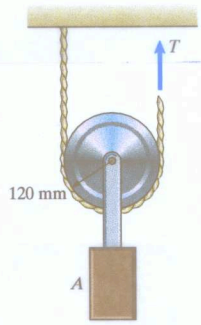
EX 18.51

Lecture 29 (Makeup class) - April 2, 2012

Problem 18.51 The mass of the suspended object A is 8 kg. The mass of the pulley is 5 kg, and its moment of inertia is 0.036 kg-m^2. If the force $T = 70$ N, what is the magnitude of the acceleration of A?



Solution

1. Draw free-body diagram

Data:

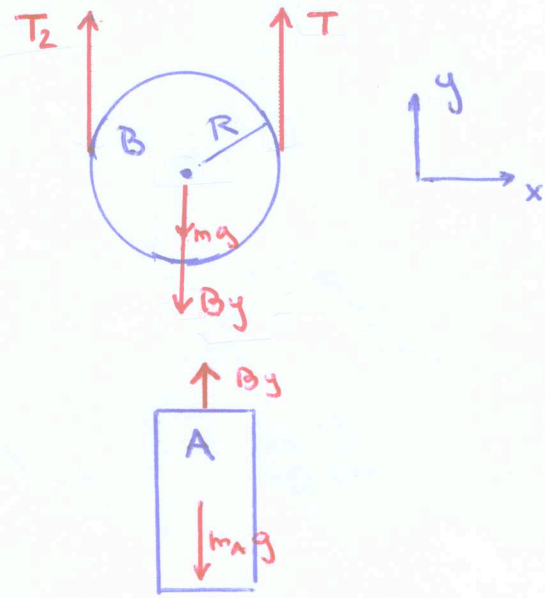
$m_A = 8\,kg$

$m_B = 5\,kg$

$I_B = 0.036\,kg\cdot m^2$

$R = 0.12\,m$

$T = 70\,N$



2. Equations of motion

(i) $\sum F_{yB} = T_2 + T - m_B g - B_y = m_B a_{By}$

(ii) $\sum F_{yA} = B_y - m_A g = m_A a_{Ay}$

(iii) $\sum M_B = (-R\hat{i} \times T_2\hat{j})\cdot\hat{k} + (R\hat{i} \times T\hat{j})\cdot\hat{k} = I_B \alpha_B$

$\Longrightarrow \quad -T_2 R + TR = I_B \alpha_B$

3. Kinematic constrains

(a) $a_{By} = a_{Ay}$

(b) $a_{By} = R\alpha_B$

<u>Solve</u>:

(ii) $\Longrightarrow$ $B_y = m_A g + m_A a_{Ay}$

(iii) $\Longrightarrow$ $T_2 = T - \frac{I_B \alpha_B}{R}$

(i) $\Longrightarrow$ $T - \frac{I_B \alpha_B}{R} + T - m_B g - m_A g - m_A a_{Ay} = m_B a_{By}$

$$2T - (m_A + m_B) g = \left(m_A + m_B + \frac{I_B}{R^2}\right) a_{Ay}$$

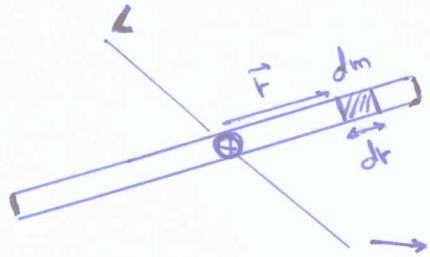
$$\boxed{a_{Ay} = \left(\frac{2T - (m_A + m_B) g}{m_A + m_B + \frac{I_B}{R^2}}\right)}$$

$$\boxed{a_{Ay} = 9.805 \text{ m/s}^2}$$

Moments of Inertia

$$I = \int_m r^2 \, dm$$

Slender bar



bar length much greater than width.

perpendicular axis through center of mass

l = length

A = cross sectional area (uniform along bar)

m = mass

Assume material is homogereous:

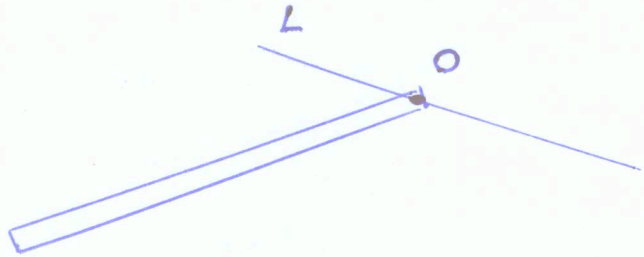
$$dm = \rho A \, dr$$

$$\Longrightarrow \quad I = \int_m r^2 \, dm = \int_{-l/2}^{l/2} \rho A r^2 \, dr = \rho A \frac{r^3}{3} \Big|_{-l/2}^{l/2} = \frac{1}{12} \rho A l^3$$

$$m = \rho A l \quad \Longrightarrow \quad \boxed{I = \frac{1}{12} m l^2}$$

Note: this is an approximation, assuming the bar width is small and the mass dm is concentraded on the bar axis.

