Thermal Inertia of Straw Bale Walls

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ABSTRACT

This paper demonstrates a theoretical argument that straw bale walls have a substantial thermal inertia potential based on wall thickness, heat conductivity, mass density, and specific heat capacity. Several different types of wall materials are compared with regard to these factors, and a ratio of wall thickness to penetration depth is established showing the heat storage potential of the different wall types. The data analyzed for straw bales are taken from previously published literature and contain a wide range of values for the thermophysical properties. From these data, a range of values are calculated for straw bales. Even at the low end, the thermal inertia potential for straw bales is comparable in value to adobe, compacted straw boards, and wood log construction.

INTRODUCTION

The popularity of straw bales as a wall material has been increasing in recent years. This increase is particularly noticeable among environmentally conscious designers, builders, and clients. The reasons for this increase are not difficult to find: it provides a use for a waste product; it is a renewable resource; and the walls, reportedly, provide enough thermal resistance without any additional insulation.

Straw, which is the stalks remaining after the harvest of grain, is an unwanted waste product. Straw does not retain any nutritional value after the removal of the grain and cannot be used for animal fodder. Straw also has a slow rate of deterioration that creates disposal problems (Steen et al. 1984). The appropriation of the existing straw as a building component provides a solution to the disposal problems and supplies to the building industry a material where no additional manufactured or natural resources are depleted.

Straw bale construction was first used on the Nebraska plains, shortly after the invention of mechanical hay baling machines in the later 19th century. The local farmers used what was on hand for building their own houses, and straw appeared a logical choice, often over the use of sod. The oldest standing structure is the Burke House near Alliance, Nebraska, built in 1903 (Steen et al. 1984). The many examples on the Nebraska plains have given rise to the naming of load-bearing straw bale structure as the "Nebraska style." Contemporary architects have revitalized the concept, and several cities, including Austin, Texas; Tucson, Arizona; and Santa Fe, New Mexico, have adopted straw bale construction building codes.

Straw has a high thermal resistance value, ranging from R2.4 to R3.0 per inch depending on the direction of the straw, moisture, and density. However, despite this high R-value, the current perception is that straw does not have a high thermal inertia. For example, a housing prototype developed by the Navajo Nation in conjunction with the United States Department of Energy (DOE) and the United States Department of Housing and Urban Development (HUD) (DOE 1995) combines straw bale construction with adobe construction. The adobe is added in deference to the cultural tradition of the Navajo Nation and to provide thermal mass for passive heating techniques. The designers did not consider the mass of the straw bale walls to also provide thermal inertia.

The authors suspect that the current belief that straw bale walls have a low thermal inertia may be unfounded. In fact, we expect that straw bale walls may be an excellent provider of thermal inertia. This paper develops a theoretical argument that demonstrates that straw bale walls have substantial thermal inertia potential. The thermal inertia potential of straw bale walls is large enough to rival those of even adobe and log walls.

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We will use the gamma index, γ, of thermal inertia (Arumí 1997) to show the heat storage potential of the straw bale wall. The data available in the literature are limited but sufficient to justify the postulate that straw bale walls are thermally very massive building components. This paper does not address other concerns about straw bale walls.

THERMAL INERTIA INDEX

The thermal inertia index we use for this argument follows from the exact solution of the one-dimensional heat diffusion equation for a simple harmonic driving temperature. The solution is outlined in the appendix and the index is

$$\gamma = \sqrt{\frac{\omega \rho c}{2k}} L \tag{1}$$

where

L = slab thickness (m),

ω = frequency of the harmonic term (rad/s) ($\omega = 2\pi/P$, P = period of oscillation, e.g., P = 24 hrs),

k = heat conductivity (W/[m·°C]),

ρ = mass density (Kg/m^3),

c = specific heat capacitance (J/[Kg·°C]).

We may also note the following alternative representations of the thermal inertia index

$$\gamma = L/\delta \text{ or } \gamma = t_{phase}/P \tag{2}$$

where

$\delta = \sqrt{\frac{2\alpha}{\omega}}$ penetration depth (m),

$\alpha = \frac{k}{\rho c}$ thermal diffusivity (m^2/s),

$t_{phase} = \frac{2\pi L}{v_{phase}}$ time the wave phase takes to cross the slab (s),

$v_{phase} = \sqrt{2\omega\alpha}$ phase velocity (m/s).

