Lecture 13 - February 15, 2012

(exercise session)

Principle of work and energy

$$U_{12} = \int_{s_1}^{s_2} \sum F_t \, ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

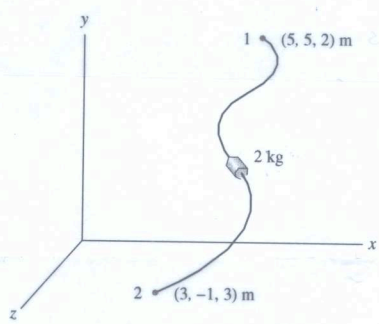
Work done by weight: $U_w = -mg\,\underbrace{(y_2 - y_1)}_{\text{change in height}}$

work done by a spring: $U_s = -\frac{1}{2} k (s_2^2 - s_1^2)$

$s = r - r_0$ stretch.

EX 15.41

Problem 15.41 The 2-kg collar starts from rest at position 1 and slides down the smooth rigid wire. The y-axis points upward. What is the magnitude of the velocity of the collar when it reaches position 2?



Solution

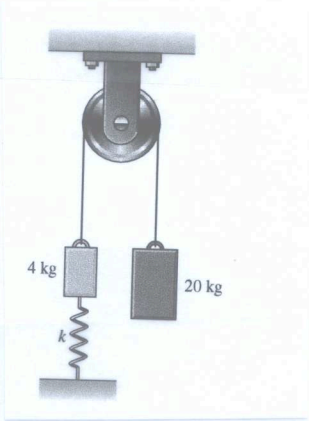
$$U_w = -mg(y_2 - y_1) = mg\underbrace{h}_{\text{height difference}} = \frac{1}{2} m v_2^2 - \frac{1}{2} m \underbrace{v_1^2}_{0}$$

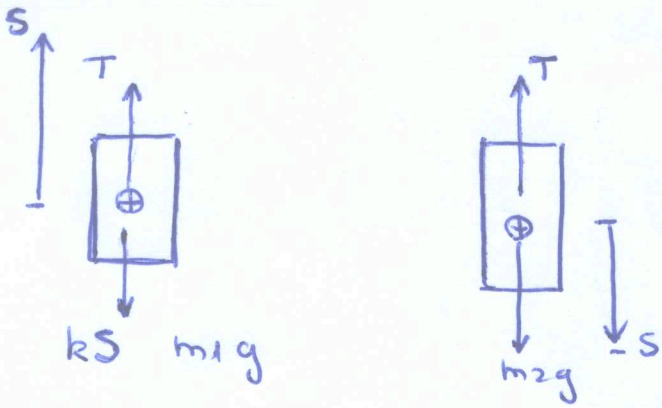
$$mgh = \frac{1}{2} m v^2 \qquad v = v_2$$

$$\boxed{v = \sqrt{2gh} = 10.85 \text{ m/s}}$$

EX 15.55

Problem 15.55 The system is released from rest with the spring unstretched. The spring constant $k = 200$ N/m. What maximum downward velocity does the right mass attain as it falls?





Body 1: $\int_0^s -kS - m_1 g + T \, ds = \frac{1}{2} m_1 v_2^2 - \frac{1}{2} m_1 \underset{0}{v_1^2}$

(i) $-\frac{kS^2}{2} - m_1 g S + TS = \frac{1}{2} m_1 v^2$ $\quad v = v_2$

Body 2: $\int_0^{-s} T - m_2 g \, ds = \frac{1}{2} m_2 v^2$

(ii) $-ST + m_2 g S = \frac{1}{2} m_2 v^2$

(i) + (ii) $\Rightarrow$ $-k\frac{S^2}{2} + (m_2 - m_1) g S = \frac{1}{2}(m_2 + m_1) v^2$ (*)

