Experimental and Theoretical Study of Natural Convection in a Superposed Air and Porous Layer When Heated from Below

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ABSTRACT

A numerical and experimental study was performed to analyze the influence of natural convection on heat transfer in a composite system comprising a porous material heated from below and an air space above. The numerical model was verified by conducting a number of experiments on a model material consisting of polystyrene pellets of cylindrical shape with hemispherical ends, made in the Wind Box. This apparatus, a guarded hot plate of large measuring area, is a prototype. In designing the measuring process, calculation of the maximum systematic error has been a control parameter; this is less than 6%.

The study included measurements in the Wind Box and calculations with a computer simulation program (CONVBOX) for three-dimensional convection problems. The agreement between the calculations and the experimental results is, in general, good.

INTRODUCTION

The existence of a fluid layer adjacent to a layer of fluid-saturated porous medium is a common occurrence in both geophysical and industrial environments. The composite systems also include such engineering applications as solar collectors with a porous absorber, journal bearings, fibrous and granular insulation where the insulation occupies only part of the space separating the heated and cooled walls, etc. Excellent reviews on the subject of natural convection in composite systems have been presented by Cheng (1978), Prasad (1991), Nield and Bejan (1992), and others.

In the building industry, the influence of natural convection on heat transfer has in many cases been disregarded in view of the fact that conduction and radiation constitute the dominant heat transfer mechanisms for thermal insulation materials. This is true for materials of high density and low permeability. Convection may, however, be a significant factor in the heat transfer in materials of low density and high permeability.

Experiments performed by Wilkes and Childs (1992) showed that convection was negligible for batt insulation but was an important heat transfer mechanism under winter conditions for some loose-fill fiberglass insulations. They observed that the thermal resistance was reduced by about 35% to 50% at large temperature differences (ca 70°F).

The main difference in construction between recent and older buildings in Sweden is that the attic floors of recent buildings have a considerably higher degree of thermal insulation. The most common thermal insulation material on attic floors, for both additional insulation and in new construction, is loose fill insulation, which has advantages because of its ease of installation. The insulation is delivered in bags and is applied to the attic floor by a special machine. The top surface of the material is left open, i.e., it has no form of protection against wind. Owing to the greater thickness of the material, with 0.5 m being a standard application, and the open top surface, there is an increased risk of air movements in the material, which will cause a reduction in its thermal resistance.

In a ventilated attic floor, the insulation material is in contact with an air space and is exposed to wind effects, which influence the thermal resistance of the material. The first limitation made in view of the magnitude of the subject is to ignore forced convection in this study. Heat transfer is studied in a composite system, shown in Figure 1, in which the inclination of the roof is not considered. We consider a horizontal porous layer bounded by an impermeable bottom surface while the top surface is open to the surrounding air. Both the bottom and the top temperatures are assumed to be uniform. The system is heated from below \((T_i > T_e)\).

The objective of this work was to determine whether natural convection occurs in isotropic porous materials and what

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its effects are on the thermal insulation capacity of the material. Both the theoretical and experimental investigation focused on the factors that influence natural convection in a configuration consisting of a porous material and an air layer.

One important goal was to determine the increase in heat flow due to convection. This is expressed by the Nusselt number, which is defined as the ratio of the heat flow, \( q \) (W/m²), with and without convection:

\[
N_u = \frac{q_{\text{with convection}}}{q_{\text{without convection}}} \tag{1}
\]

The investigation included measurements in the Wind Box and calculations with a computer simulation program (CONVBOX) for three-dimensional convection problems. The research focused on a model material consisting of polystyrene pellets of cylindrical shape. The combination of simulation and experiments performed in the Wind Box has been found to be an important instrument in understanding the complex coupled convection processes that occur in the insulation material. The completed study is described in detail in Serkitjis (1995).

**COMPUTER CALCULATIONS**

Natural convection for the case with a porous layer consisting of the model material has been theoretically studied using a simulation program, which takes into account the geometry of the Wind Box. This program, CONVBOX, is described in detail in Hagentoft and Serkitjis (1995). The program describes three-dimensional convective heat transfer in a composite system under the assumption that Darcy's law applies in the porous layer. The composite system comprises two layers, a porous layer and an air space. In the model, the air space is replaced by a thin surface layer with thermal resistance \( R \), which includes conduction, radiation, and convection. The boundary condition at the air/porous-layer interface is either permeable or nonpermeable. The model assumes that airflow rates or the values of \( R \) are small.