One of the ways in which the thermal inertia of a wall may be measured is in its ability to "store heat." Heat, of course, is not really stored. Walls heat up or cool down at different rates. When a wall is exposed to cyclically changing temperatures on the outside, the inside also responds thermally in a like cyclical manner. A cyclically changing temperature oscillates up and down over a fixed period of time. There is an average temperature typical of the cycle, and the temperature increases and decreases relative to the average value. During the heating half of the cycle, heat is diffused from the outside surface into the wall, and during the cooling half of the cycle, heat is diffused from the wall to the outside surface. The rate of diffusion is determined by the properties of the wall. If this rate is sufficiently slow, some of the "heat" will be driven back to the outside before it makes its way across to the inside surface of the wall. How far the heat makes its way into the wall depends on the period of the oscillation—if the period is shorter than the time needed for the heat to diffuse across the width of the wall, then not all of the heat finds its way inside. The measure of the distance the heat enters is the penetration depth, and it depends on the frequency (ω) of the cycle. We should also note that the thicker the wall is compared to the penetration depth, less of the heat will find its way to the inside surface. The ratio of the wall thickness to the penetration depth is γ, the thermal inertia index.

The effect of this process is illustrated in Figure 1. It shows the outside and inside surface temperatures for of a γ = 3 in. wall for two consecutive 24-hour cycles. The outside temperature, which drives the solution, is a cosine function. During half of the period, the temperature is above the average

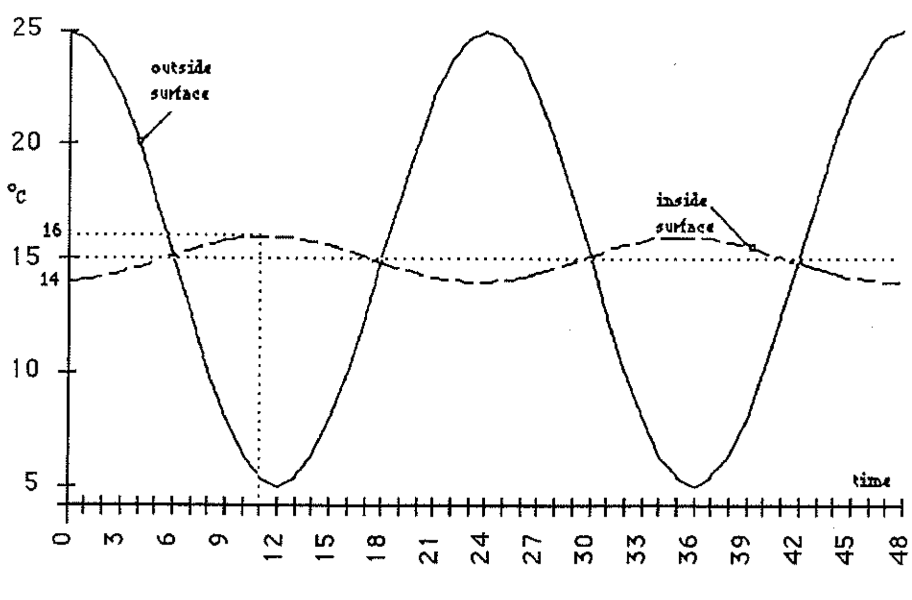


Figure 1 *Surface temperatures for a γ = 3 in. wall.*

temperature of 15°C, reaching a maximum of 25°C. This half of the period describes the daytime. The other half of the period, the temperature is below the average, reaching a minimum of 5°C. This other half of the period describes the nighttime.

The inside temperature was obtained from Equation A-11 (see Appendix). The average inside surface temperature is the same as the average outside surface temperature, which is consistent with the principle of energy conservation. It is evident from the figure that the inside surface temperature swings less than the outside and it also lags behind. The lag time is close to 11 hours, and the inside temperature swings only 10% as much as the outside surface. This reduction in the temperature swing is the attenuation of the temperature oscillation and is referred to as the temperature amplitude attenuation factor.

