A METHOD FOR CHARACTERIZING THE THERMAL PERFORMANCE OF A SOLAR STORAGE WALL FROM MEASURED DATA

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ABSTRACT

A technique is presented for characterizing the dynamic performance of a thermal-storage wall based on the data obtained from a series of temperature and heat flux measurements. It is shown that the coefficients of a transfer function model can be estimated directly from data using linear least squares regression. Data from the National Bureau of Standards (NBS) Passive Solar Component Calorimeter are used to demonstrate that the technique can be successfully applied. The transfer function techniques are potentially applicable to linear systems with time-invariant properties. It is also shown that a very simple set of parameters can be derived from the transfer function coefficients to characterize the steady-state performance. Only one parameter for each system input is required to predict long-term average thermal performance of the component.

INTRODUCTION

There is considerable interest in measuring the dynamic thermal performance of entire buildings and their components such as walls, roofs, floors, and apertures. In situ measurements have been made in residential and commercial buildings and measurements on isolated components have been made in test cells, hot boxes, and calorimeters. These data are useful in such varied activities as studies of heat transfer mechanisms, validation of mathematical models, and characterizing the performance of as-built components. Several investigators have suggested methods by which useful building thermal parameters might be estimated from data. Fang and Grot (1985) derived thermal resistance values of building envelopes from measured data. Pederson and Mouen (1973) attempted to fit a response factor model to measured data as part of an ASHRAE research project. They were unsuccessful at fitting a response factor model directly to data but found that they could estimate material thermophysical properties from the data. Sherman et al. (1982) began their analysis with the assumption that most existing models contain too many parameter to be suitable for direct analysis and then proceeded to develop a reduced set of parameters from which response factors could be derived. Time domain transfer functions can provide an efficient technique for characterizing many types of building envelope components with coefficients estimated directly from measured thermal performance data. Transfer functions have been used for a number of years as an efficient method of simulating building heat transfer. The computer program BLAST (Hittle, 1979), DOE (Lawrence Berkeley Laboratory, 1980), and TRNSYS (Klein et al. 1983) all use transfer functions in the calculation of dynamic heating an cooling loads. However, the use of transfer functions to characterize measured thermal performance is novel.

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The purpose of this paper is to present the theoretical development and application of a method for estimating transfer function coefficients from measured data. The method is applied to a data set for a double-glazed, concrete-block, thermal storage wall, which was tested in the NBS Passive Solar Component Calorimeter. The results of this analysis show that the transfer function model predictions are in good agreement with measured data. This paper also addresses the problem of prediction of long-term average performance as opposed to hourly dynamic performance.

TRANSFER FUNCTION BACKGROUND

A transfer function relates the outputs of a system to its inputs. The conduction transfer function has been used in building energy calculations to relate the present heat flux at the interior surface of a wall to a time series of present and past inside and outside temperatures and past heat fluxes. The conduction transfer function recommended by ASHRAE (1981) for use in cooling load calculations when the indoor air temperature is constant is given as

$$q_t = \sum_{i=0} b_i' T_{eq,t-i\delta} - \sum_{i=1} d_i' q_{t-i\delta} - T_{air} \sum_{i=0} c_i' \qquad (1)$$

where

q_t = heat flux at the inside surface at time t, Btu/hr-ft^2 (W/m^2)

T_{eq} = sol-air temperature at the outside surface, °F(°C)

T_{air} = inside air temperature, constant, °F(°C)

b', c', d' = transfer function coefficients for the system,

t = time

δ = time step interval

i = summation index.

The use of conduction transfer functions in building energy analysis was introduced by Stephenson and Mitalas (1971) to provide an efficient method for long-term simulation. Mitalas (1978) discusses the development and properties of the Z-transfer function. Several methods are available for determining transfer function coefficients. The ASHRAE Handbook-Fundamentals (ASHRAE, 1985, 1977) lists transfer function coefficients for 179 different construction types. These were computed using the Mitalas and Arsenault program (1972), which requires inputs of basic material properties and solves a system of partial differential equations. This method is presently implemented as part of the TRNSYS program. Hittle and Bishop (1983) developed an improved root-finding technique. Pawelski et al. (1979) and Ceylan and Myers (1980) have developed alternative techniques for determining transfer function coefficients. Pawelski's method requires construction of a finite difference model and execution of a simulation using this model. The coefficients are determined by linear regression on the simulated results. Ceylan shows that it is possible to compute transfer function coefficients directly from the finite difference formulation.