The model provides the opportunity to study both natural and forced convection in different geometries. Equations are solved numerically by a finite difference method, which means that the computation region is divided into parallelepiped computation elements. In order that good results may be obtained, temperatures are calculated analytically in three directions between adjacent nodes. Pressure distribution is calculated from an approximate temperature distribution. When the pressure distribution and thus the airflow field have been calculated, a new and more precise temperature field can be calculated. Both pressure and temperature are computed by an iterative method. Computations are continued iteratively until the result is stabilized between two global iterations. During the calculations, the result can be seen on the screen, both by means of the calculated values of the deviations from perfect mass and heat balance, temperatures, and Nusselt number and Rayleigh number and by means of figures that show cross-sectional images of temperature distribution in the three orthogonal planes.

The results of calculations can be followed up via three files, which contain the following: input data, current Nusselt number and Rayleigh number, and the computed temperature distribution. Computed temperatures can be read and displayed graphically, which makes it possible to study the number and appearance of the convection elements. A 90 MHz pentium PC computes a case with 8,000 computational elements in approximately four hours.

For all calculations, the geometry of the measuring apparatus (see Figure 7) has been used, i.e., the length is 1.2 m and the width is 0.6 m. For the thermal resistance, the value 0.2 m²·K/W has been used, which represents the thermal resistance of air spaces of thicknesses varying between 0.1 m and 0.5 m on the assumption that the air is exposed to a temperature difference of 2°C (−1°C and +1°C).

The validity of the model has been tested with reference to a horizontal space filled with a porous material, the assumption being that the boundary surfaces are nonpermeable and isothermal. Theoretical and experimental considerations (Nield and Bijan 1992) have shown that natural convection occurs when the modified Rayleigh number, \( Ra_m \), exceeds the value 40. This is also confirmed by the computer program used (see Figure 2). \( Ra_m \) is defined as

\[
Ra_m = \frac{\rho_0 c_p g \beta \lambda_m \cdot K \cdot \Delta T}{v \cdot \lambda_m} \tag{2}
\]

where

- \( \rho_0 \) = density of the air (kg/m³),
- \( c_p \) = specific heat capacity of the air at constant pressure (J/kg·K),
- \( g \) = acceleration due to gravity (m/s²),
- \( \lambda_m \) = mean thermal conductivity of the material (W/m·K),
- \( K \) = thermal conductivity of the air (W/m·K),
- \( \Delta T \) = temperature difference between the surfaces (°C),
- \( v \) = air velocity (m/s).
\[ \beta = \text{coefficient of cubic expansion (K}^{-1}) \]
\[ d_m = \text{height of porous space (m)} \]
\[ K = \text{air permeability of porous medium (m}^2) \]
\[ \Delta T = \text{temperature difference across the porous medium (K)} \]
\[ \nu = \text{kinematic viscosity (m}^2/\text{s}) \]
\[ \lambda_m = \text{thermal conductivity of the porous medium (W/m·K)} \]

The values calculated by CONVBOX are presented in Figure 2. The results of simulations made with the two-dimensional models of Delmas and Wilkes (1992), denoted “ORNL,” and those of Fryklund (1995), denoted “CHConP,” are plotted in the same figure. A comparison of CONVBOX with the results of Delmas and Wilkes and Fryklund shows good agreement.

If the top surface of the porous material is left open, natural convection—according to theory (Nield and Bijan 1992)—begins earlier, namely, when the value of the modified Rayleigh number approaches 27.1. Calculations made with CONVBOX show not only good agreement with the theory but also very good agreement with the results from the different models (see Figure 3).

Convection in the composite system is governed by several parameters. A comparison between theoretical and empirical results can be made for a certain geometry and for a certain combination of porous material and fluid. It is possible to experimentally verify the theoretical results for a specific combination, but it is not clear whether the results can be used for general calculations. Thus, there are no general correlations in the literature for the composite system. Both theoretical and experimental studies suggest, however, that the value of the critical modified Rayleigh number is below 30.

With the help of CONVBOX, calculations have been made to simulate the course of events for a porous material that is heated from below and whose top surface is in contact with an air space. The influence of an open and a covered top surface has been calculated and is shown in Figure 4.

The influence of the thermal resistance of the air space and the thickness of the porous material has been calculated and the results are plotted in Figure 5. The factors that influence convective heat transfer and that may be of interest for building technology and for experimental investigations are included in the definition of the modified Rayleigh number \( R_a_m \). The thermal conductivity, permeability, and thickness of the porous material and the temperature difference across the porous material affect the calculated value of the Rayleigh number and, thus, convective heat transfer.

