THE MEASUREMENT OF U-VALUES ON SITE

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ABSTRACT

Standard techniques for the measurement of steady-state thermal properties in the laboratory are well established, but complementary measurements on site are needed in order to investigate practical effects of the influence of weather and workmanship.

As steady-state conditions rarely exist, it is necessary to derive the U-value from time-dependent data taking account of the phase difference between temperature and heat flux which results from the thermal mass of the structure.

This can be done by observing the variation with time of the ratio of the cumulative heat flux to the cumulative temperature difference; this ratio ultimately converges to the U-value. This paper examines the behaviour of this ratio under dynamic conditions to evolve criteria which determine when sufficient data have been collected. Cyclic daily variations of internal or external temperature give rise to a damped oscillation of the ratio, whilst changes in daily mean temperature can in some circumstances give rise to an apparent convergence to an erroneous value. In the latter case, however, a correction can be applied.

The analysis technique is illustrated by data collected at the author's laboratory.

INTRODUCTION

Various laboratory techniques are available for the measurement of the basic thermal properties of building materials. These include hot plate methods for thermal conductivity and, where appropriate, hot box methods for built-up sections. National standards for these test methods exist in many countries, and are also being developed in ISO.

Such tests provide input data for calculation procedures, but there are various reasons why actual performance may differ from that predicted on the basis of laboratory tests and calculations. These reasons include effects of the environment on the basic thermal properties of materials (particularly moisture effects, and also effects of air velocity and air infiltration); complex heat-flow patterns at interfaces of different materials (including thermal bridges, etc); and differences, intentional or otherwise, between the design and the structure as built. Such considerations point to the need for a technique to be available for in-situ measurements.

One fundamental property of a building component in terms of its thermal performance is its thermal transmittance, or U-value. In the UK the U-value applies between environmental temperatures (Danter 1974), which take account of both convective and radiant exchanges, on either side of the test element. Air temperatures are often used when they are similar to the environmental temperatures. The thermal conductance applies between the corresponding surface temperatures: the difference between conductance and transmittance is of course due to the internal and external surface resistances. (The thermal resistance, or R-value, is the reciprocal of the thermal conductance.)

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In principle a U-value is defined under conditions of steady temperatures and heat flux. Such conditions rarely apply during site tests, and this is one of the most important differences between laboratory and in-situ testing of thermal performance. At the simplest level the U-value may be derived from time-dependent data by using the average heat flux and the average temperature difference, provided that the measurement is continued long enough for thermal mass effects to have a negligible influence on the result. However, under adverse circumstances of large thermal mass, low U-value and changing internal or external temperature, this period can become very long, as noted by Roulet et al. (1985). Of even greater import, it is possible for the measured U-value to converge to an apparently stable, but erroneous, value.

This paper examines the basis of U-value measurement under dynamic temperature conditions, in order to evolve criteria for determining when sufficient data have been recorded, and to establish a basis for correcting for changes in mean internal and external temperature.

EXPERIMENTAL PROCEDURES

In the present work U-values are measured using a heat flow meter and two thermocouples measuring respectively the internal and external temperatures. These are connected to a data logger, capable of running unattended for several days, which scans the sensors continuously and at each hour records the average values during the preceding hour. This averages out any rapid fluctuations caused by, for example, convective air movements. The heat flux is measured at the inside surface. In the examples in this paper, U-values are derived from measurements of internal and external air temperature.

There are several factors which must be considered when undertaking this type of measurement. These include: (1) whether the position chosen for test is representative of the component; (2) the additional thermal resistance of the heat flow meter itself, and distortions of the heat flux caused by its presence; (3) surface contact; (4) emittance of the surface; (5) radiant and convective heat transfer to the surface and the appropriate temperature to be used in defining the U-value; (6) calibration uncertainties and instrument resolution.

These factors have been discussed in more detail elsewhere (for example Anderson 1984, McIntyre 1985). This paper is concerned with one specific aspect: how variations in internal or external temperatures affect the result.

THE BASIS OF U-VALUE MEASUREMENT UNDER DYNAMIC TEMPERATURE CONDITIONS

To obtain a U-value it is necessary to know the heat flux per unit area through the component and the temperature difference across it; in steady state the U-value would simply be the ratio of these two quantities.

In practice neither the internal nor the external temperature will be held constant during the course of a test and, as a result, the heat flux will also vary; but not in phase with the temperature variations because of the mass of the structure. The analysis in this paper considers the following aspects.

- -- The external temperature varies in an approximately sinusoidal cycle with a period of one day.
- -- The magnitude of the cycle is not usually fixed for several days in succession.
- -- The internal temperature will not usually be constant. Unless closely controlled, the mean daily value is likely to change if the external temperature changes, and there may be a cycle associated with intermittent heating. The author's laboratory often undertakes measurements in occupied houses, where the occupants' choice of heating patterns have to be accepted, although these may be far from ideal for testing purposes.
- -- There may be a change in the mean daily external temperature over the course of the test. In that case some of the recorded heat flux will be stored in the wall rather than passing through it.

Cyclic External Temperature

The simplest case of a cyclic external temperature is an external temperature that varies sinusoidally while the internal temperature is held constant, so that the temperature difference across the component at time t is

$$\Delta T(t) = \Delta T_o + \Delta T_1 \cos \omega t \tag{1}$$

where

ΔT_o = daily mean temperature difference
ΔT_1 = amplitude of external temperature cycle
$2\pi/\omega$ = 24 hours.

For practical reasons it is usually most convenient to measure heat flux at the interior surface. The admittance theory, which was developed for predicting temperatures inside buildings under cyclic conditions, particularly in sunny weather (Loudon 1968), can conveniently be applied to this situation. The heat flux density at the inner surface resulting from Equation 1 is (Milbank and Harrington-Lynn 1974):

$$q(t) = U \Delta T_o + f U \Delta T_1 \cos \omega(t-\phi) \tag{2}$$

where

U = U-value of component
f = decrement factor
ϕ = time lag of decrement factor.

The decrement factor relates the magnitude of the cyclic component of the heat flux at one surface to a temperature cycle at the opposite surface. Its associated phase lag, ϕ, represents the fact that the cyclic heat flux lags behind the cyclic temperature. Values of f and ϕ are tabulated for several typical constructions in the CIBSE Guide Section A3 (CIBSE 1980).

An instantaneous measurement of q and ΔT thus gives

$$\frac{q}{\Delta T} = U \frac{1 + f \frac{\Delta T_1}{\Delta T_o} \cos \omega(t-\phi)}{1 + \frac{\Delta T_1}{\Delta T_o} \cos \omega t} \tag{3}$$

which can take virtually any value depending on the relationship of t to ϕ, and ΔT_1 to ΔT_o.

Data are shown in Figure 1 for a test on a concrete wall with an internal lining. The temperature difference varied in a nearly regular cycle for six days, as shown in Figure 1(a). The resultant heat flux, shown in Figure 1(b), also displays a daily cycle but out of phase by about four hours. The ratio of these two curves is shown in Figure 2, which indicates that the instantaneous value would be anywhere between 0.09 and 0.32 $Btu/h \cdot ft^2 \cdot F$ (0.5 and 1.8 $W/m^2 \cdot °C$) depending on just when the measurement was made.

A more reliable result is obtained from the average of many readings of heat flux density, $(1/n)\ \Sigma q$, and of temperature difference, $(1/n)\ \Sigma \Delta T$. Assuming that the frequency of readings is sufficient (in relation to the period of the temperature variation) to allow the summations to be replaced by integrals, we have

$$\frac{\Sigma q}{\Sigma \Delta T} = \frac{\int_{t_1}^{t_1+\theta} q(t)\, dt}{\int_{t_1}^{t_1+\theta} \Delta T(t)\, dt}$$

$$= U \frac{\Delta T_o\, \omega\theta + f\, \Delta T_1 [\sin \omega(t_1 + \theta - \phi) - \sin \omega(t_1 - \phi)]}{\Delta T_o\, \omega\theta + \Delta T_1 [\sin \omega(t_1 + \theta) - \sin \omega t_1]} \qquad (4)$$

for an arbitrary starting time t_1 and integration period θ. As shown in Appendix A this reduces to approximately

$$\frac{\Sigma q}{\Sigma \Delta T} = U \left\{1 + \frac{\Delta T_1}{\Delta T_o \omega\theta} [a \sin \omega\theta + b(\cos \omega\theta - 1)]\right\} \qquad (5)$$

where a and b are functions of f, ϕ and t_1, and are constant for a given test. This is a damped sinusoidal oscillation of magnitude (peak to trough) of approximately

$$\pm\, 0.2 \frac{\Delta T_1}{\Delta T_o} \frac{1}{N} \qquad (6)$$

where N is the number of days since the start of the integration period. As discussed in Appendix A this is to be considered as an uncertainty band in the measured U-value.

