. At $t = 0$, the spring is unstretched and the disk has a clockwise angular velocity of 2 rad/s. What is the amplitude of the resulting vibrations of the center of the disk?



Solution

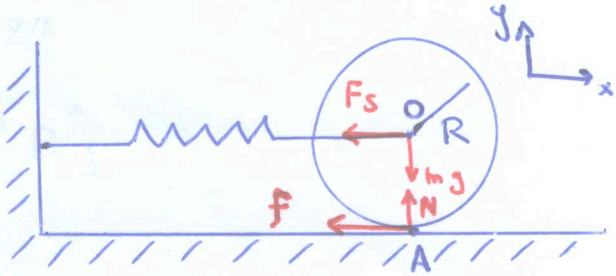
Data: $W = 20$ lb

$R = 6$ in $= 0.5$ ft

$k = 15$ lb/ft

$\vec{\dot{O}}(0) = -2$ rad/s $\hat{k}$

$x(0) = 0$



Newton's 2nd law:

$$\sum F_x = \boxed{-kx - f = m\ddot{x}} \quad (1)$$

Angular motion equation (about O

$$\sum M_O = (-R\hat{j} \times -f\hat{i}) \cdot \hat{k} + \cancel{(-R\hat{j} \times N\hat{j}) \cdot \hat{k}}^{\,0} = -I\alpha$$

$(\alpha > 0)$

$$\boxed{fR = I\alpha} \quad (2)$$

(1) + (2) $\quad m\ddot{x} + kx + \frac{I\alpha}{R} = 0$

$I_{disk} = \frac{1}{2} m R^2$

$$\Rightarrow \quad \boxed{m\ddot{x} + kx + \frac{1}{2} m R \alpha = 0} \qquad (3)$$

By kinematics

$$\vec{V}_O = \underset{\overset{\shortparallel}{0}}{\vec{V}_A} + \vec{\dot{\theta}} \times \vec{r}_{O/A} = -\dot{\theta}\hat{k} \times R\hat{j} = R\dot{\theta}\hat{i}$$

$$\boxed{\dot{x}(0) = R\dot{\theta}}$$

$$\vec{a}_O = \vec{a}_A + \vec{\ddot{\theta}} \times \vec{r}_{O/A} - (\dot{\theta})^2 \vec{r}_{O/A}$$

$$a_O\hat{i} = a_A\hat{j} - \alpha\hat{k} \times R\hat{j} - \dot{\theta}^2 R\hat{j}$$

$$\begin{cases} \ddot{x} = a_O = \alpha R \\ a_A = -R\dot{\theta}^2 \end{cases}$$

$$(3) \Rightarrow \quad m\ddot{x} + kx + \frac{1}{2} m R \frac{\ddot{x}}{R} = 0$$

$$\frac{3}{2} m\ddot{x} + kx = 0$$

$$\Rightarrow \quad \boxed{\ddot{x} + \frac{2k}{3m} x = 0} \qquad \ddot{x} + \omega^2 x = 0 \qquad \omega^2 = \frac{2k}{3m}$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

$$x(0) = 0 = B \qquad \dot{x}(0) = \omega A = R\dot{\theta} \Rightarrow A = \frac{R\dot{\theta}}{\omega} = \frac{R\dot{\theta}}{\sqrt{\frac{2k}{3m}}}$$

Amplitude $\boxed{A = \sqrt{\frac{3W}{2kg}}\, R\dot{\theta}} = 0.249 \text{ ft}$