The time lag is a consequence of the finite rate of heat diffusion. In the example from Figure 1, we have an 11-hour. time lag, which means that the inside surface temperature is highest when the outside temperature is at its lowest value during the nighttime, and the inside temperature is at its lowest value when the outside is the warmest. The temperature amplitude attenuation factor may be interpreted as a measure of the heat stored. Since the inside surface temperature rises above the average by 1°C, while the outside surface rose by 10°C, then not all of the heat that entered through the outside surface arrived at the inside surface. Likewise, the inside surface temperature drops below the average by an equal amount, 1°C, while the outside surface drops by 10°C. Thus, not all of the heat that leaves through the outside surface came from the inside surface. Therefore, it appears as if the excess daytime heat is stored for nighttime use. Since the inside temperature swings only 10% as much as the outside temperature, it appears as if 90% of the excess daytime heat is stored for nighttime use.

The temperature amplitude attenuation factor, $|\eta|$, for a uniform slab depends only on γ and it is shown in Figure 2. The functional dependence is derived from first principles in the appendix, and its explicit form is given by Equation A-10.

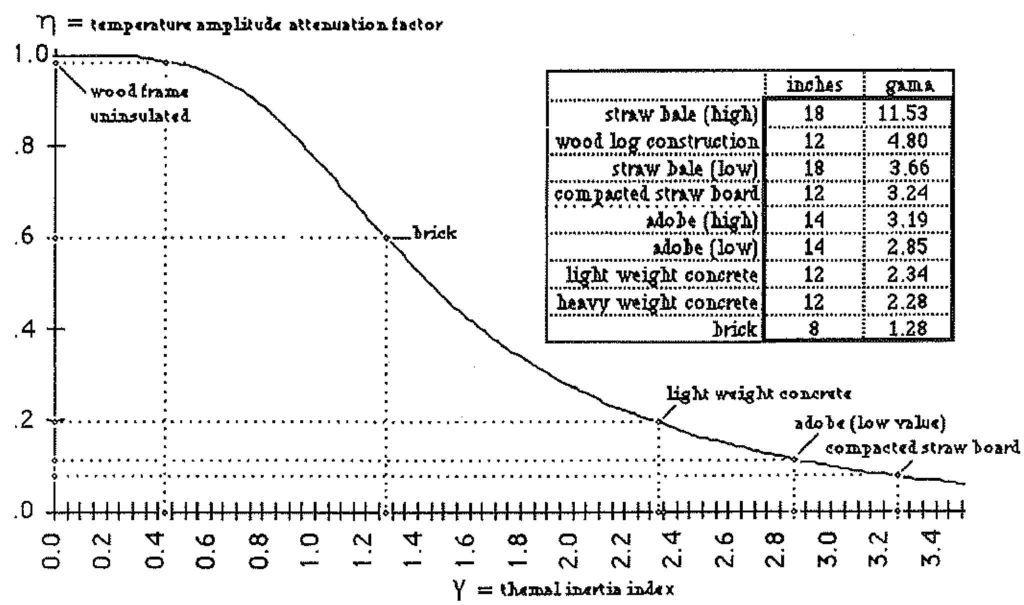
The heat storage potential, in the sense described above, is given by $1\text{-}|\eta|$.

Figure 2 includes several walls that serve as a reference to interpret the results for the straw bale walls. The insert table includes several walls made of uniform slabs of material, the order of the walls in the table is shown in descending order of thermal inertia potential. All gamma values were calculated for a period P = 24 hours. These walls are not wall systems inasmuch as they consist of a single slab of a single material. We exclude composite wall systems in order to reduce the number of parameters to only one; thus, we simplify the comparison of the thermal inertia properties of the walls.

Straw bale walls head the list with the highest gamma value. The other wall types were selected because of their well-known potential to provide thermal inertia. Other than compacted straw boards, these walls are well-known standard construction types. Of the standard walls, log construction combines the highest specific heat capacitance with low thermal conductivity of wood to produce an extremely high gamma value.