Thus an estimate of the size of the external temperature cycle, $2\Delta T_1$, and of the mean internal/external temperature difference, ΔT_o, can be used to anticipate the test period (N days) required to reduce the uncertainty band in the measured result due to the external temperature variation, to a desired value. This can be particularly useful when a test is being conducted at a remote location for which weather reports are available. (A significantly shorter time will usually suffice for very lightweight structures.)

$\Sigma q/\Sigma \Delta T$ is shown in Figure 3, for the data in Figure 1. The U-value of this structure is thus 0.176 ± 0.003 Btu/h·ft²·F (1.00 ± 0.02 W/m²·°C) ± uncertainties from other sources. Compared with an instantaneous measurement, as in the case of Figure 1, the integration technique dramatically reduces the uncertainty band (Devisme et al. 1985, McIntyre 1985).

Variations in the Cyclic External Temperature Profile

In most cases the external temperature will not vary in a regular sinusoidal cycle. It can be represented by a Fourier series, however, each term of which can be treated as in the previous section, provided the daily mean remains the same. The conclusions above still apply: the limits of the excursion of $\Sigma q/\Sigma \Delta T$ are to be regarded as the uncertainty band in the measurement, even though this may show random variations in size if ΔT_1 varies day by day.

Cyclic Variations in Internal Temperature

For a constant external temperature we may write (Milbank and Harrington-Lynn 1974)

$$\Delta T(t) = \Delta T_o + \Delta T_2 \cos \omega t$$

$$q(t) = U \Delta T_o + Y \Delta T_2 \cos \omega(t + \phi_y)$$

where

- ΔT_2 = amplitude of internal temperature cycle
- Y = admittance
- ϕ_y = phase lead of admittance.

The admittance relates the magnitude of the cyclic component of the heat flux at a surface to the temperature cycle at the same surface. Its value is determined primarily by the thermal properties of the surface layer in a multilayer structure.

As is discussed in Appendix B, the result is again a damped oscillation of the ratio $\Sigma q/\Sigma \Delta T$; but the magnitude of the oscillation, given approximately by

$$\pm \frac{1}{6} \frac{Y}{U} \frac{\Delta T_2}{\Delta T_o} \frac{1}{N}$$

is liable to be somewhat greater for a given size of temperature cycle than in the case of the external cycle. It is therefore helpful to avoid large changes in internal temperature during a test. Some typical values of admittance are given in Table 1 (abstracted from CIBSE 1980).

Changes in Mean Temperature

As distinct from the cyclic variations considered above, a change in mean internal or external temperature over any day causes a change in the heat stored in the wall at the end of the 24-hour period. This means that the measured heat flux integrated over the 24 hours will differ from the product of the U-value and the integrated temperature difference, because of this change in heat storage.

As an example, consider a double-leaf brick wall with cavity insulation, having a U-value of 0.1 Btu/h$\cdot$ft$^2\cdot$F (0.6 W/m$^2\cdot$°C) and thermal capacity of 13 Btu/ft$^2\cdot$F (0.27 MJ/m$^2\cdot$°C). If this wall is subjected to a temperature difference of 18 F (10°C) the heat flowing through it by transmission in 24 hours is

$$0.1 \times 18 \times 24 \text{ Btu/ft}^2 \quad (0.6 \times 10 \times 24 \times 3600 \text{ J/m}^2)$$
$$= 43 \text{ Btu/ft}^2 \quad (0.51 \text{ MJ/m}^2).$$

If its temperature rises by 1.8 F (1°C), the change in heat storage is 23 Btu/ft^2 (0.27 MJ/m^2), or approximately 50% of the transmitted heat. Thus the potential for error from fairly small temperature changes is large.

This is illustrated in Figures 4 and 5, which display measurements made on the same double leaf brick wall having urea formaldehyde cavity fill. The solid line in Figure 4 shows the position for the total measurement period of 15 days. Initially we see oscillations of the type discussed previously, around a U-value of 0.12 Btu/h$\cdot$ft$^2\cdot$F (0.67 W/m$^2\cdot$°C) during the first three days. As the test continues, however, there is a sharp drop in U-value, and it appears to stabilise at around 0.088 Btu/h$\cdot$ft$^2\cdot$F (0.50 W/m$^2\cdot$°C), although on a slight downward trend.

We have also examined the data from this test starting the integration at the fourth day. The solid line in Figure 5 gives $\Sigma q/\Sigma \Delta T$ which shows a stable value at 0.078 Btu/h$\cdot$ft$^2\cdot$F (0.44 W/m$^2\cdot$°C) for the final three days. With a 40% fluctuation in the apparent U-value what is the correct value?

These discrepancies arose in this instance because of changes in the mean daily temperature of the wall, and the resultant heat storage effects. Figures 6 and 7 show how the internal and external temperature changed during the 15-day test period. Initially there was a rise in both the internal and external temperature, which pushed the apparent U-value up, followed by a fall which pushed it down.

The effect of such temperature changes is considered in Appendix C, which proposes that instead of obtaining U from

$$U = \frac{\Sigma q}{\Sigma \Delta T} \tag{8}$$

it should be obtained from:

$$U = \frac{\Sigma q - \rho c \ell \left(\frac{1}{3}\,\delta T_i + \frac{1}{6}\,\delta T_e\right)}{\Sigma \Delta T} \tag{9}$$

where

Σq = integrated heat flux
$\Sigma \Delta T$ = integrated temperature difference
$\rho c \ell$ = thermal capacity of wall per unit area
δT_i = change in internal temperature since start of integration period
δT_e = change in external temperature since start of integration period

with an amendment to the correction term in Equation 9 dependent on the location of the predominant thermal mass within the structure.

A particular consequence of Equation 9 is that if T_i or T_e is continually rising each day, it is possible for the effective increase in heat storage to be as much as the accumulation in Σq for each day, so that U obtained from Equation 8 can remain high for as long as the temperature continues to rise. The converse applies to a falling temperature trend. Thus, under conditions of adverse temperature trend, the test periods necessary to obtain the correct U-value can run to several weeks. (Roulet et al. 1985 found 50 days to be necessary in one instance.)

However, if we apply the adjustment included by Equation 9 to the data in Figures 4 and 5, we obtain the broken lines shown on these Figures. The correction was obtained by applying Equation C-10 to the exterior thermal mass and Equation C-11 to the interior thermal mass, and was computed according to algorithm (C-8). The latter implies no correction during the first 24 hours, the correction being built up during the second 24 hours, and only fully applied thereafter. The U-value now stabilises very much more quickly (after two days in each case) and to a consistent value: 0.099 ± 0.002 Btu/h·ft²·F (0.56 ± 0.01 W/m²·°C) in the case of Figure 4, and 0.096 ± 0.002 Btu/h·ft²·F (0.54 ± 0.01 W/m²·°C) in the case of Figure 5. (The uncertainties quoted are from the range of U near the end of the integration period, as discussed previously; uncertainties from other sources must also be added.)

DISCUSSION

The analysis has divided the effect of temperature changes into two components: that due to cyclic internal or external temperatures, and that due to changes in the mean daily value of either temperature.

Cyclic temperature variations cause an oscillation of $\Sigma q/\Sigma\Delta T$ around the U-value; but these oscillations are damped and the test can be continued until the uncertainty associated with them is relatively small.