Another case

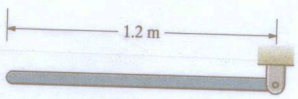


$$I_o = \int_0^{l} \rho A r^2 \, dr = \frac{1}{3} m l^2 \qquad m = \rho A l$$

$$\boxed{I_o = \frac{1}{3} m l^2}$$

EX 18.18

Problem 18.18 The 5-kg slender bar is released from rest in the horizontal position shown. Determine the bar's counterclockwise angular acceleration (a) at the instant it is released, and (b) at the instant when it has rotated 45°.

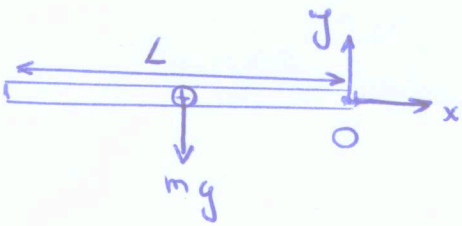


Solution

$m = 5\,kg$

$L = 1.2\,m$

(a)



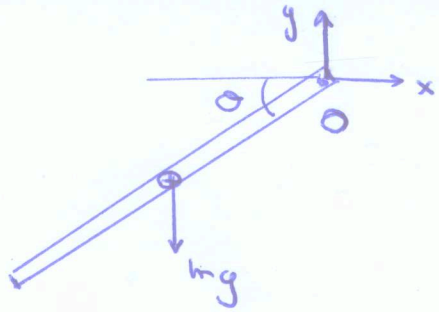
$$\sum M_o = (\vec{r} \times \vec{W}) \cdot \hat{k} = \left(-\frac{L}{2}\hat{i} \times mg(-\hat{j})\right) \cdot \hat{k} = \frac{mgL}{2}$$

$$= I_o \alpha = \left(\frac{1}{3} m L^2\right) \alpha$$

$$\Rightarrow \boxed{\alpha = \frac{3}{2}\frac{g}{L}} \Rightarrow \boxed{\alpha = 12.3 \text{ rad/s}^2}$$

(b)

$\theta = 45°$



$$\sum M_o = (\vec{r} \times \vec{W}) \cdot \hat{k} = \left[\left(-\frac{L}{2}\cos\theta\,\hat{i} - \frac{L}{2}\sin\theta\,\hat{j}\right) \times (-mg\,\hat{j})\right] \cdot \hat{k}$$

$$= \frac{L}{2} mg \cos\theta = I_o\,\alpha = \left(\frac{1}{3} m L^2\right)\alpha$$

$$\boxed{\alpha = \frac{3}{2}\frac{g}{L}\cos\theta} \Rightarrow \boxed{\alpha = 8.67 \text{ rad/s}^2}$$

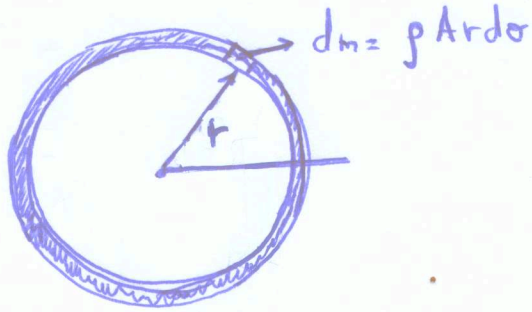
Other moment of inertia

Thin ring

A = cross sectional area

m = mass

r = radius



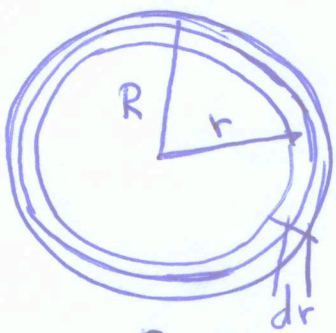
Assume material is homogeneous $\qquad dm = \rho r d\sigma A$

$$I = \int_m r^2 dm = \int_0^{2\pi} r^2 \rho A r d\sigma = \rho A r^3 2\pi$$

$$m = 2\pi r A \rho \implies \boxed{I = m r^2}$$

Circular disk

Divide disk in concentric thin rings



$$I = \int_m r^2 dm = \int_0^R \underbrace{I(r)}_{\text{moment of inertia of thin ring with radius } r.} dr = \int_0^R m(r) r^2 dr$$