It is followed by "compacted straw boards," a material made of wheat straw that is fused and compacted under heat and pressure with a continuous resin-bonded paper membrane on all sides.[1] We take particular interest in this material

[1] "Tetratech," Building Systems international, Carrolton, Tex. No longer in business.



	inches	gama
straw bale (high)	18	11.53
wood log construction	12	4.80
straw bale (low)	18	3.66
compacted straw board	12	3.24
adobe (high)	14	3.19
adobe (low)	14	2.85
light weight concrete	12	2.34
heavy weight concrete	12	2.28
brick	8	1.28

Figure 2 *Attenuation factors as a function of the thermal inertia index (see Equation A-10). All walls, except the wood-frame uninsulated wall, are single slabs made of the respective material.*

because its basic material is straw and because of its well-documented thermal properties. A 12-inch slab of compacted straw board yields a gamma value of 3.24. From Figure 2 we obtain an attenuation factor of about .08, which results in a .92 storage potential coefficient.

A 14-inch adobe wall results in a gamma value of 2.85. From the figure, we obtain an attenuation factor of about .12, which results in a .88 storage potential coefficient. Adobe bricks are not uniformly made, their thermal properties vary, and the information available is sketchy. The values used represent the upper and lower limits of what is believed to be the range of likely values (Arumí 1984).

We include in Figure 2 the attenuation factor for an uninsulated wood-frame wall. This is a composite wall and not a uniform slab of single material. The calculation of the attenuation factor follows a concatenation procedure not shown in the appendix. The result yields an attenuation factor of about .99, which means a negligible storage potential index of .01. The *effective* gamma value is about .43. Gamma values cannot be defined for composite walls; however, attenuation factors can. Therefore, the gamma value of a single slab that exhibits the attenuation factor of a composite wall may be used to define the effective gamma value of the composite wall. A large fraction of existing housing stock in the southern tier of the United States has this type of wall construction, especially among houses built before the oil embargo of 1974. Most walls used in construction since then have gamma values in the range from .5 to 1.5; the increase in gamma values comes primarily due to the use of insulation.

The graph in Figure 2 goes only as far as a gamma value of about 3.5; therefore, the log and the straw bale walls are out of range. The straw bale walls may have gamma values as high as 11.53. This extraordinarily high value is due primarily to the 18-inch thickness. The next most significant contribution is due to the very low heat conductivity. The specific heat capacitance may rival the heat conductivity in significance; however, we have no direct measurements for heat capacitance in the literature we found, and thus we have no certainty about its value.

Gamma values of this magnitude mean that the attenuation factor is practically zero and the heat storage potential is 100%. In fact, the meaningful storage potential for such a wall should be measured over weekly cycles or even longer. For example, a weekly cycle corresponds to a period of $P = 7$ days, which results in a weekly gamma value of 4.36, which, in turn, yields a heat storage potential of about 99%. For a monthly cycle, $P = 4$ weeks, the respective gamma value is 2.18, which implies a storage potential of 75%. For a yearly cycle, $P = 365$ days, the respective gamma value is .6, which implies a storage potential of about 5%.

Such a high value of heat storage potential is difficult to believe. However, we should note that by using the low end of the data for straw bale we get a gamma value of 3.66, which implies a diurnal storage potential of 95%. Only log construction outperforms it.

THERMOPHYSICAL DATA

The results of the analysis in this article are based on the experimental data from the following sources.

McCabe

McCabe (1993) carried out his measurements as part of his thesis for a master's degree. He makes no mention of the temperature at which the measurements were taken.

- Conductivity
  against the grain, $k = .014$ W/m·°C (.027 Btu/h·ft·°F)
  along the grain, $k = .019$ W/m·°C (.035 Btu/h·ft·°F)
- Specific heat capacitance not reported.
- Density values are deduced from weight and volume measurements as listed by McCabe

Weight of bales:		from 50 lb to 80 lb
Height:	14 in. to 16.5 in.	(35.6 cm to 41.9 cm)
Length:	35 in. to 47 in.	(88.9 cm to 119.4 cm)
Width:	18 in. to 23 in.	(45.7 cm to 58.4 cm)

McCabe does not say which size bale weighs 50 lb and which one weighs 80 lb. We assign the smallest bale by volume to the smallest weight and the largest bale to the heaviest; we obtain a density range:

Low value $\rho = 124.1$ Kg/m^3 (7.75 lb/ft^3)
High value $\rho = 157$ Kg/m^3 (9.8 lb/ft^3)

Sampathrajan et. al

These authors[2] did not measure straw bales but particle boards that contained straw bale residues. There is no reason to expect that the thermophysical properties of these particle boards reflect the respective properties of straw bale walls.