Changes in mean daily temperature require a correction to be made if the structure under investigation has significant thermal mass. To do this an estimate has to be made of the thermal capacity of the structure. Usually it will be possible to do this to sufficient accuracy (e.g. ±20%) for the purposes of this analysis. As an illustration, in Figure 8 we show the data from Figure 4 where the estimate of thermal capacity has been decreased and increased by 50%. Estimates of the thermal mass as poor as this are sufficient to affect the final result noticeably, but one can readily see that something is wrong. Unlike the corrected curve in Figure 4, neither "corrected" curve in Figure 8 shows daily oscillations about a stable value from the end of day 2 onwards (the time at which the correction becomes fully applied). In such cases the result be considered unreliable - an essentially level trend should be sought, with an oscillation resulting from cyclic daily temperature variations.

The correction term depends heavily on the temperatures recorded during the first 24 hours. If these are unrepresentative, the corrected function may not be level with time. In such cases the data can be re-examined, starting the integration at a different time. A consistent final result should be looked for, irrespective of starting time, as in Figures 4 and 5.

The analysis provided in Appendix C has some similarities to that of Ahvenainen et al. (1980). Ahvenainen, however, retained the first few terms of the infinite series and used hour-by-hour data to derive the time constants involved. Thus all variations - mean temperature drift, cyclic, and random - are included within the same analysis. This approach did not prove successful for measurement periods of two or three days. Roulet et al. (1985) used the same theory applied to longer measurement periods, and obtained results consistent with the basic method (Equation 8), with the necessary measurement period reduced in some, but not all, cases.

The proposals in this paper are rather simpler to apply and provide a visual appraisal of the results to judge the reliability of the final value. From analysis of several U-value tests undertaken by the author, our approach gives consistent results for the estimate of the U-value. A further example is shown in Figure 9, which is a cavity brick wall. Here, at the end of the 14-day measurement period, the uncorrected and corrected curves yield virtually the same result, but the greater stability of the corrected curve makes it more reliable for the determination of the final value.

The proposed correction factor is significant for walls having fairly large mass; problems are substantially less with walls of low mass. Nevertheless, Equation 9 can still be used, though the correction for any temperature changes will be small.

A different analysis method, involving a phase adjustment for cyclic changes, was proposed in the context of a lightweight construction by Brown and Schuyler (1982). This involved an iterative analysis to determine the phase lag ϕ, with the heat flux then adjusted in phase by that amount. This works well with lightweight structures, but is less helpful for massive structures unless both the internal and external temperature cycles are very regular. Two phase angles are involved: the phase lead of the admittance on account of the internal temperature cycle, and the phase lag of the decrement factor on account of the external temperature cycle.

Another important point, however, is raised by the work of Brown and Schuyler. They derive the thermal resistance of their wall as a function of its mean temperature. The analysis provided in the present paper assumes that the U-value is sensibly independent of temperature over the range of temperatures encountered in the test. If that is not the case it would have to be allowed for in the interpretation of the results.

CONCLUSION

A method has been provided for analysing heat flow and temperature data to derive the thermal transmittance (U-value) of a structure in-situ (which can be applied equally to a measurement of thermal conductance or thermal resistance). It is most valuable when measuring U-values of structures of relatively high mass under adverse temperature conditions. Adverse temperature conditions include relatively small internal to external temperature differences (say 20-30 F (10-15°C); lower values are not recommended at all), and significant changes in internal and/or external temperature during the course of the test. The analysis technique provides the possibility of reducing measurement periods from several weeks to some days. Criteria are provided (namely a level apparent U-value within the envelope of a damped oscillation after adjustment for thermal mass effects) for knowing when a reliable result has been obtained.

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APPENDIX A

SIMPLIFICATION OF EXPRESSION FOR SINUSOIDAL TEMPERATURE VARIATION

Equation 4 in the main text can be simplified when $\Delta T_o \omega\theta >> \Delta T_1$. As in practical conditions the temperature swing $2\Delta T_1$ will not normally exceed the mean temperature difference, this limit is reached when

$$\theta >> \frac{1}{2} \cdot \frac{24}{2\pi} \quad \text{or} \quad \theta >> 2 \text{ h}.$$

After the first few hours a binomial expansion of the denominator gives, after re-arrangement,

$$\frac{\Sigma q}{\Sigma \Delta T} = U \left\{1 + \frac{\Delta T_1}{\Delta T_o \omega\theta} \left([f \cos \omega(t_1 - \phi) - \cos \omega t_1] \sin \omega\theta + [f \sin \omega(t_1 - \phi) - \sin \omega t_1](\cos \omega\theta - 1)\right)\right\} \qquad \text{(A-1)}$$

This represents a sinusoidal oscillation damped by the $1/\theta$ factor, converging towards the U-value as θ becomes large.

The magnitude of the oscillation at any time may be found from two successive turning points of Equation A-1. Since the sine functions change much more rapidly than the $1/\theta$ factor, and t_1 and ϕ are constants, the turning points of Equation A-1 are approximately those of

$$[f \cos \omega(t_1 - \phi) - \cos \omega t_1] \sin \omega\theta + [f \sin \omega(t_1 - \phi) - \sin \omega t_1](\cos \omega\theta - 1)$$

which occur when

$$\tan \omega\theta = \frac{f \cos \omega(t_1 - \phi) - \cos \omega t_1}{f \sin \omega(t_1 - \phi) - \sin \omega t_1}$$

so that at the turning points

$$\frac{\Sigma q}{\Sigma \Delta T} = U \left\{1 + \frac{\Delta T_1}{\Delta T_o \omega\theta} \left(\pm \sqrt{1 + f^2 - 2f \cos \omega\phi} - [f \sin \omega(t_1 - \phi) - \sin \omega t_1]\right)\right\}.$$

The magnitude of the oscillation is thus

$$\pm \frac{\Delta T_1}{\Delta T_o \, 2\pi N} \sqrt{1 + f^2 - 2f \cos \omega\phi} \qquad \text{(A-2)}$$

where N is the number of days. This is independent of the starting time t_1.

Although in this case $\Sigma q/\Sigma\Delta T = U$ at each 24-hour period irrespective of the starting time t_1 ($\omega\theta = 2n\pi$ in Equation A-1), it does not in general oscillate about this value. It would only do that if t_1 were chosen such that

$$f \sin \omega(t_1 - \phi) = \sin \omega t_1. \qquad \text{(A-3)}$$

This might appear to indicate the scope for reducing the uncertainty by starting the test such that the condition represented by Equation A-3 is met. In practice, however, neither f nor ϕ may be known a priori; also the other factors discussed in the paper (daily temperature variation not precisely cyclic, effect of internal temperature cycle, effect of changes in mean temperature) make $\Sigma q/\Sigma\Delta T$ a much more complicated function. It is therefore recommended that the whole range represented by Equation A-2 be regarded as an uncertainty band.

The magnitude of the function $\sqrt{1 + f^2 - 2f \cos \omega\phi}$ is in the range 0.0 to 2.0. To obtain an indication of its likely magnitude in practice, the function has been calculated for all the constructions tabulated in the CIBSE Guide (CIBSE 1980). The constructions have U-values ranging from 0.07 to 0.58 Btu/ft²·h·F (0.4 to 3.3 W/m²·°C), and the function is shown as a function of f in Figure A-1. From this we see that the function does not exceed about 1.3. It can be lower for lightweight structures with $f \rightarrow 1$ and $\phi \rightarrow 0$; but most structures with any appreciable thermal mass have $f < 0.5$, and as a guide to the likely magnitude of the variations in $\Sigma q/\Sigma\Delta T$, the value of 1.3 is suggested in general. The magnitude of the oscillation is then

$$\pm 1.3 \frac{\Delta T_1}{\Delta T_o \ 2\pi \ N} \approx \pm 0.2 \frac{\Delta T_1}{\Delta T_o} \frac{1}{N} . \qquad \text{(A-4)}$$

Thus if the internal temperature is maintained at 68 F (20°C) and the external temperature varies between 41 and 59 F (5 and 15°C) then ΔT_o = 18 F (10°C) and ΔT_1 = 9 F (5°C), so that after five days the uncertainty band is ±0.02, or 2%.