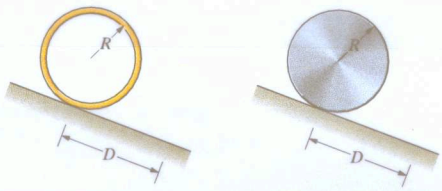
mass of ring with radius r.

$$= \int_0^R 2\pi r A \rho r^2 dr = 2\pi A \rho \int_0^R r^3 dr = 2\pi A \rho \frac{R^4}{4} = \frac{1}{2} \overbrace{(\pi R^2 A \rho)}^{M \to \text{mass of disk}} R^2$$

$$\boxed{I = \frac{1}{2} M R^2}$$

EX 18.34

Problem 18.34 A thin ring and a homogeneous circular disk, each of mass m and radius R, are released from rest on an inclined surface. Determine the ratio v_{ring}/v_{disk} of the velocities of the their centers when they ave rolled a distance D.



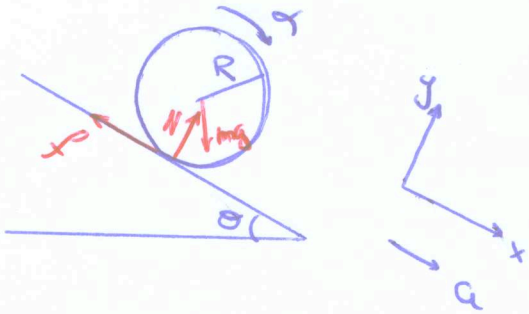
Solution

Data: m: mass (of both) given

R given

θ given

1. Draw free-body diagram



2. Motion of center of mass

(i) $\Sigma F_x = -f + mg\sin\theta = ma$

$\Sigma F_y = N - mg\cos\theta = 0$

3. Angular motion

(ii) $\Sigma M = -Rf = -I\alpha$

4. From kinematics

(iii) $a = R\alpha$

Take (i), (ii) and (iii) $\rightarrow$ unknowns f, a, α

3 equations with 3 unknowns

<u>Solve:</u>

(i) $\Rightarrow$ $-\left(\frac{I\alpha}{R}\right) + mg\sin\sigma = ma$

$$-\frac{I}{R^2}a + mg\sin\sigma = ma \quad / \cdot R^2$$

$$mgR^2\sin\sigma = a(I + mR^2)$$

$$\boxed{a = g\frac{mR^2}{I + mR^2}\sin\sigma}$$ $\leftarrow$ You can get same result directly from 18.33

For the ring $I_{ring} = mR^2 \Rightarrow a_{ring} = \frac{g}{2}\sin\sigma$

For the disk $I_{disk} = \frac{1}{2}mR^2 \Rightarrow a_{disk} = \frac{2}{3}g\sin\sigma$

From kinematics $v^2 = \underset{0}{v_0^2} + 2a(s - \underset{0}{s_0})$

$$v = \sqrt{2aD}$$

$$v_{ring} = \sqrt{2a_{ring}D} = \sqrt{gD\sin\sigma}$$

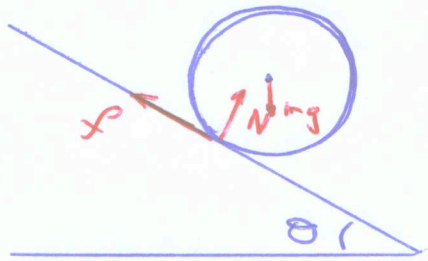
$$v_{disk} = \sqrt{2a_{disk}D} = \sqrt{\frac{4}{3}gD\sin\sigma}$$

$$\boxed{\frac{v_{ring}}{v_{disk}} = \sqrt{\frac{3}{4}}}$$

Note:

In problem 18.33 :

we found



$$\mu_s = \frac{I}{I + mR^2} \operatorname{tg}\theta$$

$$\alpha = \frac{g}{R}\left(\sin\theta - \mu_s \cos\theta\right)$$

$$a = R\alpha = g\left(\sin\theta - \mu_s\cos\theta\right) = g\left(\sin\theta - \frac{I}{I+mR^2}\operatorname{tg}\theta\cos\theta\right)$$

$$= \frac{g}{I+mR^2}\left(\sin\theta\,(\cancel{I}+mR^2) - I\cancel{\sin\theta}\right)$$

$$\boxed{a = g\frac{mR^2}{I+mR^2}\sin\theta}$$