- Conductivity, $k = .019$ W/m·°C (.036 Btu/h·ft·°F)
- Specific heat capacitance, $c = 232.1$ J/Kg·°C (.180 Btu/lbm·°F)
- Density, $\rho = 190$ Kg/m^3 (11.9 lb/ft^3)

Desjarlais

The thermophysical properties of "compacted straw boards" were measured and Desjarlais (1984) wrote the report of the measurements. As mentioned in the previous section, these boards are made of wheat straw that is fused and compacted under heat and pressure.

- Conductivity, $k = .024$ W/m·°C (.046 Btu/h·ft·°F)
- Specific heat capacitance, $c = 645.2$ J/Kg·°C (.5 Btu/lbm·°F)
- Density, $\rho = 259.5$ Kg/m^3 (16.2 lb/ft^3)

2. Sampathrajan, A., et al., "Mechanical and Thermal Properties of Particle Boards Made from Farm Residues," *Bioresource Technology* 0960-8254/92, quoted by McCabe 1993).

House of Straw

These results (DOE 1995) were reported on the Navajo housing project. For a 23-inch bale, the authors report:

- Resistance, 56.5 h·ft^2·°F/Btu
- Weight, 21.4 lb/ft^2
- Heat capacity, 6.4 Btu/ft^2·°F)

Converted to intensive units, this becomes:

- Conductivity, k = .018 W/m·°C (.034 Btu/h·ft·°F)
- Specific heat capacitance, c = 387.1 J/Kg·°C (.3 Btu/lbm·°F)
- Density, ρ = 179.4 Kg/m^3 (11.2 lb/ft^3)

Summary of Data

The data from these sources, arranged in ascending order of value, are as follows.

- Mass density:
  McCabe, low value, ρ = 124.1 Kg/m^3
  McCabe, high value, ρ = 157 Kg m^3
  House of Straw, ρ = 179.4 Kg/m^3
  Sampathrajan, ρ = 190 Kg/m^3
  Desjarlais, ρ = 259.5 Kg/m^3

- Heat Conductivity:
  McCabe, against the grain, k = .014 W/m·°C
  House of Straw, k = .018 W/m·°C
  McCabe, along the grain, k = .019 W/m·°C
  Sampathrajan, k = .019 W/m·°C
  Desjarlais, k = .024 W/m·°C

- Specific heat capacitance:
  Sampathrajan, c = 232.1 J/(Kg·°C)
  House of Straw, c = 387.1 J/(Kg·°C)
  Desjarlais, c = 645.2 J/(Kg·°C)

These results show significant variability. The source of the differences may be both systematic and random. Some of the systematic sources are evident: Desjarlais' data are for hay that has been heated and compressed; Sampathrajan's data are for particle boards with only residues of straw mixed with other materials; and straw bales made of rice may have different properties than those made of wheat. And finally, one hay baling machine may not produce bales identical to those produced by other machines.

Based, therefore, on these data alone, we cannot make a unique assessment of the thermal inertia of straw bale walls; however, we can define a range of possible values for the thermal diffusivity. By using the lowest heat conductivity and the largest volumetric heat capacitance (ρc), the lower limit is obtained, and by using the highest heat conductivity and the lowest volumetric heat capacitance, the upper limit is obtained.

$$8.6 \times 10^{-8}\ \text{m}^2/\text{s} < \alpha < 8.5 \times 10^{-7}\ \text{m}^2/\text{s}$$

The lower the diffusivity, the higher the thermal inertia. Table 1 displays the thermophysical properties of the various construction materials used in the wall constructions mentioned earlier in Figure 2. The list is sorted in descending order of intensive thermal inertia (γ/cm.) The values for straw bales span this range of materials.