APPENDIX B

INTERNAL TEMPERATURE CYCLE

The analysis is formally the same as the external temperature cycle, with f replaced by Y/U and $-\phi$ replaced by ϕ_y, giving a similar expression to Equation A-1. The magnitude of the oscillation is, from Equation A-2,

$$\pm \frac{\Delta T_2}{2\pi \ N \ \Delta T_o} \sqrt{1 + \left(\frac{Y}{U}\right)^2 - 2 \frac{Y}{U} \cos \omega\phi_y} .$$

Here the phase lead of the admittance ϕ_y is relatively small (one to two hours), and most structures have Y >> U (CIBSE 1980). An indication of the likely size of the uncertainty band is then

$$\pm \frac{1}{6} \cdot \frac{Y}{U} \cdot \frac{\Delta T_2}{\Delta T_o} \cdot \frac{1}{N}. \qquad \text{(B-1)}$$

If, for example, a wall such as brick/insulation/brick has

Y = 0.70 Btu/ft²·h·F (4.0 W/m²·°C)

U = 0.07 Btu/ft²·h·F (0.4 W/m²·°C)

and

ΔT_o = 18 F (10°C)

ΔT_2 = 9 F (5°C)

the uncertainty band after five days is ± 0.17 or ± 17%. This is much greater than the uncertainty band associated with an external temperature cycle of the same size (see Appendix A). The test would have to be continued rather longer in such circumstances, unless the internal temperature can be controlled.

APPENDIX C

CHANGE IN MEAN TEMPERATURE

This Appendix examines the effect of a change in mean daily external or internal temperature, on the integrated heat flux measured at the inside surface.

External Temperature Change

Consider a uniform wall of thickness ℓ, made from material with density ρ, thermal conductivity λ, thermal capacity c, and thermal diffusivity $\kappa = \lambda/\rho c$, having initially zero temperature throughout. Denote distance through the wall by x, and ignore any effect of surface resistance.

Consider a step change δT_e at one surface, such that for $t > 0$, $T = \delta T_e$ at $x = \ell$, while T is maintained at zero at $x = 0$. This problem can be solved using the technique of Laplace transforms, described more fully in standard texts such as Carslaw and Jaeger (1959). Using a bar over a variable to denote its Laplace transform, then for $t > 0$,

$$\frac{d^2\overline{T}}{dx^2} - q^2\overline{T} = 0 \qquad \text{(C-1)}$$

where

$q^2 = p/\kappa$
p = Laplace parameter.

The boundary conditions are $\overline{T} = 0$ at $x = 0$ and $\overline{T} = \delta T_e/p$ at $x = \ell$, for which the solution of Equation C-1 is

$$\overline{T} = \frac{\delta T_e \sinh qx}{p \sinh q\ell}.$$

Let H be the total quantity of heat per unit area which has crossed the plane $x = 0$ from $t' = 0$ to $t' = t$. Then

$$H = -\lambda \int_0^t \left[\frac{\partial T}{\partial x}\right]_{x=0} dt'$$

so that

$$\overline{H} = -\lambda \frac{1}{p} \left[\frac{\partial \overline{T}}{\partial x}\right]_{x=0}$$

$$= -\frac{\lambda\, \delta T_e\, q}{p^2 \sinh q\ell}.$$

This can be solved by contour integration to give

$$H = -\frac{\lambda\, \delta T_e}{\ell}\left(t - \frac{\ell^2}{6\kappa} - \frac{2\ell^2}{\kappa\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-\kappa n^2\pi^2 t/\ell^2}\right). \qquad \text{(C-2)}$$

We shall use this result to consider the effect on the heat flux due to a temperature change δT_e between one day and the next. The exponential terms may be dropped since, for all practical building materials, the terms involving exponentials become negligible after a few hours, giving

$$H = -\frac{\lambda\, \delta T_e}{\ell}\left(t - \frac{\ell^2}{6\kappa}\right).$$

Now the U-value of this wall is $U = \lambda/\ell$, so that

$$H = -U\, \delta T_e\, t + \frac{1}{6}\rho c \ell\, \delta T_e. \qquad \text{(C-3)}$$

H is the integrated heat flux, Σq, at the interior surface due to a change in temperature δT_e at the exterior surface. The first term of Equation C-3 is the heat flux resulting from steady-state heat flow and the second term is that due to the change in heat storage. Any pre-existing temperature difference may be incorporated in the first term, giving

$$\Sigma q = U\, \Sigma\Delta T + \frac{1}{6}\rho c\ell\, \delta T_e. \qquad \text{(C-4)}$$

Internal Temperature Change

A step change δT_i in internal temperature can be treated similarly by solving Equation C-1 with the boundary conditions $\overline{T} = \delta T_i/p$ at $x = 0$ and $\overline{T} = 0$ at $x = \ell$. This leads to

$$\overline{T} = \frac{\delta T_i (\sinh q\ell \cosh qx - \cosh q\ell \sinh qx)}{p \sinh q\ell}$$

$$\overline{H} = \frac{\lambda \, \delta T_i \, q \cosh q\ell}{p^2 \sinh q\ell}$$

and

$$H = \frac{\lambda \, \delta T_i}{\ell} \left(t + \frac{\ell^2}{3\kappa} - \frac{2\ell^2}{\kappa\pi^2} \sum_{n=1}^{\infty} e^{-\kappa n^2 \pi^2 t/\ell^2} \right). \quad \text{(C-5)}$$

Dropping the exponential terms the integrated flux is

$$\Sigma q = U \, \Sigma\Delta T + \frac{1}{3} \rho c\ell \, \delta T_i. \quad \text{(C-6)}$$

Effect on Measured U-value

Equations C-4 and C-6 may be combined to give

$$\Sigma q = U \, \Sigma\Delta T + \rho c\ell \left(\frac{1}{3} \delta T_i + \frac{1}{6} \delta T_e \right)$$

so that the best estimate of U is obtained from

$$U = \frac{\Sigma q - \rho c\ell \left(\frac{1}{3} \delta T_i + \frac{1}{6} \delta T_e \right)}{\Sigma\Delta T} \quad \text{(C-7)}$$

where

Σq = integrated heat flux
$\Sigma\Delta T$ = integrated temperature difference
$\rho c\ell$ = thermal capacity of wall per unit area
δT_i = change in internal temperature since start of integration period
δT_e = change in external temperature since start of integration period.

As mentioned previously, the temperature changes δT_i and δT_e to be used are those occurring between one day and the next. This is so that (1) the exponential terms in Equations C-2 and C-5 can be dropped, and (2) we do not attempt to adjust for the cyclic temperature change discussed in Appendices A and B for which the present theory is not valid without these exponential terms.

This means that no correction is applied during the first 24 hours. Subsequently it is advantageous in obtaining a relatively smooth curve of apparent U-value against time, to make an adjustment at each reading by reference to the temperature 24 hours previously, rather than apply a correction each 24 hours based on mean daily temperature changes. Thus for data collected at hourly intervals for a total of N hours, a suitable computing method is:

```
SQ = 0, ST = 0
for h = 1 to N
   SQ = SQ + q(h)
   ST = ST + Ti(h) - Te(h)
   if h > 24 then SQ = SQ - (FI*(Ti(h) - Ti(h-24)) + FE*(Te(h) - Te(h-24)))/24
   U(h) = SQ/ST
next h
```

(C-8)

U(h) is then plotted against h. The factors FI and FE are 0.333*$\rho c\ell$ and 0.167*$\rho c\ell$ respectively for a uniform single layer wall, or the coefficients of δT_i and δT_e respectively in the formulae below for other walls. $\rho c\ell$ must be obtained in units consistent with SQ; that depends on the time interval between data points.

Extension to Practical Structures

The above theory has considered a uniform wall with zero surface resistance. The existence of surface resistance does not significantly affect the conclusions provided that, as is usually the case, the surface resistance is much less than the resistance of the wall. The theory can be extended in the following instances to give more general applicability.

