THERMAL PERFORMANCE OF RIGID CELLULAR FOAM INSULATION AT SUBFREEZING TEMPERATURES

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ABSTRACT

Rigid cellular foams exhibit pronounced variations in k-value as operating temperatures change. The almost universal practice of utilizing a fixed k-value to predict the performance of insulating systems regardless of operating temperature leads to serious errors, particularly in subfreezing applications such as refrigeration buildings and equipment.

This paper presents a straightforward method for designing insulation systems, taking into account variations in k-value as operating temperatures change. A computer model is created based on temperature variation through the insulation. This model generates data that may then be used to compute the expected heat flow under any given set of operating conditions. Experimental verification of this method is provided by testing, under simulated operating conditions, extruded polystyrene foam and polyurethane foam. The test method and test data are discussed in the paper.

INTRODUCTION

Rigid cellular foam insulation products that depend on fluorocarbons as blowing agents may have particularly sharp changes of conductivity at temperatures near 32°F. Such foams are the mainstay of the design of cold storage warehouses and processing buildings, and of trucks and rail cars used to transport frozen products. Understanding the behavior of such materials in the subfreezing environment in which they are used will contribute to a more accurate design of the insulation envelope.

GENERAL COMMENTS

In cold storage applications, the cold side of the insulation is at a relatively unchanging temperature, regardless of the temperature of the warm side. The heat flux rises and falls as the temperature on the warm side changes, and this changes the temperature drop across the air film at the cold side. Practically speaking, however, these changes are negligible; the insulation next to the cold side operates at a relatively constant temperature and has the k-value characteristic of the insulation at the cold-side temperature. Conversely, the warm-side temperature varies enormously throughout the year. The insulation near the warm side may experience wide variations in k-value as consequence.

The operating temperature of the center of the insulation depends upon the type of insulation, its thickness, the heat flux density, and the temperatures of the cold and warm sides of the insulation.

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THE COMPUTER MODEL

A computer model developed by the authors computes a plot of temperature versus positions through the insulation beginning with the temperature at the cold side and ending with the temperature of the warm side. This temperature profile is developed for any insulating material whose k-value variation with temperatures is known. The user is cued to input the specific operating conditions, thickness, cold-side temperature and warm-side temperature. After computing the temperature profile, the computer model computes the overall R-value for these conditions using the conventional formula: \( R = \frac{(\text{warm-side temperature} - \text{cold-side temperature})}{\text{heat flux density}} \).

This model is based on a uniform infinite slab of insulation (i.e., no transverse heat flow) and steady-state conditions (i.e., no time variations of temperature). The insulation is assumed to be homogenous and of uniform thermal properties. Air films are not included here because they are not included in the experimental test method used to verify the model.

Figure 1 shows two temperature profiles developed by the computer utilizing k-value vs temperature data from ASHRAE, (ASHRAE, 1985) for one-inch-thick polyurethane and one-inch-thick extruded polystyrene. Temperature, on the vertical scale, ranges from a cold-side temperature of -30°F to a warm-side temperature of +50°F. Distance through the slab is on the horizontal scale. Note the heat flux density is not the same for the two cases because the overall R-values of the two materials are different.

Similar (ASHRAE data) temperature profile pairs have been generated for warm-side temperatures of 30°F, 70°F, and 90°F. A plot of the overall R-values as a function of warm-side temperature is given in Figure 2. Note that at a warm-side temperature near 50°F, both materials have the same R-value, while the effective R-values for extruded polystyrene exceed those of polyurethane at low warm-side temperatures.

THEORY

The formula for steady-state conduction in a homogenous medium is, \( q = k \times a \frac{dT}{dx} \), where

\[
q = \text{heat flow rate, Btu/h} \\
k = \text{thermal conductivity, Btu/h x F x ft} \\
a = \text{cross section area ft}^2 \\
\frac{dT}{dx} = \text{thermal gradient F/in}
\]

Consider the elemental section cut from an indefinitely large insulation slab, shown in Figure 3. The section is dx thick along lines of uniform heat flow, and has a unit area normal to this direction. Assuming that the insulation is uniform within this element,

\[ H = \frac{(k)}{dx} (T_2 - T_1) \]

where \( H = \text{heat flux density (Btu/h/ft}^2) \)

\( dx = \text{thickness} \)

\( k = \text{thermal conductivity of the element} \)

\( T_1, T_2 = \text{temperatures at the unit element surfaces} \)

In general, the value of k will be a function of location, and orientation of the element within the insulation slab as well as of the temperature of the element. If these k-values are known, an integration along the lines of heat flux will give the total temperature difference (degrees Fahrenheit across the slab and the R-value of the slab \( F/h \times Btu \)).

For this paper we consider only the effect of different temperatures on k-value. We assume an arbitrarily chosen thickness of homogenous insulating slab extending indefinitely in directions perpendicular to the heat flow. A unit area through this slab will be made up of \( N = TK/dx \) elements, where \( TK \) is the thickness of the slab and \( dx \) is the thickness of the elements. If the
temperatures of the two faces of the slab are different, heat will be conducted through the slab and the heat flux density, \( H \), will be proportional to the temperature difference.

