Engineering Optimization: From Intuition to Systematic Techniques

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Abstract

- Engineers used to search for “optimum” solutions using intuition and parametric analysis
  - Experience level
  - Limited number of design variables
  - Two objectives at most
- Designs are getting more complex
  - More variables are being considered
  - More alternatives are being introduced
  - More constraints and objectives are imposed
  - Hence: require systematic techniques
- Case study: real HVAC&R engineering optimization problem

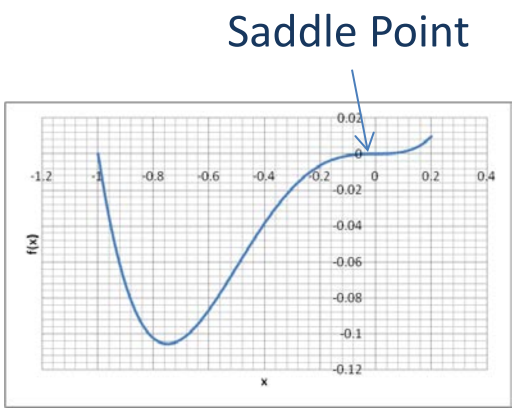
Optimization: a Definition

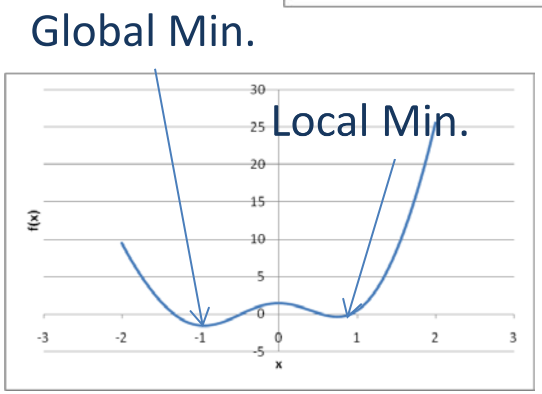
- "an act, process, or methodology of making something (as a design, system, or decision) ***as fully perfect, functional, or effective as possible***; *specifically* **:** the mathematical procedures (as finding the maximum of a function) involved in this" – Merriam-Webster Online Dictionary.
- "In mathematics and computer science, optimization, or mathematical programming, refers to ***choosing the best element from some set of available alternatives***." - Wikipedia
- "A branch of mathematics which encompasses many diverse areas of minimization and optimization." – Wolfram Mathworld.

Single Objective Optimization

- Minimize or maximize a single function (objective)
- Results in a single global optimum solution
  - Local optima
  - Saddle point
- Mathematical formulation:

$$\begin{aligned} &\underset{x}{\text{minimize}} \quad f(x) \\ &\text{subject to } g_i(x) \le 0 \qquad i = 1,\ldots,M \\ &\qquad\qquad\quad h_j(x) = 0 \qquad j = 1,\ldots,N \\ &\qquad\qquad\quad x_k^L \le x_k \le x_k^U \qquad k = 1,\ldots,d \end{aligned}$$