For all calculation cases, convection elements of cylindrical shape and parallel to the long side have been formed at low values of the Rayleigh number. These change into a square shape (see Figure 6) when the Rayleigh number increases. The slope of the Nusselt number curve changes (bumps in Figures 4 and 5) during this transition from cylindrical to square-shaped convection cells.

**EXPERIMENTS**

The influence of natural convection on heat transfer through a porous material has been studied for different configurations in the Wind Box. The significance of various

![Covered top surface](image)

**Figure 2** Comparison of the values computed by the program CONVBOX with the two-dimensional simulation programs of Delmas and Wilkes (ORNL) and Fryklund (CHConP). The top surface is covered. The aspect ratio is the ratio between the width and the height. For calculations made with CONVBOX, the ratio between the length and the width is equal to 2.
Figure 3  Comparison of values calculated by the program CONVBOX with the two-dimensional simulation programs of Delmas and Wilkes (ORNL) and Fryklund (CHConP). The top surface is open. The aspect ratio is the ratio between the width and the height. For calculations made with CONVBOX, the ratio between the length and the width is equal to 2.

Figure 4  The influence of boundary conditions on convective heat transfer in a composite system comprising an air space and a porous material. The calculations refer to a porous material with $d_m = 0.2$ m, $K = 6 \times 10^{-5}$ m$^2$, and $\lambda = 0.044$ W/m$^\circ$K.
Figure 5  The influence of the thermal resistance of the air space and the thickness of the porous material on heat transfer in a composite system comprising an air space and a porous material.

Figure 6  Typical CONVBOX plots showing three cross-sectional images (in the x, y, and z directions) of temperature distribution. In this case, two square-shaped convection elements are formed.
factors has been examined with reference to theory and the construction of the measuring apparatus.

Experiments have been carried out on horizontal insulation layers. The conditions for the top surface, open or covered, have been especially investigated. The effect of an overlying air layer with different thicknesses has also been studied. Some of the results are presented in this paper. Other parameters studied include thickness, permeability, ambient temperature, and the temperature difference across the sample, as well as different combinations of these parameters. All the measurements may be regarded as relative (comparative) measurements, which means that the effect of systematic error can be ignored.

The principal criteria in choosing the material used in the experiments are that it have a simple geometry, low density, and high permeability. The material should not be hygroscopic or be prone to settlement. The sample material used in the experimental investigation presented in this paper focuses on a model material that comprises cylindrical polystyrene pellets with hemispherical ends. In order to find out more about the sample material and to provide a sound basis for the calculations, the relevant material properties have been investigated. The results of this investigation are provided in Table 1.

### Table 1: Summary of the Sample Material Properties

<table>
<thead>
<tr>
<th>Ø (mm)</th>
<th>Porosity (%)</th>
<th>ρ (kg/m³)</th>
<th>K (m²)</th>
<th>λ (W/m·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.2</td>
<td>33.2</td>
<td>13 - 13.5</td>
<td>5.5·10⁻⁸ - 5·10⁻⁸</td>
<td>0.046 - 0.044</td>
</tr>
</tbody>
</table>

Owing to the variation in packing density, there has been a certain amount of uncertainty in determining permeability and thermal conductivity.

The measuring apparatus, a Wind Box, is a prototype. The Wind Box is the development of a hot plate apparatus and may be described as a horizontal guarded hot plate apparatus of large measuring area, which is an advantage as insulation thicknesses increase.

Thermal resistance is determined by measuring temperature drops and heat flow rates in a horizontal sample. The thermal resistance can also be determined when air passes over the top surface at different velocities. The general arrangement of the apparatus is shown in Figure 7. The heat supplied to the warm side of the apparatus per unit time can be determined. In order to obtain a measure of the actual heat flow rate, it is necessary to ensure that the flow of all the heat supplied through the sample is one-dimensional. The same temperature is set on the balancing heating foil, the guard sections, and the measuring sections so that all power supplied to the measuring sections passes straight through the sample.

The sample material on the measuring section and the guard sections can be separated by a thin expanded plastic frame, which prevents convective heat transfer between these sections. The entire apparatus is enclosed in a chamber equipped with a cooling unit controlled by a contact thermometer. In this way, the temperature of the air surrounding the test apparatus can be adjusted to the mean temperature of the sample (normally +10°C).