Internal Insulation Layer Against Thermally Massive Outer Layer. Here we take the mass as being concentrated entirely in the outer leaf, and the thermal resistance entirely in the inner leaf. Then, if the internal temperature changes there is no change in temperature of the thermal mass, and therefore no adjustment. If the external temperature changes by δT_e, then both faces of the thermal mass change by δT_e; however, this will be achieved by heat flux from the exterior surface in this limit so that the heat flux at the interior surface will be unaffected. Thus no correction is applicable.

External Insulation Layer Against Thermally Massive Inner Layer. In this case the temperature of the thermal mass is unaffected by a change in external temperature, but the whole thermal mass rises by δT_i as a result of an internal temperature change δT_i. In this limit the temperature rise of the thermal mass is achieved by heat flux at the interior surface so that

$$\Sigma q \rightarrow \Sigma q - \rho c \ell \, \delta T_i . \qquad \text{(C-9)}$$

Two Layer Structure, Both Layers Having Finite Resistance, the Inner Layer Having Negligible Thermal Mass (Compared to the Other). Let the resistance of the outer leaf be R_1, its thermal mass per unit area be $\rho c \ell$, and the resistance of the inner leaf be R_2. This situation can be solved by the Laplace transform technique to obtain

$$\Sigma q \rightarrow \Sigma q - \rho c \ell \left[\frac{1}{3} \left(\frac{R_1}{R_1+R_2} \right)^2 \delta T_i + \frac{R_1}{R_1+R_2} \left(\frac{1}{6} + \frac{1}{3} \frac{R_2}{R_1+R_2} \right) \delta T_e \right] . \qquad \text{(C-10)}$$

Two Layer Structure, Both Layers Having Finite Resistance, the Outer Layer Having Negligible Thermal Mass (Compared to the Other). Let the resistance of the inner leaf be R_1, its thermal mass per unit area be $\rho c \ell$, and the resistance of the outer leaf be R_2. The solution in this case is

$$\Sigma q \rightarrow \Sigma q - \rho c \ell \left[\left(\frac{R_2}{R_1+R_2} + \frac{1}{3} \left(\frac{R_1}{R_1+R_2} \right)^2 \right) \delta T_i + \frac{R_1}{R_1+R_2} \left(\frac{1}{6} + \frac{1}{3} \frac{R_2}{R_1+R_2} \right) \delta T_e \right] . \qquad \text{(C-11)}$$

Insulation Between Two Thermally Massive Low-Resistance Layers. This can be treated by application of Equation C-10 in respect of the exterior thermal mass, and Equation C-11 in respect of the interior thermal mass, the factors FI and FE in computing algorithm C-8 being the sum of the coefficients of δT_i and δT_e respectively.

Where the composition of the structure is uncertain the basic formula represented by Equation C-7 is recommended. An examination of the effect of Equations C-7, C-9, C-10 and C-11 in practical situations indicates that internal temperature changes usually give rise to greater correction factors than external temperature changes of the same magnitude.

TABLE 1
Typical Values of Admittance

Inner leaf of structure	Admittance Btu/hr·ft²·F	W/m²·°C
Brick or dense concrete with dense plaster	0.70 - 0.90	4.0 - 5.0
Brick or dense concrete with light plaster	0.50 - 0.70	3.0 - 4.0
Brick or dense concrete with plasterboard lining	0.35 - 0.45	2.0 - 2.5
Lightweight concrete with light plaster	0.35 - 0.45	2.0 - 2.5
Insulated internal lining	0.15 - 0.25	1.0 - 1.5

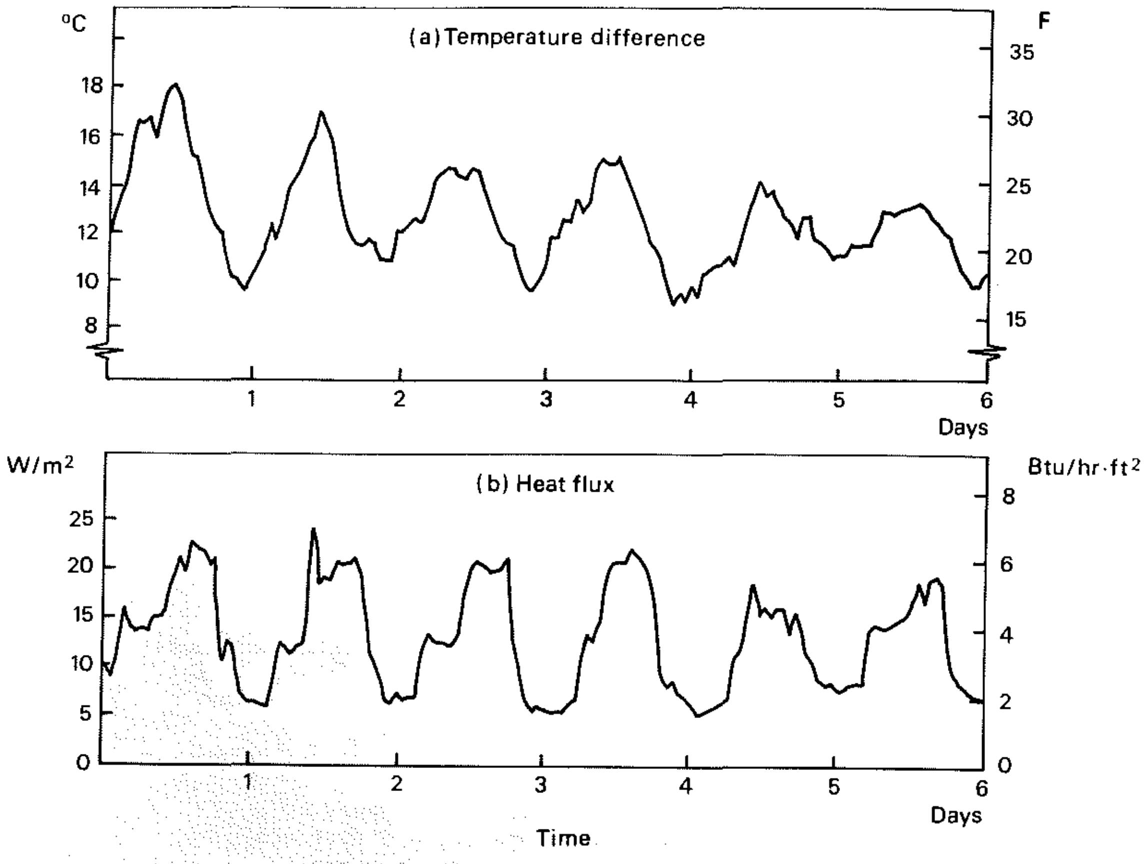

Figure 1. Test with cyclic variation in temperature difference (top) and resultant heat flux (bottom)