Multi-Objective Optimization

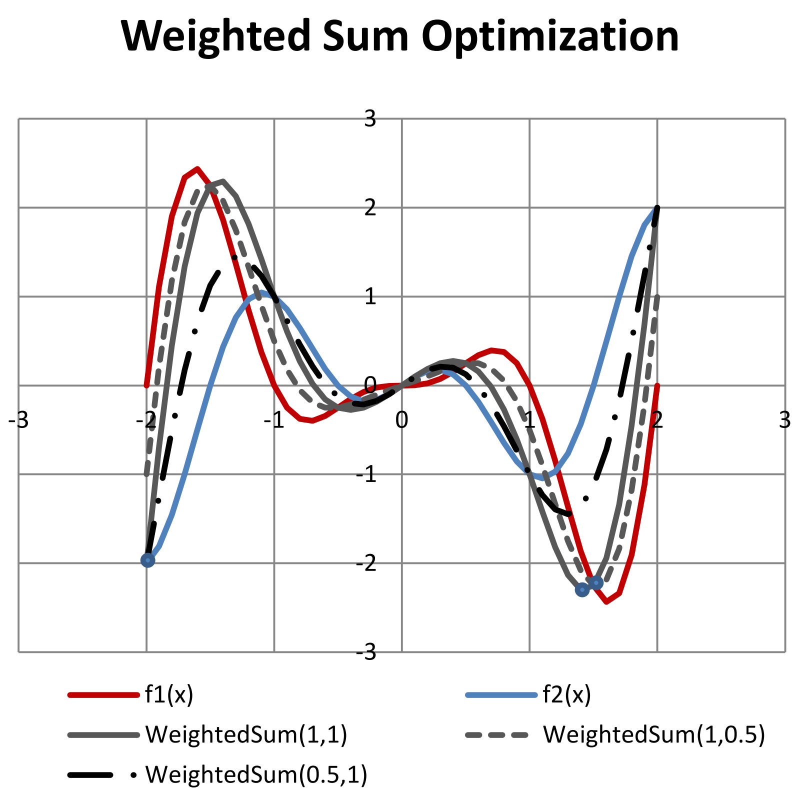
- Minimize and/or maximize 2 or more conflicting objectives
- Available techniques
  - Weighted Sums (Linear aggregation)
    - A single objective function is constructed
    - Optimum design depend on weighing factors
  - Pareto Optimization
    - Simultaneous min/max of **multiple** objective functions
    - Results in "Pareto" optimum points presenting the tradeoff between the conflicting objectives (Pareto curve, Pareto surface, or Pareto hypersurface)

$$
\begin{aligned}
\underset{x}{\text{minimize}}\quad & \breve{f}(x) = \sum_{l} \alpha_l f_l(x) & \\
\text{subject to}\quad & g_i(x) \le 0 & i = 1,\dots,M \\
& h_j(x) = 0 & j = 1,\dots,N \\
& x_k^L \le x_k \le x_k^U & k = 1,\dots,d
\end{aligned}
$$

$$
\begin{aligned}
\underset{x}{\text{optimize}}\quad & f_l(x) & l = 1,\dots,L \\
\text{subject to}\quad & g_i(x) \le 0 & i = 1,\dots,M \\
& h_j(x) = 0 & j = 1,\dots,N \\
& x_k^L \le x_k \le x_k^U & k = 1,\dots,d
\end{aligned}
$$