Transfer functions have typically been used to model walls, roofs, and floors for which the predominant heat transfer mechanism is conduction. In principle, transfer functions can model any linear system with time invariant properties (Stephenson and Mitalas, 1971). The heat transfer system of interest in this paper is a nonvented, concrete masonry thermal-storage wall adjoining a constant temperature room. Based on the known inputs of solar irradiance and ambient temperature, a proposed transfer function model for the thermal-storage wall is

$$q_t = \sum_{i=0}^{N} a_i I_{t-i\delta} + \sum_{i=0}^{N} b_i T_{t-i\delta} - c_o T_{air} - \sum_{i=0}^{N} d_i q_{t-i\delta} \qquad (2)$$

where

q_t = heat flux at the inside surface of the thermal-storage wall at time, t, Btu/hr-ft^2 (W/m^2)

I = vertical solar irradiance, Btu/hr-ft^2 (W/m^2)

T = ambient dry-bulb temperature, °F (°C)

T_{air} = indoor air dry-bulb temperature, constant, °F (°C)

a,b,c,d = transfer function coefficients to be determined for the system.

Transfer functions appear to be a useful and efficient method for characterizing data. No direct knowledge of material thermophysical properties or heat transfer mechanisms is required. The effects of the actual heat transfer mechanisms are implicitly modeled with a best fit linear approximation. The adequacy of the assumptions of linearlity and time invariance are determined by comparison of measured test results with those predicted by the model.

THERMAL-STORAGE WALL EXPERIMENTS

The National Bureau of Standards Passive Solar Component Calorimeter, shown in Figure 1, was used to test a concrete block thermal-storage wall in order to provide data for the transfer function analysis. The facility is particularly well suited for testing a thermal-storage wall (as opposed to whole buildings or test cells) because an accurate measurement of heat flux through a specific building component can be made. This measurement was accomplished by placing a heat-metering chamber behind a component installed in the south wall of the Passive Solar Test Building located at the NBS Annex. The outdoor side of the test article is exposed to the natural environment, and the inside thermal conditions are carefully controlled. The air temperature inside the metering chamber is held constant by adding thermal energy with an electric heater coil and withdrawing energy by means of a chilled water coil. The temperatures of inside surfaces of the calorimeter with which the test article exchanges radiant energy can be similarly controlled. Heat transfer from the test wall can be determined by measuring the power to the fan, power to the heaters, energy transferred from the cooling coils, and heat loss from the metering chamber, which is measured during a separate calibration test. The average inside air temperature was measured with a six juction pair thermopile. Total solar irradiance on the vertical south-facing surface was measured with a precision pyranometer. The ambient air temperature was measured at two locations near the aperture with type T thermocouple inside ventilated white wooden boxes. McCabe et al. (1982) describe details of the calorimeter facility design and operation.

The thermal-storage wall tested was built of low density 100 lb/ft^3 (1600 kg/m^3) solid concrete blocks. The blocks were stacked without mortar and the joints between blocks were sealed with silicone adhesive. Nominal dimensions of the wall are 8 in. (0.2 m) thick by 48 in. (1.2 m) wide by 80 in. (2 m) high. No thermocirculation vents were included. A selective surface foil (α = .92, ε = .12) was applied to the front surface of each block. A sealed double-glazing unit 80 in. (2 m) x 48 in. (1.2 m) with a 1/2 in. (13 mm) air gap covered the aperture. The glass was 1/8 in. (3.2 mm) thick. The distance between the front surface of the wall and the inside surface of the glazing was 1.5 in. (38 mm). The test started on April 1 and lasted until April 10, 1984. A varied range of weather conditions was experienced. The solar irradiance and the ambient air temperature are shown in Figures 2 (a) and (b), respectively.

ESTIMATING TRANSFER FUNCTION COEFFICIENTS FROM DATA

The analysis involves fitting an appropriate transfer function model to a subset of the experimental data. Linear least squares regression is applied to the data to estimate the transfer function coefficients. A primary objective is to determine the number of time steps required to adequately model the system. Additional terms for previous time steps are included in the transfer function equation until the result of an F statistic test (Horton, 1978) indicates that the addition of the last term was insignificant. The actual steps taken to arrive at the proper model are described as follows.