The computer model cues for the total thickness of the slab, and for the number of elements, \( N \). The first element has as its outer side the outer side of the slab, the temperature of which is \( T_c \). The other side of the first element has as its temperature,

\[
T_l = T_c + \frac{H}{k_1} \times \frac{TK}{N}
\]

where
- \( T_c \) = cold-side temperature
- \( H \) = heat flux density
- \( k_1 \) = k-value at the temperature \( \frac{(T_o + T_l)}{2} \)
- \( TK \) = thickness of slab
- \( N \) = number of elements

Note that the thickness of each element is \( \frac{TK}{N} \).

The second element begins \( \frac{TK}{N} \) distance from the cold side of the slab and has a temperature of \( T_l \) on one side and a temperature \( T_2 \) on the other:

\[
T_2 = T_l + \frac{H}{k_2} \times \frac{TK}{N}
\]

The temperature at element \( M \) is,

\[
T_m = T_c + \frac{\frac{H}{k_1} \times \frac{TK}{N} + \frac{H}{k_2} \times \frac{TK}{N} + \ldots}{1}
\]

where
- \( k_1 \) is the k-value of the first element,
- \( k_2 \) is the k-value of the second element,
- \( k_m \) is the k-value of the \( M \)th element,
- \( T_m \) is the temperature of the \( M \)th element.

Extracting the common factors and rearranging,

\[
T_m - T_c = \frac{TK}{N} \times \frac{H}{1} \times \left( \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_m} \right)
\]

Note that, as in conventional practice, the temperature difference across the insulation depends on the thickness, the heat flux density, and the k-value of the material. Note, also that this equation is the conventional way of expressing the overall k-value of a series of pieces of insulation having different k-values. The boundary conditions require that \( T_N = T_w \) (warm-side temperature) and \( T_o = T_c \) (cold-side temperature). The computer cues for these values and develops a first temperature profile using \( H = 0.115 (T_w - T_c) \) (an arbitrary starting value).

If \( T_N \) does not equal the temperature \( (T_w) \) of the warm-side of the slab after the computer has developed its first temperature profile, what then? Since \( TK \) and \( N \) are fixed, the only way to make \( T_N = T_w \) is to change the value of \( H \). We do so by multiplying the first chosen value of \( H \) by the ratio of \( T_w - T_c \) divided by \( T_N - T_c \) and repeating the computation of \( N \) steps. Note that this process may change the value of each k. By imposing a requirement that \( T_N \) lie between \( T_w +/- \) any desired accuracy limit, the computer must work until it gets the right \( T_N \). After \( T_N \) is within prescribed limits the program prints the temperature profile, heat flux, and R-value.

In principle, this computer model will develop with precision the temperature profile through an insulating slab of any chosen thickness. However, this precision is limited by the accuracy with which the relationship between k-value and operating temperature is known. In practice, k-values are measured on relatively thick slabs \((0.5 - 2 \text{ in})\) and the temperature across the slab is usually 50F. Published values for k versus temperature tend to smooth out any
discontinuities in this relationship. Furthermore, most suppliers do not provide test data at temperatures other than 75°F or 40°F. The most comprehensive data is available in the ASHRAE Handbook - 1985 Fundamentals, volume 3, page 23.17. This gives k-values for various types of foam from -50°F to 100°F. The identical data appears in the 1981 edition, so we are persuaded that little reliable data exists for generating our model. We have, however, used the ASHRAE data in generating Figure 1 and Figure 2 curves. In Figure 2, data supplied by manufacturers was used to update the ASHRAE data in our computer formula, resulting in the higher R-values modeled for current materials.

The computer program used here employs a formula to relate k-value to operating temperature (a slight modification would enable use of a look-up table instead). Figures 1 and 2 were computed using curves fitted to data from the ASHRAE Handbook - 1985 Fundamentals, page 23.17. The formulas and data are given in Figure 4.

U-VALUES

The computer model was extended to include the effects of air films on the warm and cold sides. The same language was used, assuming a windless inside surface (R=0.61) and a windy outside surface (R=0.167) per ASHRAE Handbook - 1985 Fundamentals, page 32.14. This extended program (not shown) can be used to provide U-values for an insulated wall under expected operating conditions.

The authors believe that this model is equivalent to that described in ASTM 680-82. A further refinement, to include the wind and emissivity relationships of ASTM 680 can be accomplished.

EXPERIMENTAL VERIFICATION

Tests were run on samples of extruded polystyrene and modified polyisocyanurate material. The test samples were cut from two-inch-thick slabs to the one-inch test thickness. The samples were then tested using a guarded hot plate. The configuration of the extruded polystyrene sample is:

- Dimensions: 12 in x 12 in
- Thickness: 1.03 in
- Density: 1.35 lb/ft³
- Blowing Agent: F12
- Aged over 90 days at room temperature.

The configuration of the polyisocyanurate sample is:

- Dimensions: 12 in x 12 in
- Thickness: 1.0 in
- Density: 1.74 lb/ft³
- Blowing Agent: F11
- Aged over 90 days at room temperature.