A lot of care has been taken with calibration and elimination of interference to ensure that the measurements are of high quality. Any errors have been reduced or eliminated when possible. One means of control in designing the measuring process was the calculation of the maximum systematic error (Serkitijs 1995), and this is less than 6%.

For a measurement to be considered as correct, it is necessary that the balance measured between the measuring plates and guard sections and between the measuring plates and the balancing heating foils does not exceed 0.1°C. Further, the difference between the calculated thermal conductivity and three consecutive measurements at 30-minute intervals shall not exceed 2% or exhibit a tendency to change.

In order to create favorable conditions for convective heat transfer, all measurements were made with upward heat flux. The contributions of conduction and radiation to heat transfer were kept at a constant level by making all measurements at the same mean temperature (10°C) in the sample material. In order to reduce boundary losses, the ambient air temperature was maintained constant at 10°C, i.e., at the mean temperature of the sample material.

The following procedure was applied:

The necessary total thickness is obtained by placing wooden frames measuring 2 m x 1.2 m x 0.1 m on top of one another. The same number of layers of 0.1 m thick polystyrene sheets are placed on the guard sections, and the thermocouples are fixed by strings stretched between the edges of the test box. Any joints between the polystyrene sheets are taped. After this, the sample material is poured over the measuring sections until the desired thickness is reached. The area above the measuring sections is covered with a 1.22 m x 0.62 m x 0.002 m black painted plate that is taped to the polystyrene sheets. A plastic fabric is stretched over the entire top surface to prevent any ingress of the air circulating above.

When steady conditions are attained and the mean temperature of the sample is 10°C, the first reading is made. This is checked with respect to the systematic error of the measuring apparatus and the reference value of the thermal conductivity of the sample material. If everything is in order, the next temperature differential step (usually 2°C) can be set. The value given for each measuring section is the mean over a period of ten hours. A mean value is calculated for all the measuring sections. At the end of the measurement series, the density of the sample material is determined by weighing.

### Horizontal Space Filled with the Sample Material

Natural convection for a porous material placed between two impermeable plates, the lower one of which is at a higher temperature than the top one, occurs when the modified Rayleigh number, $Ra_m$, exceeds the value $Ra_m = 40$. This crit-
Figure 7  General arrangements of the measuring apparatus. All dimensions in mm.
ical limit has been known since 1945 and has been verified both experimentally and theoretically by many researchers (see Subsection 2.3.4, Serkitjis 1995). The value 40 can thus be used as a typical value in the calculations and measurements. This limit is reached when the temperature difference across the material in question ($\lambda = 0.044$ W/m·K and $K = 5.5 \times 10^{-8}$ m$^3$/Pa·s) is ca 36°C, assuming that its thickness is 0.3 m.

The definition of $R_m$ (see Equation 2) includes the permeability $K$ of the porous material, its thermal conductivity $\lambda_m$, thickness $d_m$, and the temperature difference across the material. This means that in measurements on a certain material (of given $K, \lambda_m$, and $d_m$), natural convection can be brought about by increasing the temperature difference across the material. In this way, a critical temperature difference can be identified and $R_m$ calculated with reference to this.

A measurement series with a 0.3 m thick layer ($d_m = 0.3$ m) of pellets and a covered top surface was carried out in order to check the function of the measuring apparatus. Steady conditions were attained after ca five days. Measurement results obtained for increasing values of the temperature difference across the material are plotted in Figure 8.

It is evident from Figure 8 that the readings for the three measuring regions are well bunched for temperature differences up to 35°C and deviate widely for larger temperature differences. An increase in temperature difference beyond 35°C gives rise to an increase in thermal conductivity. This suggests that natural convection commences between a 35°C and 37°C temperature difference across the material, which according to calculations corresponds to $R_m = 40$, entirely in accordance with the theory. An investigation of temperature fields inside the material indicates that four two-dimensional cylindrical convection cells are formed parallel to the short side of the measuring apparatus. This is also in accordance with the theory but not with the program, according to which only two cylindrical convection cells are formed parallel to the long side of the apparatus. The alignment of the convection cells does not appear to have an appreciable effect on the calculated value of the Nusselt number.

In the calculation section, results obtained with the computer program CONVBOX are compared with other experimental and theoretical results. Agreement is very good. The calculation and measurement results obtained in this investigation are plotted in Figure 9. The Nusselt number for the measurements has been calculated as the ratio of the heat flux with and without natural convection at the same temperature difference.