Select a Linear, Time-Invariant Model

The selection of a linear and time-invariant transfer function equation is based upon an analysis of the heat transfer mechanism for the component under consideration. An appropriate form for the equation is frequently apparent after considering which variables are driving functions for the system. In the case of the thermal-storage wall tested in the calorimeter, it is clear that solar irradiance, outside air temperature, and inside air temperature constitute a set of inputs for the system and the appropriate form is given by Equation 2.

Select a Time Step, δ

The time step for the transfer function model depends upon the detail desired in simulating the dynamics of the system and upon the material properties for the component. Transfer function equations with 1/4-, 1-, 2-, and 4-hour time intervals are developed for the thermal-storage wall data to investigate the effects of the time step interval on accuracy of the simulation results. For this study, the measured data were averaged over the period of the time step.

Set Up Time Series Data

A set of time series equations must be generated from the measured data as input for the regression analysis. Begin with one past time step and add one more at each iteration until the F-Test (step 5) indicates that the addition of the last time step did not significantly improve the fit. To illustrate the method for a problem with two past time steps and a 1-hour time interval ($N = 2$, $\delta = 1$), Equation 2 is expanded to yield

$$q_2 = a_0 I_2 + a_1 I_1 + a_2 I_0 + b_0 T_2 + b_1 T_1 + b_2 T_0 - c_0 T_{air} - d_1 q_1 - d_2 q_0 \tag{3}$$

$$q_3 = a_0 I_3 + a_1 I_2 + a_2 I_1 + b_0 T_3 + b_1 T_2 + b_2 T_1 - c_0 T_{air} - d_1 q_2 - d_2 q_1 \tag{4}$$

$$q_4 = a_0 I_4 + a_1 I_3 + a_2 I_2 + b_0 T_4 + b_1 T_3 + b_2 T_2 - c_0 T_{air} - d_1 q_3 - d_2 q_2 \tag{5}$$

$$\vdots$$

$$q_m = a_0 I_m + a_1 I_{m-1} + a_2 I_{m-2} + b_0 T_m + b_1 T_{m-1} + b_2 T_{m-2} - c_0 T_{air} - d_1 q_{m-1} - d_2 q_{m-2} \tag{6}$$

The coefficients a_0, a_1, a_2, b_0, b_1, b_2, c_0, d_1, and d_2 will be estimated in the regression analysis. Note that m must be greater than (preferrably much greater than) the total number of coefficients being estimated.

Perform the Regression

Regress the present output on the present and past inputs and past outputs using the method of linear least squares. A package routine is available (Ryan et al. 1976).

Test the Significance of the Model Order with the F Statistic

Compare the predictions of the transfer function model with N-1 past time steps to those of the model with N past time steps. Use the coefficients determined in the preceding step to predict the value of q_t at each time step from Equation 2. Calculate the sum of the squares of the residuals between measured and predicted output:

$$SSR_N = \sum_{i=N}^{m} (q_{meas,i} - q_{pred,i})^2. \tag{7}$$

Next calculate the F statistic (Horton, 1978)

$$F = \frac{\dfrac{SSR_N - SSR_{N-1}}{NC_N - NC_{N-1}}}{\dfrac{SSR_N}{m-N}} \tag{8}$$

where NC_N is the number of coefficients in the model with N past time steps and NC_{N-1} is the number of coefficients in the model with N-1 past time steps.

Consult a table of the F distribution to determine if the reduction in the residual sum of squares is significant. If the reduction was significant, add one additional time step to the model and repeat the parameter estimation process starting at step 3. If the reduction in residuals was not significant, then use the model with N-1 time steps.

Check the Model

Plot the measured and predicted output to determine whether the dynamics of the system are correctly modeled by the assumed transfer function equation.

RESULTS

The preceding step-by-step procedure was applied to the thermal-storage wall data. Models were developed for four different time intervals: =1/4, 1, 2, and 4 hours. The analytical procedure was started with one past time step (N=1) in Equation 2 and one additional time step was added at each iteration. The decision of how many previous time steps to include in the best fit model can be made from Table 1. This table lists the computed F-value and the critical F-value for each increase in the order of the model. If $F_{critical}$ is greater than $F_{calculated}$ then the model order is too large. Increasing the order of the model beyond this point does not significantly improve the fit of measured and predicted outputs. The models selected by the F test are 12 pasttime steps for a 1/4-hour time interval, 3 pasttime steps for 1-# and 2-hour time intervals, and 2 pasttime steps for a 4-hour time interval. Table 2 shows the residual mean square for each time interval and model order. In general, the residual mean square is initially reduced by the addition of more time steps until a minimum is reached.