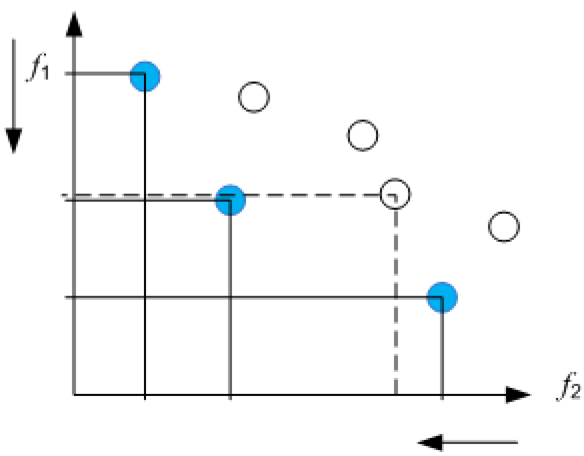
Multi-Objective Optimization

Weighted Sum

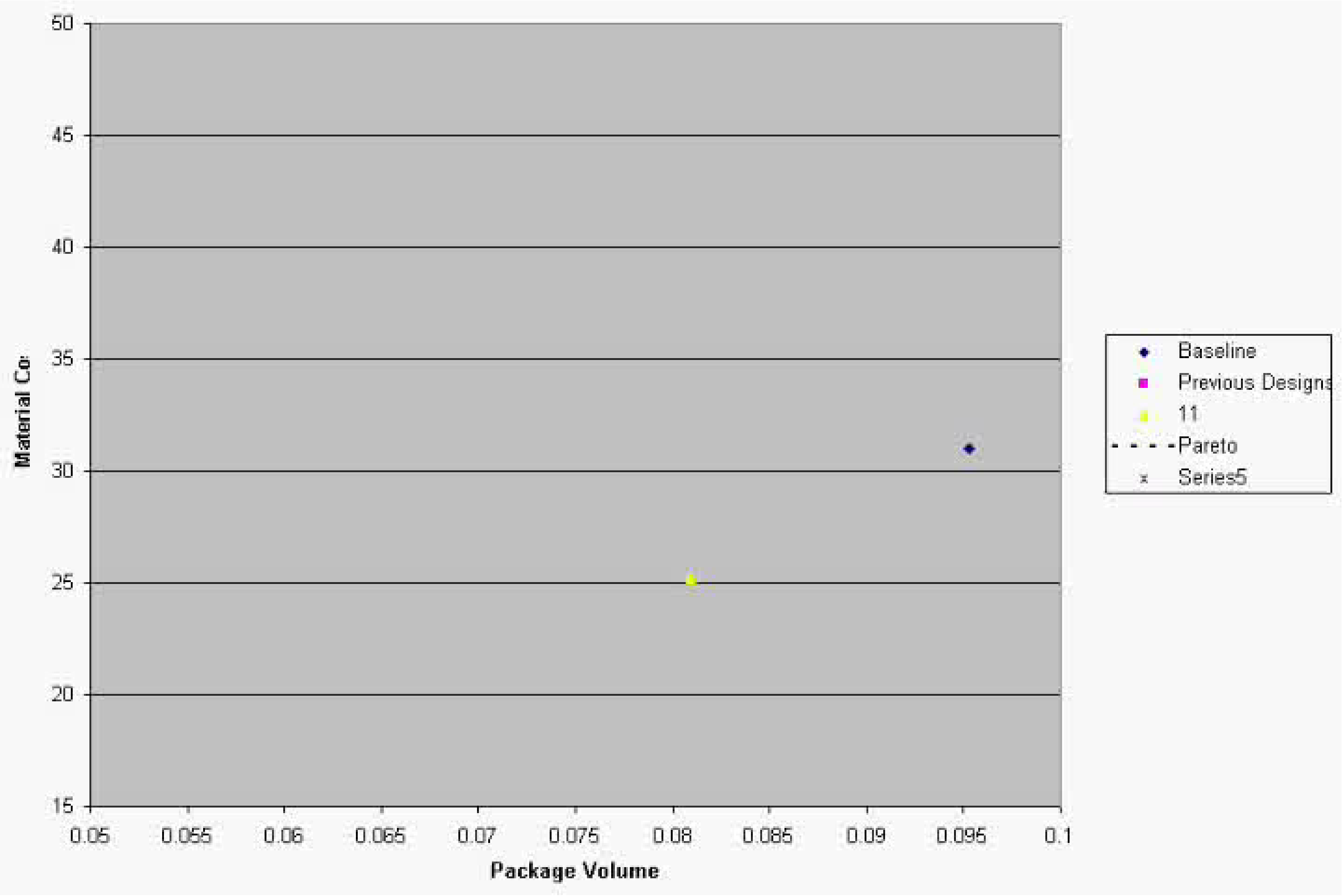


Pareto

- Tradeoff between the objective functions
- The design space is globally searched to find this tradeoff
- “Non-dominated” designs



Non-Dominated Sorting - MOGA



Why Optimize?

- Engineers always seek optimum solution:
  - Performance improvement (e.g. higher EER)
  - Price reduction (e.g. less amount of materials)
  - Quality improvement (e.g. better IAQ, better reliability, etc.)
  - …
  - And Making our life easier!

How to Optimize?

- In the old days
  – Intuition (only experienced engineers can)
  – Parametric analysis (limited number of parameters – time consuming)
- Mathematically rigorous practices:
  – Using optimization algorithms
    - Gradient based solvers
    - Stochastic approaches
  – Post processing results using scatter matrix plots and multi-dimensional data representation

Intuition

- A near optimum solution can be obtained using intuition for problems with a single variable
- Pressure drop minimization: use the larger possible diameter

– $\Delta P = f(D^{-4})$ laminar flow, $\Delta P = f(D^{-5})$ fully turbulent flow