A comparison of theory with computer calculations and measurements in the Wind Box shows that the model works and the measuring apparatus is reliable.

**Combination of Air Layer and Porous Material**

In order to be quite certain regarding the influence of the air space and to find when convection commences, a further four measurement series were performed (see Table 2). Three series were carried out with constant material thickness ($d_m = 0.2$ m) and variable air space thickness ($d_f = 0.1, 0.2, $ and 0.3 m). The material was not removed from the apparatus between the measurement series. Its density was determined after the last series and was found to be 13.3 kg/m$^3$. In the fourth series, the combination of 0.2 m air space above a 0.3 m thick layer of material was investigated. The density of the material in this series was 13.4 kg/m$^3$.
The results obtained in measurement series 1 through 3 are plotted in Figure 10. The increase in thermal conductivity at temperature differences greater than 30°C indicates that natural convection occurs in the system. This is confirmed by the nonlinear temperature distributions in the material.

On comparing the measurement series (Figure 10), it is seen there is a difference that is due to the air space above the material. This can be confirmed by expressing thermal conductivity in relation to the temperature difference across the material (see Figure 11). In view of the measured temperatures, it is evident that the temperature drop across the air space is, in principle, constant, approximately 2°C, which means that the thermal resistance of the air space may be considered constant (see Figure 10). Natural convection thus occurs in the material. On the basis of measured temperatures, it was possible to identify three-dimensional convection cells (probably square in shape).

Assuming that the first measured values contain no convective contribution, the results can be plotted in a diagram, with, e.g., the Nusselt number as a function of the temperature difference either across the system or across the material.

Reference to the temperature difference across the material is advantageous because $Ra_m$ can be calculated, and this facilitates comparisons between calculations and measurements.

On the basis of measured temperature differences across the sample material and the estimated permeability for $d_m = 0.2$ m ($K = 5.5 \times 10^{-8}$ m²) and for $d_m = 0.3$ m ($K = 5.10^{-8}$ m²) and $\lambda = 0.044$ W/m·K, $Ra_m$ can be calculated. The results are set out in Figure 12 where Nu is plotted as a function of $Ra_m$ for both the calculations and all the measurements. It is evident from Figure 12 that $Ra_m = 20$ according to the experimental investigation and $Ra_m = 24$ according to theoretical calculations. The cause of this difference may be that the estimated values of permeability and thermal conductivity are not correct.

**CONCLUSIONS**

Comparisons with experimental results for natural convection in a homogeneous horizontal insulation material show good agreement. The calculations predict both the right magnitude of the Nusselt number and the critical temperature difference, i.e., the temperature difference for the onset of convection, that have been found in the experiments. The results also show good agreement with results found in the literature in terms of critical Rayleigh numbers.

On the basis of calculations and measurements regarding the influence of the air space on the convective component of heat transfer in the underlying insulation, the following can be stated:

- The occurrence of natural convection in the air space has only a marginal effect on heat transfer in the underlying insulation.
Figure 10 Measurement series 1-3, \(d_m = 0.2\) m and \(d_f = 0.1, 0.2,\) and \(0.3\) m. Thermal conductivity is a function of the temperature difference across the composite system. The individual plots represent the means of the three measuring regions.

Figure 11 Measurement series 1-3, \(d_m = 0.2\) m and \(d_f = 0.1, 0.2,\) and \(0.3\) m. Thermal conductivity is a function of the temperature difference across the specimen (°C). The individual plots represent the means of the three measuring regions.
The thermal resistance of the air space may be assumed constant, which means that the air space can be replaced in theoretical models by a constant thermal resistance.

Alteration of the boundary condition of a covered top surface, $R_{am} = 40$, to an open top surface in contact with an air space halves the critical modified Rayleigh number.

The ratio between the thicknesses of the air space and the thermal insulation $d_f / d_m$ has no critical significance for heat transfer in the insulation.

It may be considered that the available data regarding the validity of the theoretical model are of limited extent for an assessment to be made on whether it can be applied more generally. From the comparisons of calculations with measurements on polystyrene pellets, it is evident that the theoretical model holds for this material. It is possible that the geometry of the material may have some influence, which means that the model ought to be tested on other porous materials, such as anisotropic fibrous materials.

The case with forced convection has so far not been studied experimentally. It will be done in a continuation of the project. Both forced and natural convection will be investigated using mineral wool as insulation material.

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