Table 3 presents a listing of the a, b, c, and d coefficients of the selected models for 1-, 2-, and 4-hour time intervals. The total number of coefficients required for each model are 12, 12, and 9, respectively. The 1/4-hour time interval model required a total of 39 coefficients and these results were not tabulated.

Plots of the measured and predicted heat transfer rate are made to determine whether the transfer function correctly models the dynamics of the system. Figures 3a, b, and c show the plots for 1-, 2-, and 4-hour time interval models, respectively, for nine days of data. Figure 4 is a plot of the difference between the measured and predicted heat flux versus time for 1-hour time intervals. These differences are not found to be random and independent but are positively correlated. The positive correlation is inherent in the transfer function model since predicted heat fluxes from past time intervals are used to predict the present heat flux. Thus, once an error arises, it tends to be propagated to future predictions. Lack of randomness and independence violates an assumption of the F statistic test. The consequence of violating the assumptions of the F test are discussed at some length by Horton (1978). For the types of errors found in this analysis, the violation of assumptions tends to make the F test more conservative than the nominal significance level. That is, the number of past time steps included in the final model may be too large.

The effect of using time intervals other than 1-hour can be observed in Figures 5a, b, and c. Figure 5a shows a plot of 1-hour and 1/4-hour predictions versus time. These two models predict almost identical heat fluxes, which implies that there is no advantage in using time intervals shorter than one hour for this component. Figure 5b show 1- and 2-hour predictions versus time. Only very small differences are found between these predictions implying that a model with 2-hour time intervals is capable of modeling the full dynamics of the thermal-storage wall. Figure 5c shows that if 4-hour predictions are compared to 1-hour predictions, the larger time interval model misses some of the detailed behavior of the system.

STEADY-STATE PARAMETERS FROM TRANSFER FUNCTION COEFFICIENTS

The transfer function equation can be used to model the dynamics of a thermal-storage wall to a satisfactory level of detail. It is frequently desirable, however, to have a simple method of predicting the average or steady-state system response. From Equation 1, it can be shown that the steady-state conductance, U, of a wall can be expressed as a ratio of the summation of transfer function coefficients:

$$U = \frac{\sum_{i=0}^{N} b_i}{\sum_{i=0}^{N} d_i} = \frac{\sum_{i=0}^{N} c_i}{\sum_{i=0}^{N} d_i} . \tag{9}$$

Following a similar development, for Equation 2, the following ratios of summations of transfer function coefficients can be formed for a thermal-storage wall:

$$R_a = \frac{\sum_{i=0}^{N} a_i}{\sum_{i=0}^{N} d_i} \qquad R_b = \frac{\sum_{i=0}^{N} b_i}{\sum_{i=0}^{N} d_i} \qquad R_c = \frac{\sum_{i=0}^{N} c_i}{\sum_{i=0}^{N} d_i} \tag{10}$$

The average heat flux through the thermal-storage wall is

$$\overline{q} = R_a\overline{I} + R_b\overline{T} - R_c\overline{T_{air}} \tag{11}$$

where the barred quantities are long-term averages.

Each of these ratios has a physical interpretation. R_a is the fraction of solar irradiance that is transferred as heat at the inside surface. R_b and R_c have units of $W/m^2 °C$ and are the equivalent conductance of the component. Theoretically, R_b and R_c should be equal. These ratios provide a very simple technique for characterizing test results and may be useful as a standard method for rating either passive solar components or conventional building walls.

The ratios were computed for the 1-, 2-, and 4-hour transfer function coefficients of Table 3 and also for a set of 1/4-hour coefficients. These results are shown in Table 4. The values of R_a, R_b, and R_c appear to be independent of the set of coefficients from which they were derived. Note that there is a small difference between the values of R_b and R_c. This difference may be due to measurement error or due to nonlinear effects that are not modeled.