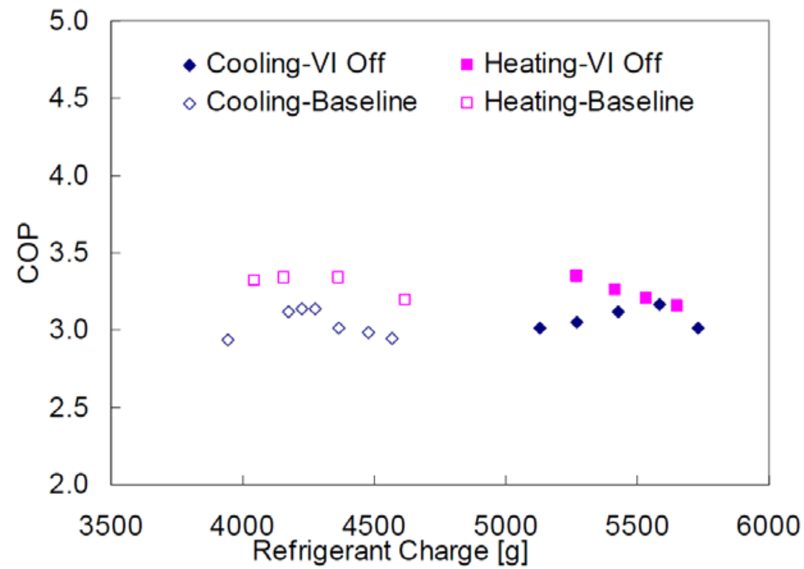
– Not valid if larger diameter caused transition from laminar to turbulent

– Does not account for:

    - Impact on heat transfer rate (loss/gain)
    - Impact on cost, (e.g. raw material, tooling, refrig. charge)
    - Geometric constraints

Parametric Analysis – 1 variable

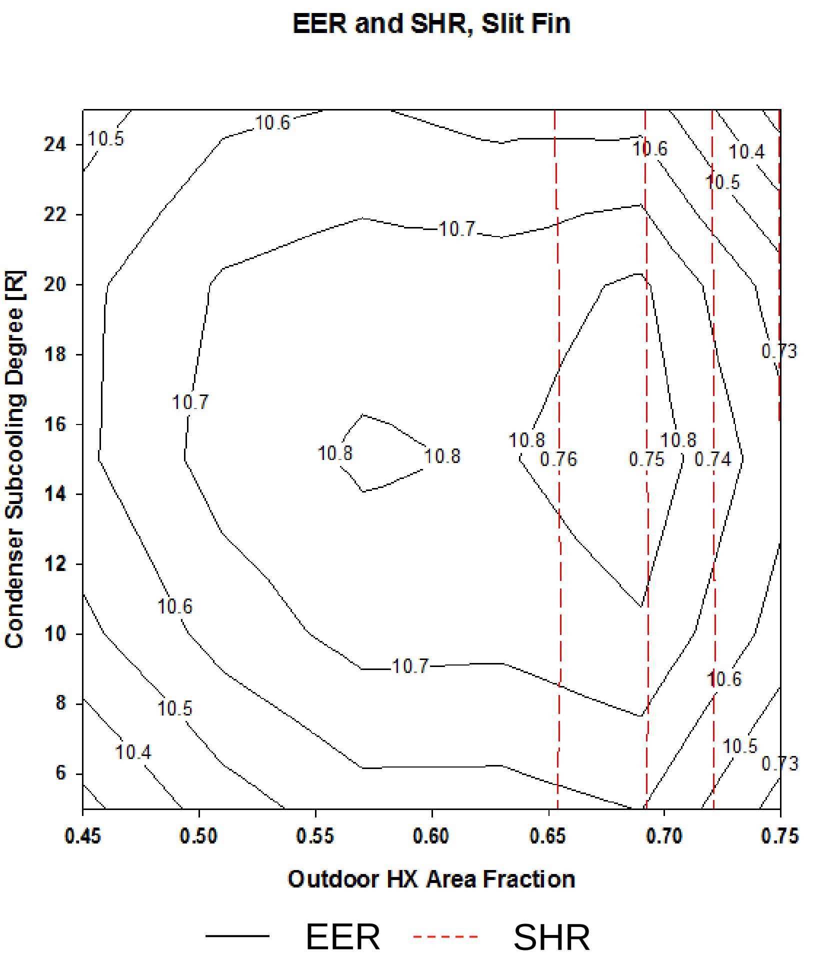
- A local optimum can be obtained
- Good for limited range with a smooth function: e.g. charge optimization
  - Usually performed at the design conditions
  - Performance is plotted against charge and optimum is identified
  - Does not account for seasonal performance



