
Heat Loss to Building Foundations: An Analysis Based on a Theory of Dynamic Thermal Networks

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ABSTRACT

The calculation of heat flow in the ground, in particular the time-dependent heat loss through the foundation, poses particular problems due to three-dimensional flow and very long time scales. In this paper a new tool of analysis called “dynamic thermal networks” is presented briefly and used to analyze the time scales for the time-dependent heat loss for different types of foundations: a slab-on-ground, a cellar, and a split-level foundation.

The heat loss may be divided into a transmittive and an absorptive component. The transmittive and absorptive step-response fluxes and the corresponding weighting functions provide a precise description of the time scales of the heat flow to the ground. It is shown that the transmittive component has time scales of a decade and even more, while the absorptive component has a time scale of days only.

INTRODUCTION

The modeling of heat flow in the thermal envelope coupled to energy balances for buildings is a main task in building physics. The calculation of heat flow in the ground, in particular the time-dependent heat loss through the foundation, is the part that is most difficult to calculate, depending on three-dimensional heat flow and very long time scales.

In this paper a new tool of analysis called “dynamic thermal networks” is presented briefly and used to analyze the time scales for the time-dependent or dynamic heat loss for different types of foundations: a slab-on-ground, a cellar, and a split-level foundation.

The heat flux from the floor to the foundation depends on the current indoor and outdoor temperatures and on the preceding sequence of values for these boundary temperatures. A key question is how far back in time we have to account for these boundary temperatures. This paper presents a methodology to answer this question within the conceptual framework of dynamic thermal networks.

The theory of dynamic thermal networks for heat flow in solid regions such as ground, walls, and roof is formulated to fit into energy balance models in a handy way. This is briefly

presented, and a way to minimize the storage and use of preceding boundary temperatures is indicated.

There is a vast literature with many publications about the problem of the relations between boundary heat fluxes and boundary temperatures. The basic mathematics is found in Carslaw and Jaeger (1959). The response factor method, introduced by Mitalas in the late 1960s, is summarized together with a large list of references in ASHRAE (1997). Other important papers are Davies (1997), where the analysis is based on ramp responses, and Kossecka and Kosny (1998), where so-called thermal structure factors for composite walls are used.

DYNAMIC THERMAL NETWORKS

We consider heat conduction in a solid volume with two boundary surfaces with the temperatures $T_1(t)$ and $T_2(t)$, respectively. An example is the thermal envelope of a building with roof, walls, foundation, and surrounding ground between indoor and outdoor boundary temperatures. The theory, which is briefly presented here without any derivations, is discussed in more detail in Claesson (2003, 2002).

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Basic Relations

The time-dependent or *dynamic* relations between boundary heat fluxes and boundary (air) temperatures may be written in the following way:

$$Q_1(t) = K_1 \cdot [T_1(t) - \bar{T}_{1a}(t)] + K_{12} \cdot [\bar{T}_{1a}(t) - \bar{T}_{2a}(t)] \quad (1)$$

$$Q_2(t) = K_2 \cdot [T_2(t) - \bar{T}_{2a}(t)] + K_{12} \cdot [\bar{T}_{2a}(t) - \bar{T}_{1a}(t)] \quad (2)$$

Here, K_{12} (W/K) is the (steady-state) thermal conductance between the two surfaces. The factor K_1 (W/K) is the *surface thermal conductance* for surface 1. It is equal to the surface area times the heat transfer coefficient between air and solid surface. The conductances are multiplied by temperature differences involving the boundary temperatures at the considered time t and at preceding times $t - \tau$, $\tau > 0$, as described below.

The second right-hand term in Equations 1 and 2 is the same except for the sign. It may be called the *transmittive* heat flux between the surfaces. The first right-hand term may be called the *absorptive* heat flux for surface 1 and surface 2, respectively. This separation of the dynamic boundary fluxes $Q_1(t)$ and $Q_2(t)$ into an absorptive component for each surface and a common transmittive component is quite intricate. It is discussed further in Claesson (2003). The basic equations 1 and 2 are represented graphically as a *dynamic thermal network* in Figure 1.

Transmittive and Absorptive Mean Temperatures

In Equations 1 and 2, we use mean values of boundary temperatures backward in time, which are indicated by a bar: \bar{T} . The transmittive mean temperatures for the two boundary temperatures are

$$\bar{T}_{1a}(t) = \int_0^{\infty} \kappa_{12}(\tau) \cdot T_1(t-\tau) d\tau \quad \bar{T}_{2a}(t) = \int_0^{\infty} \kappa_{12}(\tau) \cdot T_2(t-\tau) d\tau \quad (3)$$

Here, $\kappa_{12}\tau$ is the transmittive weighting function defined below. There is an absorptive weighting function for each boundary surface and a corresponding absorptive mean temperature:

$$\bar{T}_{1a}(t) = \int_0^{\infty} \kappa_{1a}(\tau) \cdot T_1(t-\tau) d\tau \quad \bar{T}_{2a}(t) = \int_0^{\infty} \kappa_{2a}(\tau) \cdot T_2(t-\tau) d\tau \quad (4)$$

The integrals are extended backward in time, until the weighting function is sufficiently small so that the rest of the integral (to infinity) is negligible. These backward integrals become sums in the discrete formulation. In the graphical representation of Figure 1, these backward averages are indicated by summation sign Σ .