CONCLUSION

A method for characterizing the measured thermal performance of a thermal-storage wall by use of transfer functions has been demonstrated. The method has been shown to be quite successful for modeling the dynamic performance of a particular thermal-storage wall. An investigaton of the effect of time interval on simulation results has shown that time steps of up to fur hours can adequately model this component. A step-by-step procedure has been presented as a guide for obtaining appropriate transfer function models from data for other types of building components. In addition, a simplified method for determining the long-term average response of the system has been demonstrated. The method presented for characterizing the dynamic and steady-state performance of a thermal-storage wall may be especially important in the development of standard methods for testing and rating of both passive solar components and conventional building walls.

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TABLE 1
Calculated Values of the F Statistic for the Transfer Function Prediction (Equation 5)*

Model Order Comparison	$\delta=1/4$ $F_{crit}=2.62$	$\delta=1$ $F_{crit}=2.66$	$\delta=2$ $F_{crit}=2.74$	$\delta=4$ $F_{crit}=2.88$
1-2	49.1	774.	853.	17.0
2-3	64.3	200.	3.9	1.1
3-4	132.2	-3.6	-0.1	
4-5	209.1			
5-6	176.2			
6-7	159.7			
7-8	180.5			
8-9	92.8			
9-10	96.9			
10-11	91.5			
11-12	12.3			
12-13	-11.9			

*F_{crit} based on 95% confidence interval (Ryan, et al, 1976).

TABLE 2
Mean Square of the Error for the Transfer Function Prediction (Equation 4)

N	$\delta=1/4$	$\delta=1$	$\delta=2$	$\delta=4$
1	1409.6	557.2	173.6	12.5
2	1132.1	33.39	4.8	5.2
3	856.5	6.56	4.2	4.9
4	514.4	7.14	4.3	4.2
5	250.4	6.33		
6	132.5	5.89		
7	73.42	5.55		
8	38.35			
9	26.08			
10	17.48			
11	11.93			
12	11.25			
13	12.99			
14	18.60			

TABLE 3
Transfer Coefficients For Selected Models

Transfer Function Coefficients	$\delta=1$	$\delta=2$	$\delta=4$
a_0	-0.001613	-0.005623	0.005817
a_1	0.002450	0.02738	0.06217
a_2	0.01044	0.01565	0.01321
a_3	0.008898	0.00947	
b_0	-0.04877	0.01763	-0.09242
b_1	0.1867	-0.3522	0.1404
b_2	-0.3630	0.2715	0.1403
b_3	0.3224	0.1305	
c_0	0.1056	0.2432	0.4251
d_1	0.6981	-0.7366	-0.6457
d_2	-0.4793	-0.05230	-0.04810
d_3	0.2538	-0.03172	

TABLE 4
Ratios of Summations of Transfer Function Coefficients

Time Step	Model Order	R_a Dimensionless	R_b $W/m^2 {}^\circ C$	R_c $W/m^2 {}^\circ C$
1/4	12	0.265	1.29	1.40
1	3	0.264	1.27	1.38
2	3	0.261	1.26	1.36
4	2	0.265	1.28	1.39