CONCLUSION

Due to the insufficient data specifically measuring the mass density, heat conductivity, and specific heat capacitance

TABLE 1

Thermal Properties of Various Materials*

	Density Kg/m^3	Conductivity W/m·°C	Capacitance J/Kg·°C	Diffusivity m^2/s	Depth cm	Gamma per cm	Phase cm/h
Wood	593	3.7E-02	774	8.1E-08	3.3	0.30	0.9
Straw bale "high mass"	260	1.4E-02	645	8.6E-08	3.4	0.29	0.9
Compacted straw boards	260	2.5E-02	645	1.5E-07	4.5	0.22	1.2
Lightweight concrete	641	5.3E-02	258	3.2E-07	6.7	0.15	1.7
Brick	1922	2.2E-01	258	4.5E-07	7.9	0.13	2.1
Glass fiber insulation	91	1.3E-02	258	5.7E-07	8.8	0.11	2.3
Adobe "high mass"	1874	1.7E-01	150	6.2E-07	9.2	0.11	2.4
Straw bale "low mass"	124	2.5E-02	232	8.5E-07	10.8	0.09	2.8
Heavyweight concrete	2243	5.3E-01	258	9.2E-07	11.3	0.09	2.9
Adobe "low mass"	1874	2.1E-01	95	1.2E-06	12.8	0.08	3.4

* The values for wood, concrete, brick, and glass fiber were taken from the *1997 ASHRAE Handbook—Fundamentals*, page 25-10 (Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.).

of straw bales, it is difficult to predict accurately the exact gamma value for comparing straw bale construction to other types of wall construction. However, based on available data from previous literature, the largest diffusivity value estimated for straw bales still results in a very high thermal inertia potential. Based on this estimate, straw bales are at least equivalent to adobe, compacted straw boards, and log wall construction types for storing heat. Further research of this material would provide more accurate data to test our initial conclusion that straw bale construction provides a substantial heat storage ability.

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APPENDIX

This appendix develops the exact solution to the one-dimensional heat diffusion equation for a simple harmonic time dependence. The heat diffusion equation has the form

$$\rho c\frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x} \text{ where } q = -k\frac{\partial T}{\partial x} \quad \text{(A-1)}$$

The independent variables are

t = time (s),

x = position (m).

The dependent variables are

T = temperature (°C),

q = heat flux (W/m^2).

The physical properties of the material in the homogeneous-layer are

ρ = mass density (kg/m^3),

c = specific heat capacitance (J/kg·°C),

k = heat conductivity (W/m·°C).

The overall procedure starts with the solution for one homogeneous layer, which relates the temperature and heat flux on one boundary to the respective values on the other

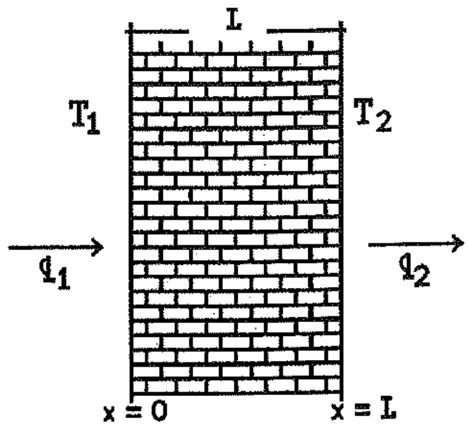

Figure A-1

boundary. The corresponding solution for a compound wall, one constructed of successive homogeneous layers, is obtained by the progressive concatenation of the matrices for each respective layer. The surface coupling by radiation and convection is derived in terms of an equivalent "homogeneous layer" matrix that is concatenated on the respective side of the wall matrix. Walls in contact with the ground and interior walls are considered separately.

Expand the time dependence of the temperature in a Fourier series such that

$$T = \sum_{n=0}^{\infty} T_n e^{in\omega t} \qquad \left(i = \sqrt{-1}\right) \quad \text{(A-2a)}$$

where

$$T_n = \frac{1}{2\pi}\int_0^{2\pi} Te^{-in\omega t}\, d(\omega t) \quad \text{(A-2b)}$$

is the nth Fourier coefficient of the series and $\omega = 2\pi/P$, where P is the period of analysis measured in units of time (s).