From Wang, X., 2008, PERFORMANCE INVESTIGATION OF TWO-STAGE HEAT PUMP SYSTEM WITH VAPOR-INJECTED SCROLL COMPRESSOR , Ph.D. Dissertation, UMD
http://hdl.handle.net/1903/7863

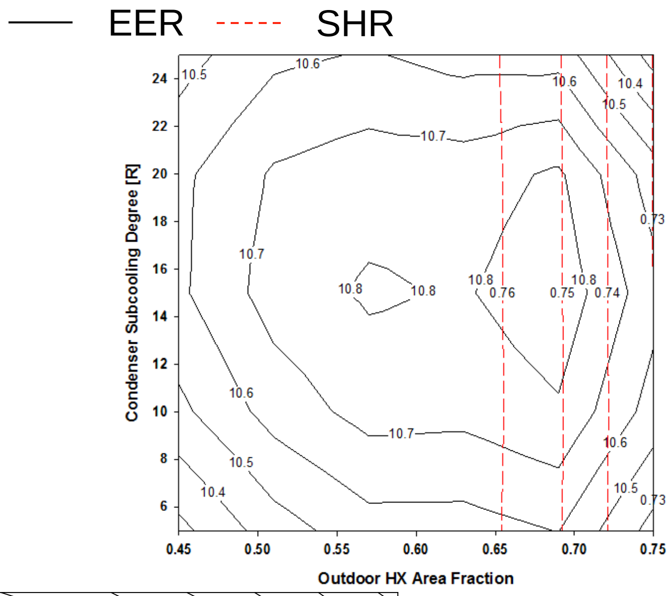
Parametric Analysis – 2 Variables

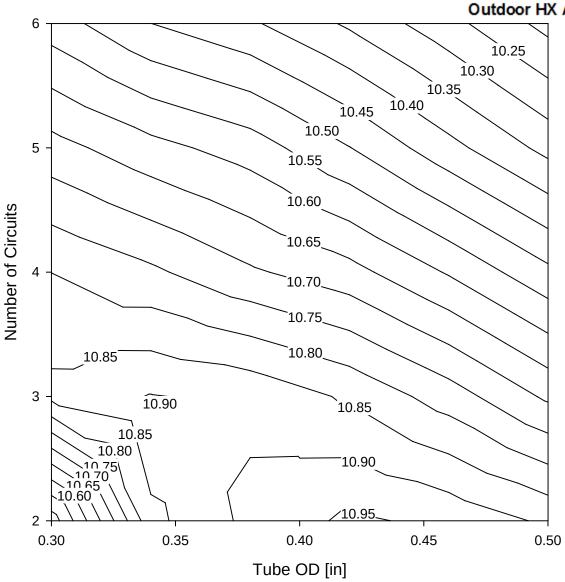
- Contour plots are required to identify optimum
- Good for smooth functions only
- Constraints handling is not as easy as systematic optimization techniques
- Advantages:
  - Simple: reasonable engineering time requirement
  - Provides visual feedback of predicted performance trends
  - Same results can be used for multiple optimization studies
- Limitations:
  - Solution depend on ΔX_i
  - Less efficient function evaluation



Parametric Analysis – n Variables

- Computationally expensive (# of experiment = $_{i=1}\Pi_n[\Delta X_i]$
  - For 6 variables each discretized into 10 → 10^6 experiments are needed
  - Inefficient constraint handling
- Post processing required beyond simple visualizations
- Non-dominated sorting algorithms are needed to identify Pareto solutions
  - to identify optimal paths among competing objectives