Note: we found $v = v(S)$

To find the maximum velocity, we differentiate the equation by s and set $\frac{dv}{ds} = 0$

$$-kS + (m_2 - m_1) g = (m_2 + m_1) v \frac{dv}{ds} = 0$$

$$\boxed{S = \frac{(m_2 - m_1) g}{k}}$$

Substitute this result into (*)

$$-\frac{k}{2}\left(\frac{(m_2-m_1)g}{k}\right)^2 + (m_2-m_1)\,g\,\frac{(m_2-m_1)g}{k} = \frac{1}{2}(m_2+m_1)v^2$$

$$\cancel{\frac{1}{2}}\frac{(m_2-m_1)^2 g^2}{k} = \cancel{\frac{1}{2}}(m_2+m_1)v^2$$

$$v = \frac{(m_2-m_1)\,g}{\sqrt{(m_2+m_1)\,k}}$$

$$v = 16\,kg\;\; 9.81\,m/s^2\sqrt{\frac{1}{24\,kg\;\; 200\,N/m}}$$

$$\boxed{v = 2.27\,m/s}$$

Potential energy and conservation of energy

$$U_{12} = \int_{\vec{r}_1}^{\vec{r}_2} \sum \vec{F} \cdot d\vec{r} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Suppose we can find a function $V(\vec{r})$ such that

$$U_{12} = V(\vec{r}_1) - V(\vec{r}_2)$$

(we will talk more about this next class)

Then we have

$$\underbrace{V(\vec{r}_1)}_{\text{Potential energy}} + \underbrace{\frac{1}{2} m v_1^2}_{\text{kinetic energy}} = V(\vec{r}_2) + \frac{1}{2} m v_2^2$$

$$\Downarrow$$

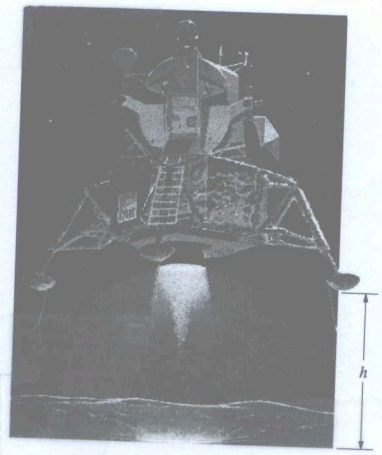
Conservation of energy!

Potential energy for weight $V = V(y) = mgy$

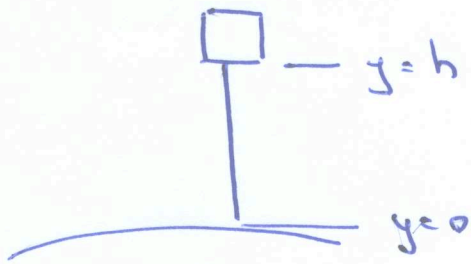
Potential energy for spring $V = V(s) = \frac{1}{2} k s^2$

EX 15.80

Problem 15.80 The Lunar Module (LM) used in the Apollo moon landings could make a safe landing if the magnitude of its vertical velocity at impact was no greater than 5 m/s. Use conservation of energy to determine the maximum height h at which the pilot could shut off the engine if the vertical velocity of the lander is (a) 2 m/s downward and (b) 2 m/s upward. The acceleration due to gravity at the moon's surface is 1.62 m/s^2.



By conservation of energy



$$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2 + 0$$

$$h = \frac{1}{2g_M}\left(v_2^2 - v_1^2\right)$$

$$h_{max} = \frac{1}{2g_M}\left(v_{max}^2 - v_1^2\right)$$

$$h_{max} = \frac{1}{2 \cdot 1.62\ m/s^2}\left((5\ m/s)^2 - (2\ m/s)^2\right)$$

$$\boxed{h_{max} = 6.48\ m}$$

Note: (1) The kinetic energy doesn't depend on the direction of velocity

(2) What if the initial velocity is higher than 2 m/s? $\Rightarrow$ There is no such h!