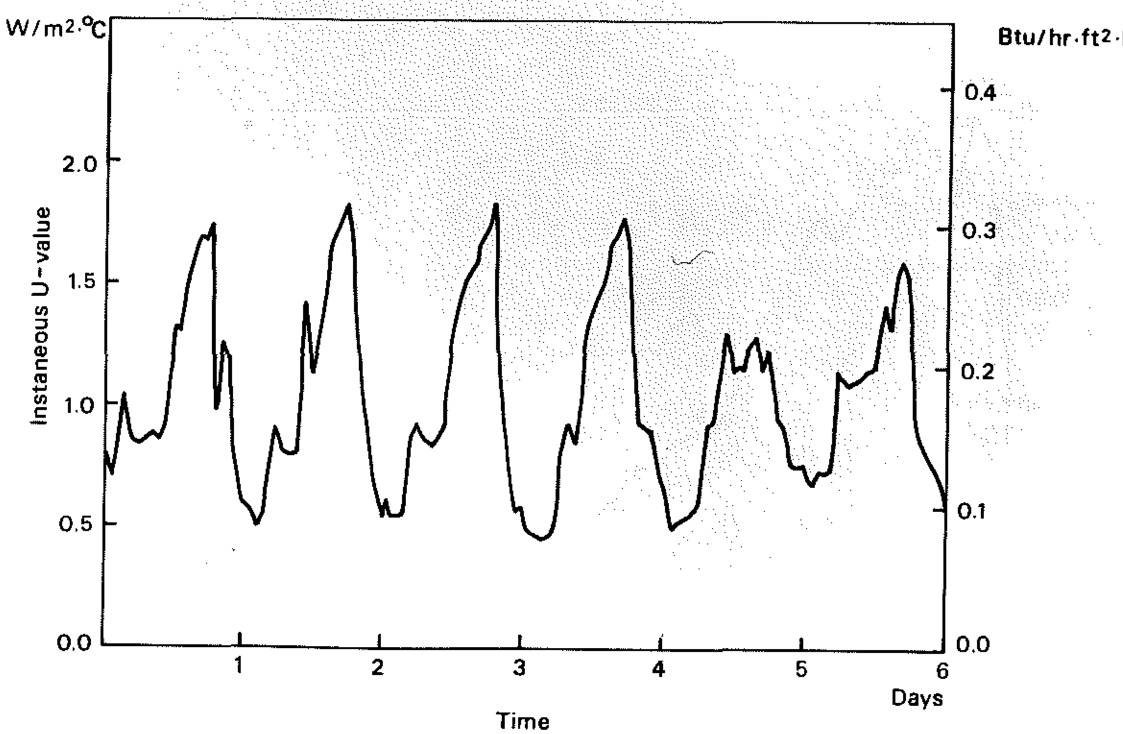

Figure 2. Variation in instantaneous U-value obtained from data in Figure 1

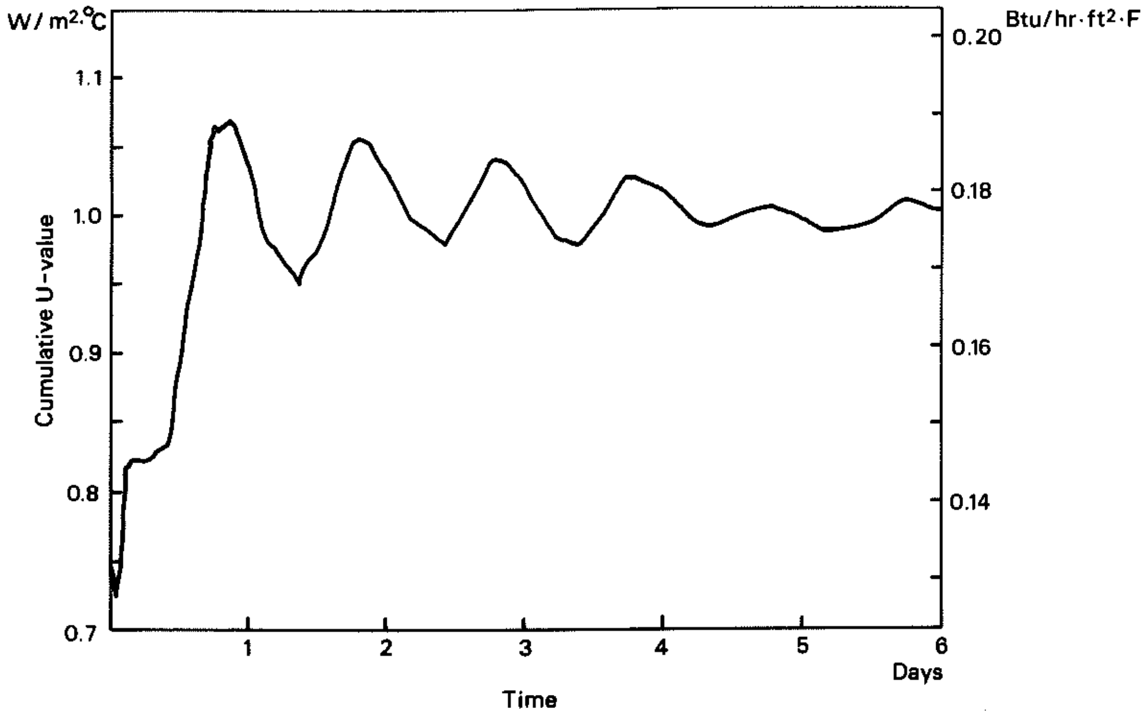

Figure 3. Cumulative heat flux divided by cumulative temperature difference plotted against time for data in Figure 1

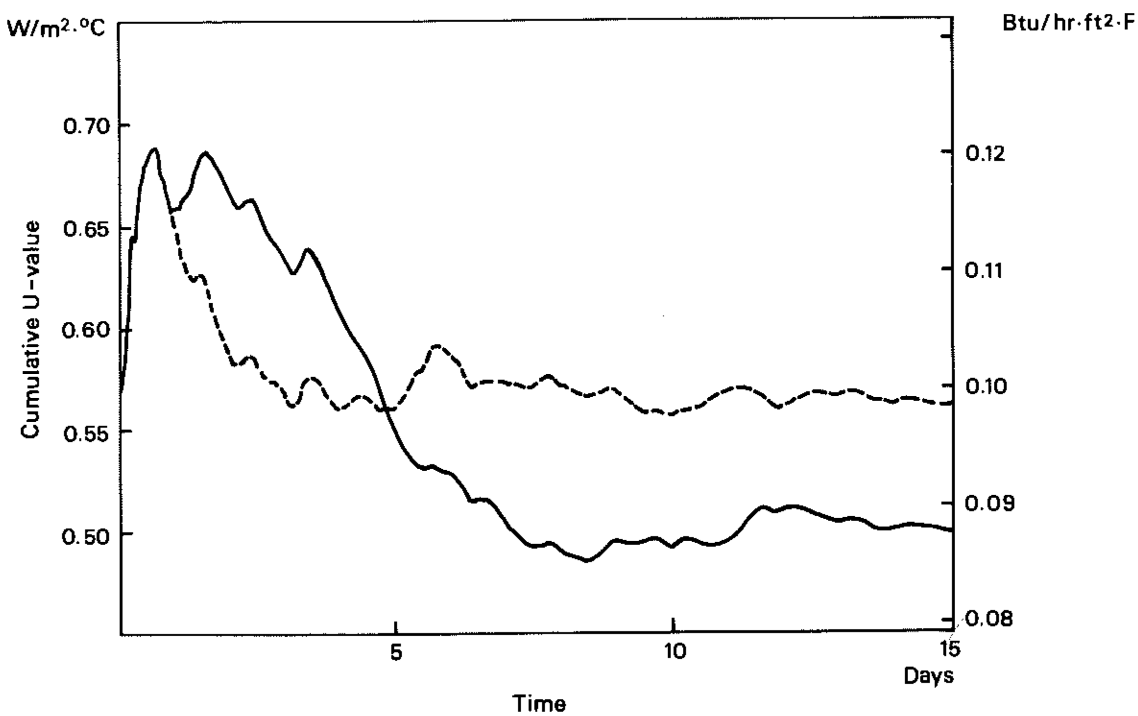

Figure 4. Cumulative U-value for brick wall with cavity fill, uncorrected (solid line) and corrected (broken line) for changes in internal and external temperature