Systematic Optimization

Deterministic

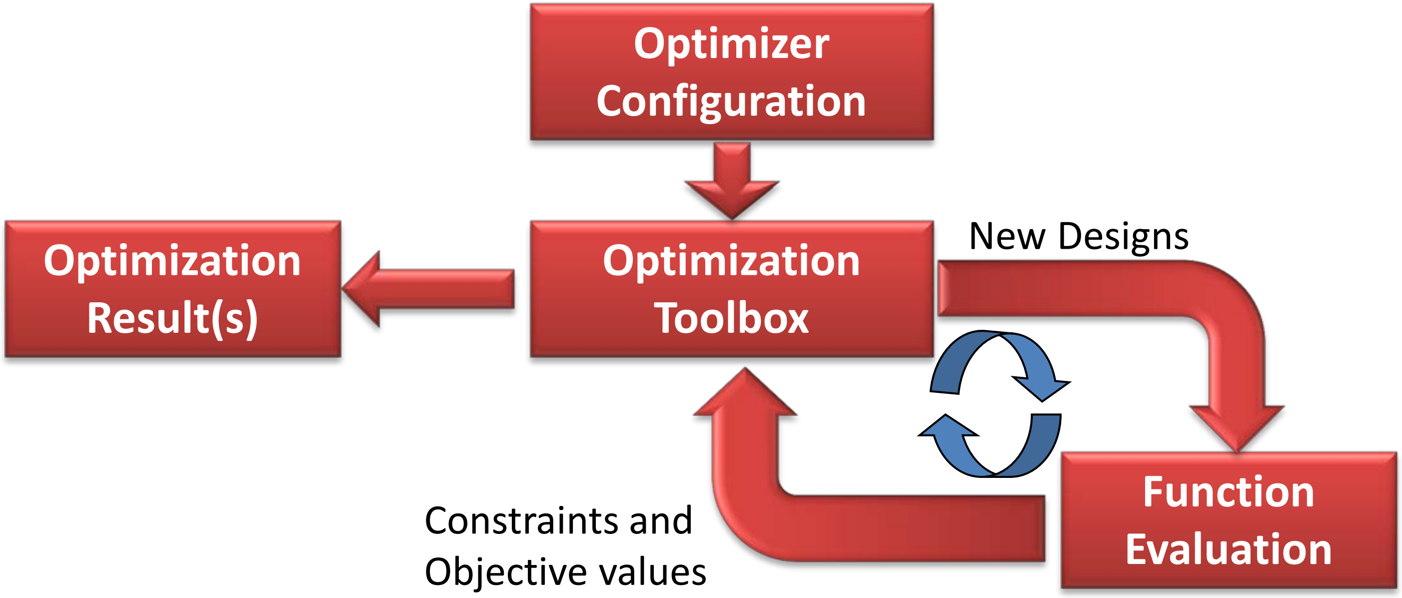
- Gradient Based (local optimum)
  - Gradient descent
  - Newton’s method
  - Etc.
- Branch and Bound
  - Discrete, or combinatorial problems
- Algebraic Geometry
- State Space Search (AI)

Probabilistic

- Stochastic
  - Monte Carlo
  - Simulated Annealing
  - Etc.
- Heuristic
  - Genetic Algorithms
  - Evolutionary Algorithms
  - Swarm Based: particle swarm, ant colony
  - Differential Evolution
  - Etc.

Optimization Tools

- Several tools exist for optimization
- Usually assume “Black box” function evaluation
- Coupling efforts vary significantly
- Some are readily implemented for specific software



A CASE STUDY: PARAMETRIC ANALYSIS VS. SYSTEMATIC OPTIMIZATION

Optimization Problem

- Problem devised from 1997 paper by Rice
- Maximize the EER of an R-410A air conditioner for
  - Fixed Design Capacity = 2.67 TR (9.38 kW)
  - Fixed total HX finned area
  - Fixed total heat pump fan power = 513 W
  - Constant superheat = 5.6 K (10 R)
  - Maximum allowable sensible heat ratio = 0.75
- Variables
  - Tube outer diameter and # circuits
  - Fraction of the total HX finned area in the outdoor coil
    - adjusted so as to maintain air-side pressure drops constant
  - Condenser subcooling
  - Compressor size adjusted automatically to maintain constant capacity
- Parameters
  - High temperature design conditions: $DBT_{outdoor}$ = 308.15 K (95°F); DBT/WBT_{indoor} = 299.85/292.55 K (80/67°F)