The x dependence of each Fourier coefficient for a homogeneous slab takes the form

$$i\frac{n\omega\rho c}{k}T_n = k\frac{\partial^2 T_n}{\partial x^2}, \quad \text{(A-3)}$$

which has the general solution

$$T_n = A_n e^{\beta_n x} + B_n e^{-\beta_n x} \quad \text{(A-4a)}$$

$$q_n = -k\beta_n(A_n e^{\beta_n x} - B_n e^{-\beta_n x}) \quad \text{(A-4b)}$$

where

$$\beta_n = (1+i)\sqrt{\frac{n\omega\rho c}{2k}} \quad \text{(A-5)}$$

A_n and B_n are integration constants that are evaluated from the boundary conditions on sides 1 and 2 of the slab. To simplify the notation, we ignore the subscript n, which identifies the harmonic, provided, of course, we bring them back into use when we complete the harmonic series. On side 1 ($x = 0$), let $T = T_1$ and $q = q_1$, and on side 2 ($x = L$), let $T = T_2$ and $q = q_2$. Therefore, we can solve for the integration constants A and B either from the T or the q equation, and we obtain the heat flux on either side of the slab as it is related to the temperature on either side by

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = C \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \quad \text{(A-6)}$$

where $C = k/L$ is the heat conductance of the slab, and the α coefficients are

$$\alpha_{11} = \beta L \frac{\left(1 + e^{-2\beta L}\right)}{\left(1 - e^{-2\beta L}\right)} \quad \textit{and} \quad \alpha_{21} = 2\beta L \frac{e^{-\beta L}}{\left(1 - e^{-2\beta L}\right)} \quad \text{(A-7a)}$$

and

$$\alpha_{22} = -\alpha_{11} \quad \textit{and} \quad \alpha_{12} = -\alpha_{21}. \quad \text{(A-7a)}$$

The product βL appears repeatedly in the solution. It is convenient to express it in terms of γ, which is a real, dimensionless quantity that plays the role of the thermal inertia index for the nth harmonic of a homogeneous layer.

$$\gamma = \sqrt{\frac{n\,\omega\rho\,c}{2k}}\,L$$

and

$$\beta L = (1 + i)\gamma. \quad \text{(A-8)}$$

For this paper, assume that the outside surface temperature is known and that it follows a cosine time dependence with a period of 24 hours.

$$T_{outside} = T_0 + \Delta T cos(\omega t) \; (\omega = 2\pi/24 \text{ hrs.})$$

Therefore, we have only two possible value for n, namely, 0 and 1.

Option 1: The inside surface temperature is held at a fixed temperature.

$$T_{inside} = T_c$$

Then the heat flux delivered by the inside surface takes the form

$$q = C\,(T_{eff} - T_c),$$

where the effective outside temperature becomes

$$T_{eff} = T_0 + \Delta T\,|\alpha_{21}|\,\cos[\omega(t - t_{lag})] \quad \text{(A-9a)}$$

where

$$\alpha_{21} = |\alpha_{21}|e^{-i\phi}$$

$$\tan(\phi) = -\text{Im}(\alpha_{21})/\text{Re}(\alpha_{21})$$

and

$$t_{lag} = \phi/\omega\,.$$

$|\alpha_{21}|$ is the heat flux attenuation factor, and ϕ is the lag angle that yields the heat flux time lag.

Option 2: The inside surface temperature is free to float (for $n = 1$ and $q_2 = 0$)

$$T_{inside} = T_0 + \Delta T\,|\eta|\,\cos[\omega(t - t_{lag})] \quad \text{(A-9b)}$$

where

$\eta \quad = \alpha_{21}/\alpha_{22}$

$\eta \quad = |\eta|e^{-i\phi}$

$\tan(\phi) \quad = -\text{Im}(\eta)/\text{Re}(\eta)$ and

$t_{lag} \quad = \phi/\omega.$

$|\eta|$ is the amplitude attenuation factor, and ϕ is the lag angle that yields the temperature time lag. The text of the paper illustrates the thermal inertia effects through the amplitude attenuation factor and the inside surface temperature. Their explicit equations are

$$|\eta| = \frac{2e^{-\gamma}}{\sqrt{\left[1 + 2e^{-2\gamma}\cos(2\gamma) + e^{-4\gamma}\right]}} \quad \text{(A-10)}$$

$$T_{inside} = T_o + 2e^{-\gamma}\frac{\left[\cos(\gamma - \omega t) + e^{-2\gamma}\cos(\gamma + \omega t)\right]}{\left[1 + 2e^{-2\gamma}\cos(2\gamma) + e^{-4\gamma}\right]} \quad \text{(A-11)}$$

Clearly, they depend only on the γ value.