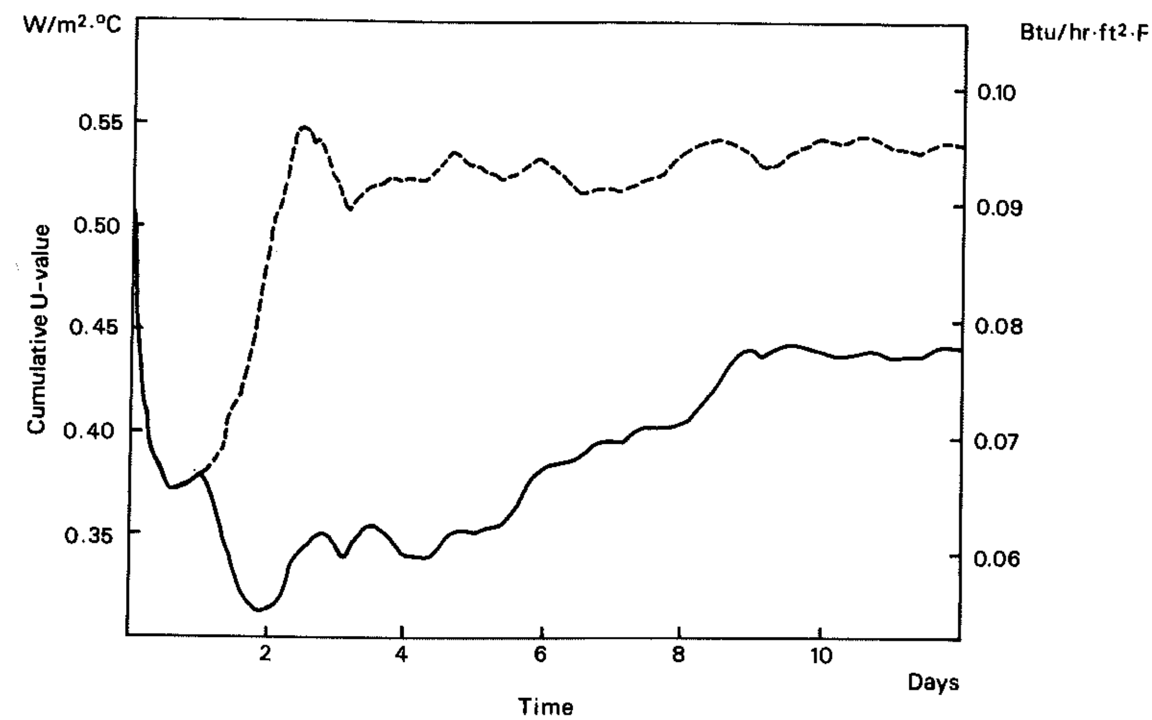

Figure 5. Cumulative U-value for the same test in Figure 4, but with integration starting at the end of day four

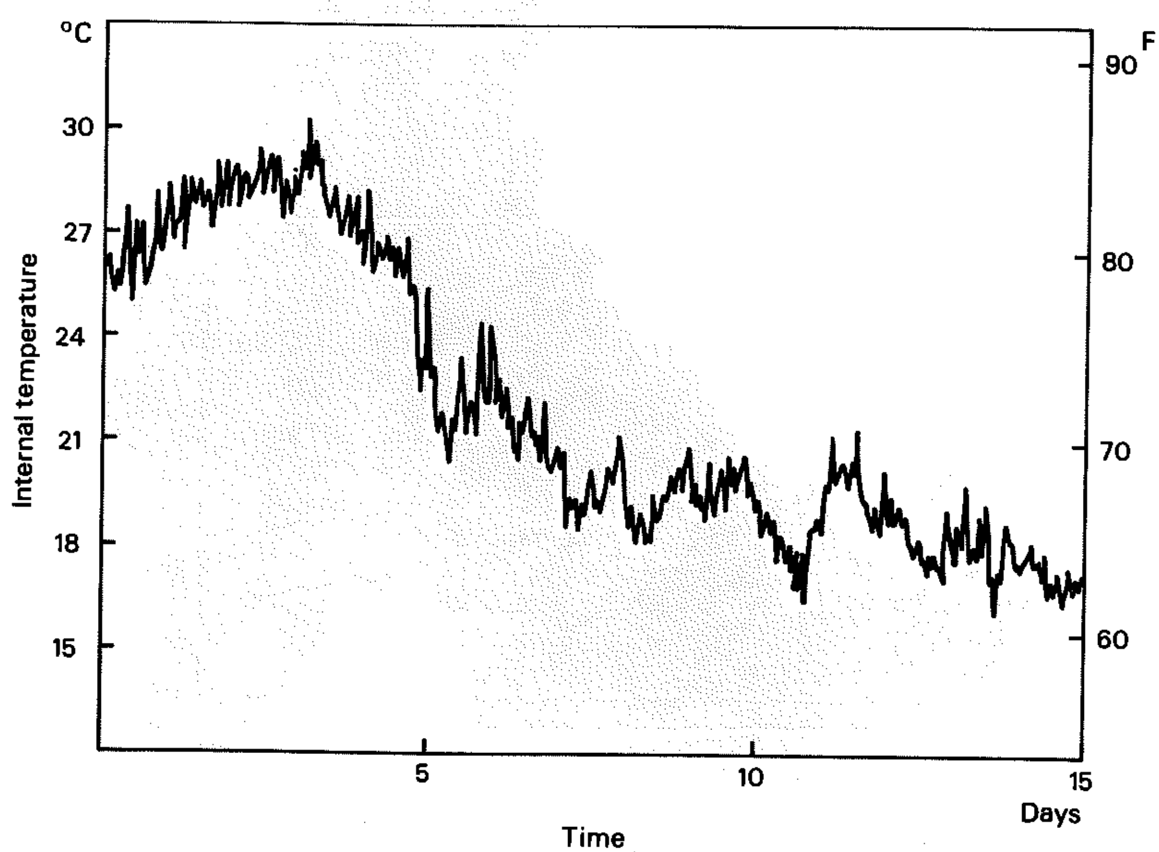

Figure 6. Variation of internal temperature during course of test on cavity-filled brick wall

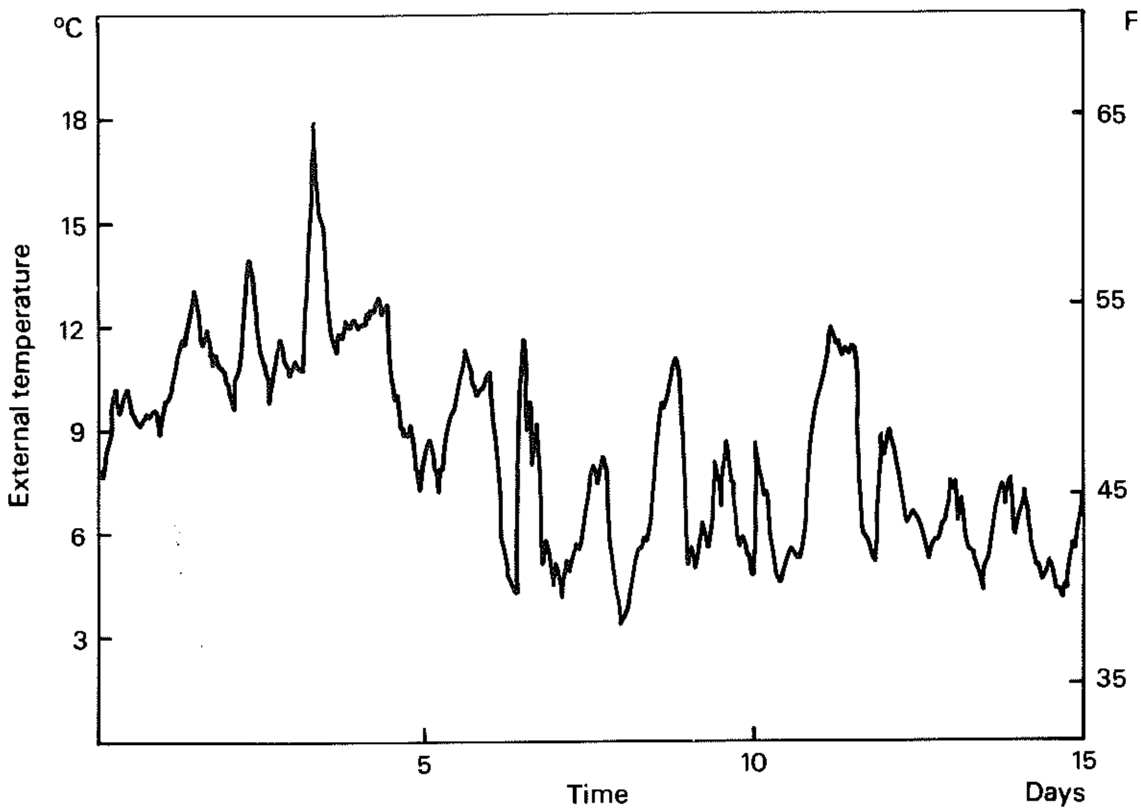

Figure 7. Variation of external temperature during course of test on cavity-filled brick wall