Each sample was tested at four operating conditions. The cold side was maintained at approximately -30°F for all tests. Warm-side temperatures were approximately +25°F, +65°F, +75°F and +100°F. Thus, the operating temperature differences were 55°F, 95°F, 105°F and 130°F, fairly typical of seasonal variations throughout the year across the USA. The results are summarized in Table 2.

CONCLUSION

The extruded polystyrene material test results agreed remarkably well with the
computer model which used manufacturers-supplied data. The R-values were about 12% higher than the model using ASHRAE data predicted. This is no doubt due to the small cell size of newer extruded foam.

The computer model used to generate the R-value per inch curves of Figure 2 for ISO, employed a formula for k-value temperature dependence similar to the formula used to develop the ASHRAE curve, but modified to match the 75F data supplied by the manufacturer. It is clear that test data points do not agree with the model at low temperatures. More up to date data on low temperature performance of polyisocyanurate foam is clearly needed to explain these differences.

REFERENCES


BIBLIOGRAPHY


APPENDIX 1

Computer program to generate temperature profile through an insulating slab where k-value is temperature dependent.

LIST

10 DIM T (200)
20 PRINT"PROGRAM NAME EXPHEAT"
30 PRINT"PANEL THICKNESS INCHES"
40 INPUT TK
45 REM: number of steps is limited by DIM statement to 200
46 REM: Little change in results occur for N greater than 10
50 PRINT"NUMBER OF STEPS"
60 INPUT N
70 PRINT"COLD SIDE TEMPERATURE, DEGREES FAHR.";
80 INPUT TC
90 PRINT"WARM SIDE TEMPERATURE";
100 INPUT TW
105 REM: H=k times temperature difference
106 REM: .115 is arbitrary choice of k to begin program
110 H=.155*(TW-TC)
120 FOR A = 1 TO N
125 REM: start at cold face of slab
130 T(0) = TC
135 REM: compute k-value using formula k=(.15 + .0004(50-T))
136 REM: compute temperature at Ath step
140 T(A) = T(A-1) + H*(TK/N)*1/(.15 + 4.000001E-04*(50 + (T(A-1) + T(A))/2))
145 NEXT A
150 REM: revise value of H if TN differs from Tw
155 H = H*(TW-TC)/(T(N)-TC)
160 REM: repeat loop if TN is out of chosen limits
165 REM: use + .001 as limits
170 IF T(N)<.999*TW or T(N)>1.001*TW THEN 120 ELSE 180
180 PRINT"DISTANCE", "TEMP"
190 FOR Y = 0 TO N
200 PRINT USING"###.###"; Y; TK/N;
205 PRINT TAB(13) USING "###.###"; T(Y)
210 NEXT Y
220 PRINT"PANEL R VALUE"; (TW-TC)/H
230 PRINT"HEAT FLUX DENSITY, BTU/Hr/SqFt"; H
240 REM: run a sample program using 10 steps
RUN
PROGRAM NAME EXPHEAT
PANEL THICKNESS INCHES? 2
NUMBER OF STEP? 10
COLD SIDE TEMPERATURE, DEGREES FAHR.? 30
WARM SIDE TEMPERATURE? 50

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<th>DISTANCE</th>
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PANEL R VALUE 11.49329
HEAT FLUX DENSITY, BTU/Hr/SqFt 6.960581
TABLE 1
Data for Figure 1

Panel thickness inches? 1
Number of Steps? 20
Cold side temperature, degrees Fahr.? -30
Warm side temperature? 50

Temperature profiles, ASHRAE data

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ASHRAE data

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<td>Extruded</td>
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TABLE 2
THE APPARENT THERMAL CONDUCTIVITY AND THERMAL RESISTANCE
OF THREE SPECIMENS OF PLASTIC FOAM INSULATION MATERIALS

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<th>Specimen</th>
<th>Test Thickness</th>
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<th>Thermal Resist.</th>
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<td>kg m⁻³ lbs ft⁻³</td>
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<td>Wm⁻¹ K⁻¹ Btu in hr⁻¹ ft⁻² F⁻¹</td>
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Reference: UCI-17
October, 1985
Figure 1. Temperature profiles: computed temperature vs. position for polystyrene and polyurethane foams

Figure 2. R-value vs. warm-side temperature, computed for cold-side temperature = -30 F
\[
q = k \frac{dH}{dx}
\]
\[
H = \frac{q}{a}
\]
\[
dt = \left(\frac{H}{k}\right) dx
\]
\[
T(a) = T(a-1) + \left(\frac{H}{k}\right) \left(\frac{Tk}{N}\right)
\]

Where \(k\) is evaluated at:
\[
T(a) + T(a-1) / 2
\]

**Figure 3.** Elemental cell

---

**Figure 4.** K-value vs. temperature. ASHRAE data points: Δ extruded polystyrene, ○ polyurethane

\[
K = 0.159 + 6.3 \times 10^{-5} (T-62.5)^2 \quad \text{for } T>0
\]
\[
K = 0.183 \quad \text{for } T<0
\]