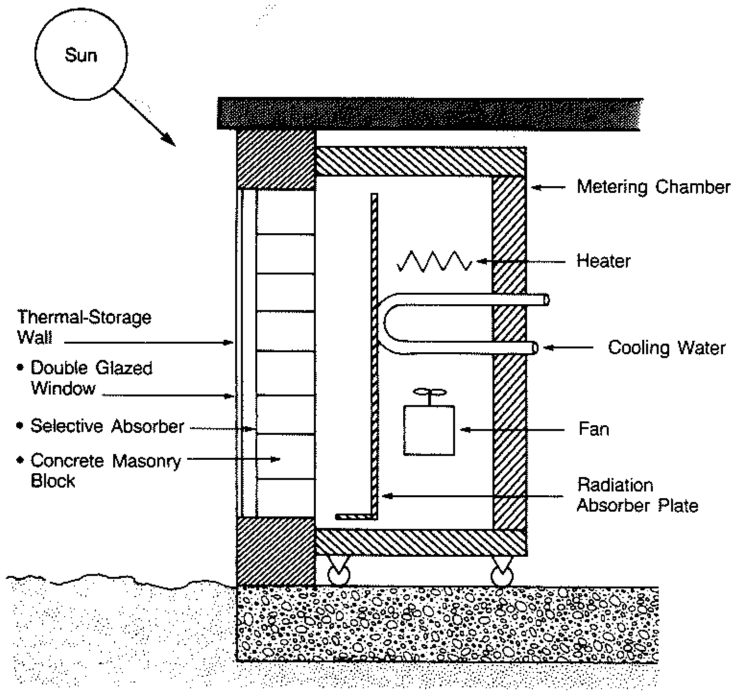

Figure 1. Schematic drawing of NBS passive solar calorimeter

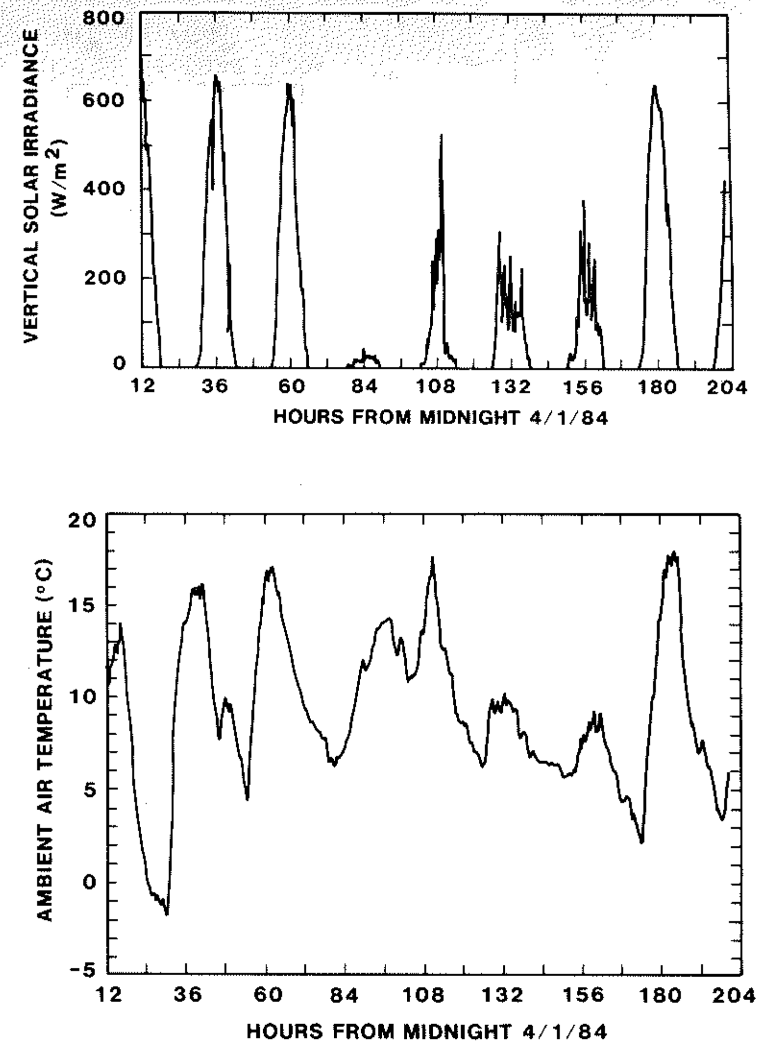

Figure 2. Climatic conditions measured during thermal-storage wall test: solar irradiance (top), ambient air temperature (bottom)