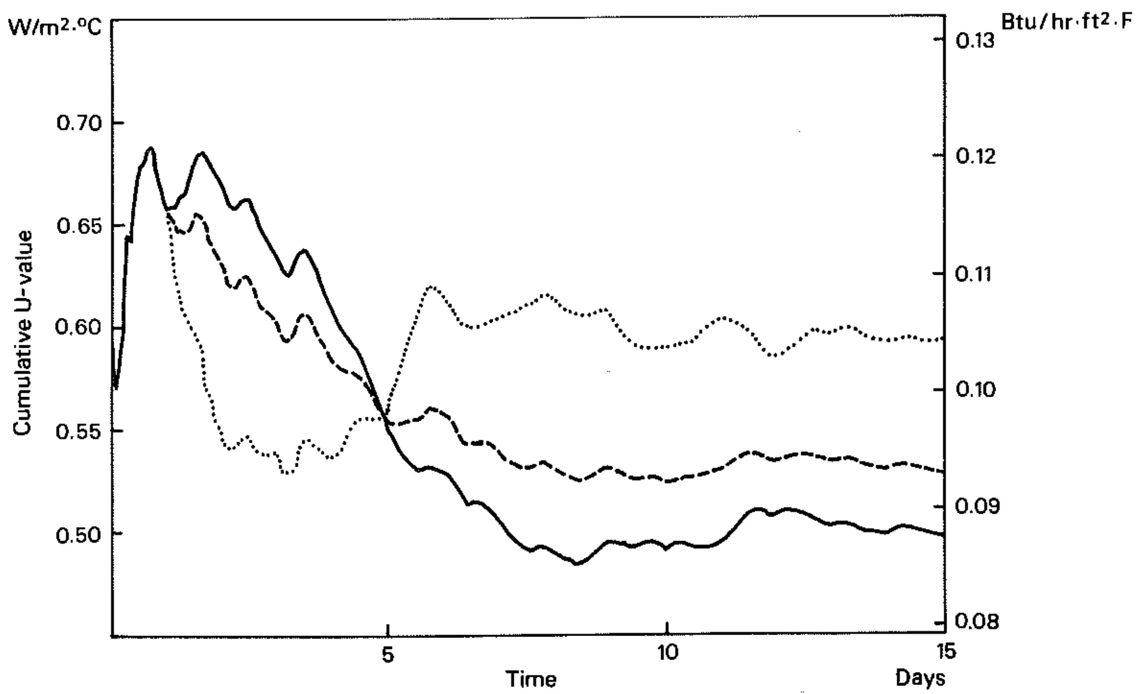

Figure 8. Cumulative U-value for data in Figure 5, uncorrected (solid line), corrected with 50% underestimate of thermal mass (dashed line), corrected with 50% overestimate of thermal mass (dotted line)

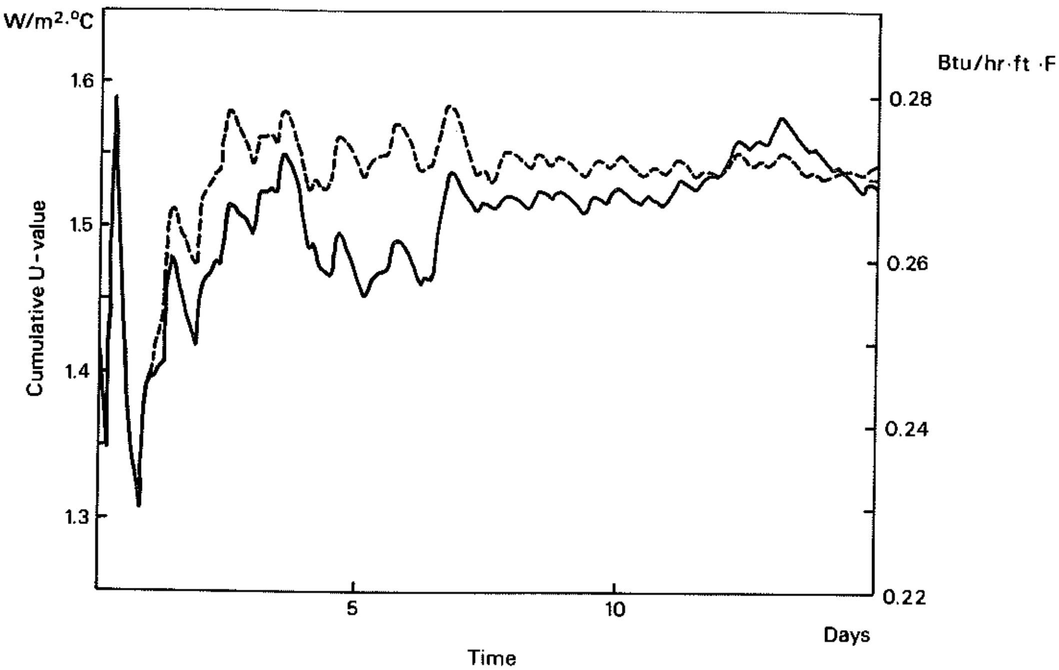

Figure 9. Cumulative U-value for unfilled cavity brick wall, uncorrected (solid line) and corrected (broken line) for changes in internal and external temperature

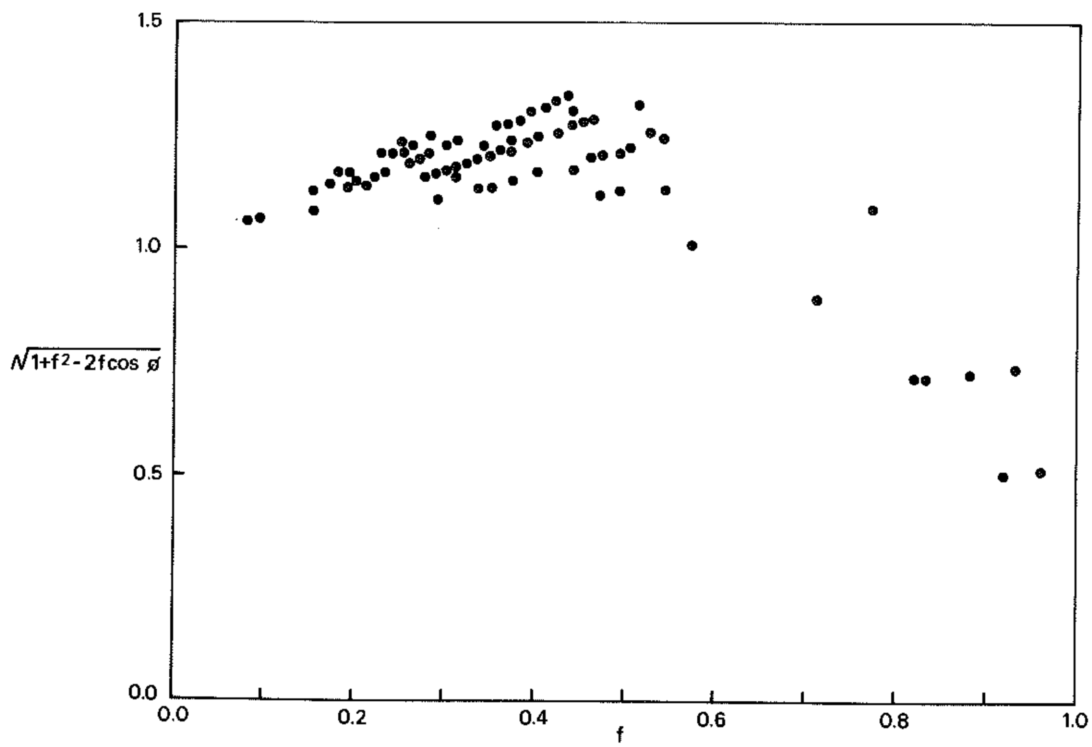

Figure A-1. Function $\sqrt{1 + f^2 - 2f \cos \omega \phi}$ over the range of decrement factor ϕ. Limits are 1.0 as $f \to 0$ and 0.0 as $f \to 1$ ($\phi \to 0$ as $f \to 1$)