Approach

Conventional – Model with Built-In Parametric Analysis Capability

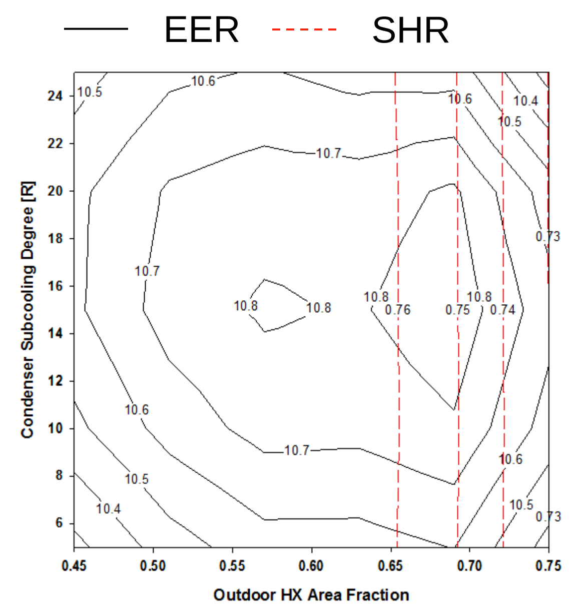
- The ORNL Heat Pump Design Model (HPDM) is readily capable of running parametric analyses
- Results are displayed as contour plots for 2 design variables at a time (slices)
  - Optimum points are identified w.r.t. 2 variables at a time

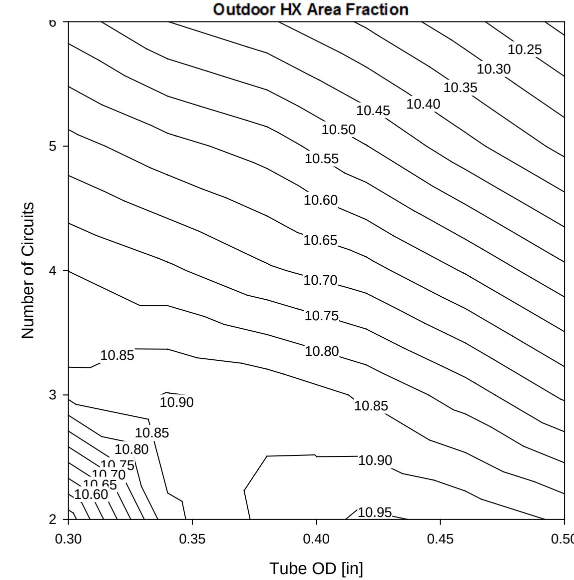
Systematic Optimization - GenOpt

- GenOpt: optimization program for the minimization of a cost function evaluated by an external simulation program
- A wrapper was developed to provide seamless coupling between GenOpt and HPDM

Conventional

- First, a 2-D parametric study is performed by changing the condenser subcooling and outdoor coil HX area fraction
  - Optimum was found to be for 15 R of subcooling with an outdoor HX size ratio of 0.69
- Second, the number of circuits and tube outer diameter for the outdoor coil were varied (another 2-D parametric study) using optimum subcooling and outdoor HX area fraction
  - Optimum was found to be for D = 0.42” and 2 circuits
- The optimum Design:
  - Subcooling = 15 R, outdoor coil surface area fraction = 0.69, outdoor coil D = 0.42”, and outdoor coil of 2 circuits
  - EER = 10.96, Refrigerant Charge = 7.825 lb_m, SHR = 0.751





Systematic Optimization - GenOpt

Optimization algorithm	Outdoor coil face area fraction [-]	Sub-cooling [R]	Outdoor coil tube outer diameter [in]	Outdoor coil, equivalent # of circuits [-]	EER [BTU/ W.hr]	SHR	Ref-rigerant charge [lb]
GPS – Hooke Jeeves (88 function evaluation)	0.694	17.82	0.325	3	10.89	0.75	6.0345
GPS – Coordinate Search (160 function evaluation)	0.694	15.94	0.4375	2	10.96	0.75	8.382
4-D Parametric	0.69	15	0.42	2	10.96	0.751	7.825

Optimization for 6 Variables

- Considering indoor coil tube size and number of circuits as additional design variables
- For the parametric analysis, a new 2-D parametric analysis was performed
  - D_{out} was varied between 0.2 and 0.5”
  - Number of equivalent circuits were varied between 2 and 6
- Optimum was found to be for D_{out} = 0.42” and 3 circuits
  - EER = 11.14, SHR = 0.73, Refrigerant charge = 9.16 lb_m

GenOpt Results

- 421 simulation runs

Outdoor coil face area fraction [-]	Sub-cooling [R]	Outdoor coil tube outer diameter [in]	Outdoor coil, # of circuits [-]	Indoor coil tube outer diameter [in]	Indoor coil, # of circuits [-]	EER [BTU/W.hr]	SHR	Charge [lb]
0.675	15	0.42	2	0.375	3.5	11.171	0.746	8.686
0.675	15.5	0.42	2	0.36	4	11.153	0.747	8.605
6-D parametric analysis								
0.69	15	0.42	2	0.42	3	11.14	0.73	9.16