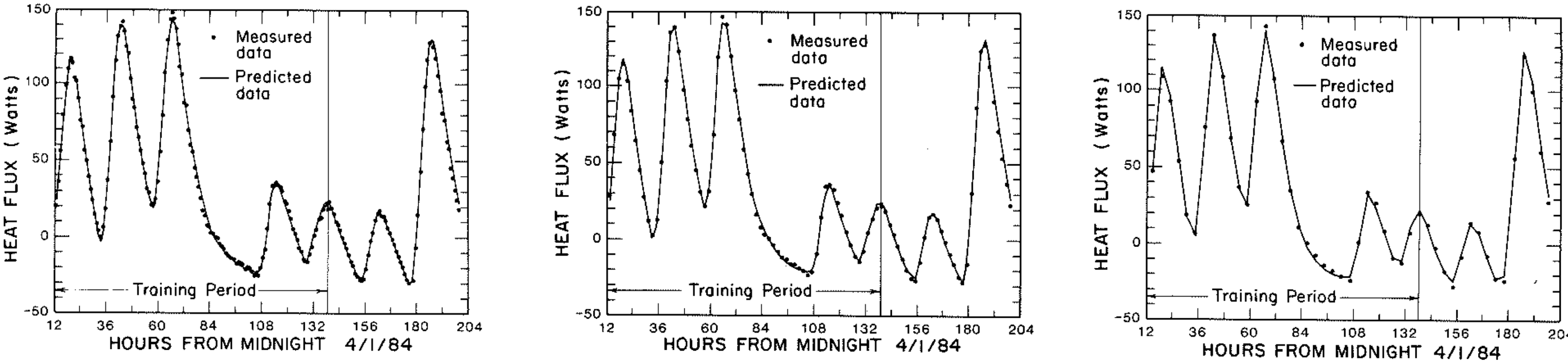

Figure 3. Measured vs. predicted heat flux for different time intervals: δ = 1 h (left), δ = 2 h (middle), δ = 4 h (right)

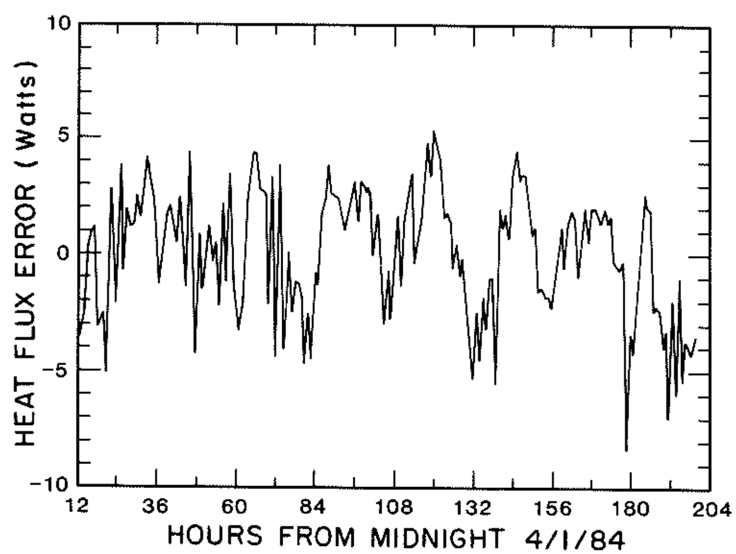

Figure 4. Error plot for one-hour time-step interval

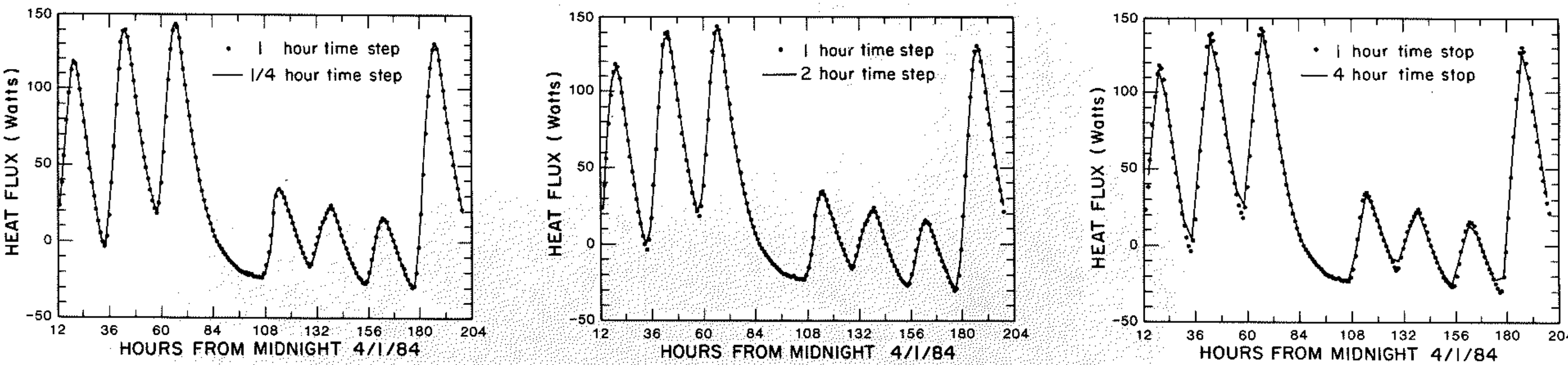

Figure 5. Comparison of predicted heat flux to one-hour time-step interval and other time-step intervals: δ = 1/4 h (left), δ = 2 h (middle), δ = 4 h (right)