Computational/Engineering Effort

Conventional – Built-In HPDM Parametric Analysis Capability

- Design was optimized on 2 steps
- After the first step, the design and parametric configuration files were adjusted manually (engineering time)
- Each step require some engineering time for post processing
- ~ 2 man hours were required to reach slightly sub-optimal solution

Systematic Optimization - GenOpt

- Using the GPS coordinate search algorithm, the optimizer required 281 simulations
- Post processing was trivial: the design parameters of the optimum point were specified by the optimizer
- No need for contour plots
- No intermediate engineering time is required

Multi-Objective Optimization

GenOpt: Weighting Sum

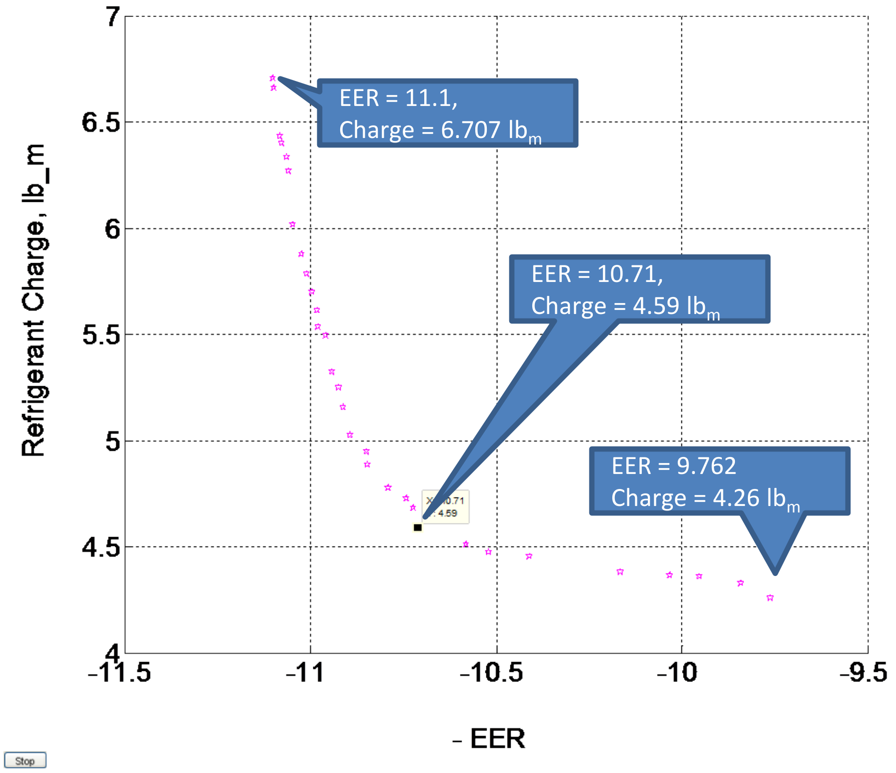
- A weighting sum function was constructed such that:

$$\underset{x}{\text{minimize}} \;\; \breve{f}(x) = -\alpha \times EER + \beta \times \text{Charge}_{\text{refrigerant}}$$

α	β	EER	Refrigerant Charge [lb_m]
1	0	11.15	8.605
0	1	5.56	4.343
1	1	10.658	4.196
1	0.5	10.8483	4.473

Multi-Objective GA

- Using the Pareto approach:



Conclusions

- Parametric analysis can become computationally prohibitive as number of variables are increased
- Parametric analysis with 2 variables at a time might miss the optimum for multivariable problems
- Available optimization toolboxes can be coupled with other simulation tools
  - Need for interface development
- Mathematical Optimization requires fewer function evaluations and are more efficient in handling larger number of variables
  - Local optima
    - Deterministic: redo the optimization with different initial guess
    - Probabilistic: re-run the optimization multiple times
- Multi-objective optimization problem
  - Weighted-Sum approach results depend on the weighting factors
  - Pareto approach allow the engineer to use his judgment for selecting the best design

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