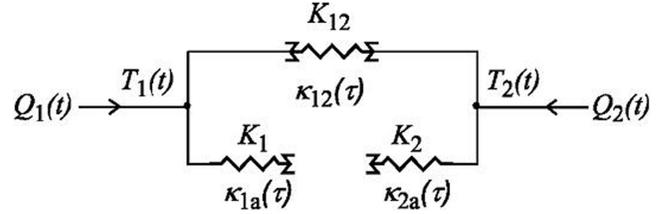


Figure 1 Dynamic thermal network to represent the relations in Equations 1 and 2.

The transmittive component is represented by the conventional resistance symbol with the thermal conductance K_{12} written above the component. To the resistance or conductance symbol, we add summation signs on both sides. The signs signify that we take an average of the node temperatures backward in time according to Equation 3. The left-hand summation sign is reversed to indicate the symmetric character of the flux and that summation concerns the values at the left-hand node. The weighting function $\kappa_{12}\tau$ may be written below the resistance symbol.

The two absorptive components are represented by resistance signs with the surface thermal conductance (K_1 and K_2) written above. A summation sign is added at the *free* end after the surface conductance to indicate averages backward in time according to Equation 4. There is not any summation sign on the node side since the *present* node temperature $T_1(\tau)$ is to be used at this side in accordance with the first right-hand terms of Equations 1 and 2.

Step-Response Heat Fluxes

The weighting functions are derivatives of certain basic step-response heat fluxes as described below. There are two step-response problems with a unit temperature step at surfaces 1 and 2, respectively. We use τ and not t as the time variable in step-response problems. Then the weighting functions have τ as an independent variable, while the backward temperatures are taken for $t - \tau$. In the first step-response problem, the temperature outside surface 1 is raised from 0 to 1 at zero time and kept at this value for $\tau > 0$. All temperatures are zero for $\tau < 0$. There is an *admittive* heat flux $Q_{11}(\tau)$ into surface 1 and a cross-flux or transmittive flux $Q_{12}(\tau)$ out through surface 2. The corresponding step-response fluxes for a step at surface 2 are $Q_{22}(\tau)$ and $Q_{21}(\tau)$. The cross-fluxes are equal due to a general symmetry principle. We have three basic step-response fluxes:

$$Q_{11}(\tau) \quad Q_{22}(\tau) \quad Q_{21}(\tau) = Q_{12}(\tau) \quad (5)$$

The general character of the two admittive fluxes and the transmittive flux are shown in Figure 2. They all approach the steady-state flux K_{12} as τ tends to infinity. The admittive fluxes start from the surfaces conductances K_1 and K_2 , respectively,

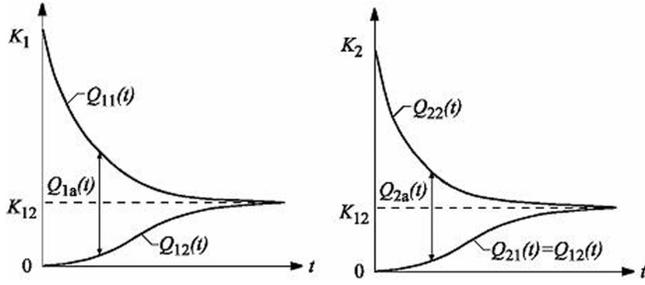


Figure 2 Character of the basic step-response heat fluxes.

and decrease to the steady-state value. The transmittive flux starts from zero and increases steadily to steady state.

The absorptive step-response fluxes for the two unit steps are given by the absorbed heat, i.e., the difference between the admittive and transmittive fluxes. We have

$$Q_{1a}(\tau) = Q_{11}(\tau) - Q_{12}(\tau) \quad Q_{2a}(\tau) = Q_{22}(\tau) - Q_{21}(\tau) . \quad (6)$$

These two fluxes start from the surface conductance and decrease steadily to zero for a long time as shown in Figure 2.

Weighting Functions

The weighting functions are time derivatives of the step-response functions. The transmittive weighting function, $\kappa_{12}(\tau)$, is the derivative of $Q_{12}(\tau)$ multiplied by $1/K_{12}$, so that the integral (Equation 9) becomes equal to 1. We have

$$\kappa_{12}(\tau) = \frac{1}{K_{12}} \cdot \frac{dQ_{12}}{d\tau} . \quad (7)$$

The adsorptive weighting function, $\kappa_{1a}(\tau)$, is given by the factor $-1/K_1$ multiplied by the time derivative of $Q_{1a}(\tau)$. We have for the two absorptive weighting functions,

$$\kappa_{1a}(\tau) = -\frac{1}{K_1} \cdot \frac{dQ_{1a}}{d\tau} \quad \kappa_{2a}(\tau) = -\frac{1}{K_2} \cdot \frac{dQ_{2a}}{d\tau} \quad (8)$$

The character of the weighting functions is shown in Figure 9. They are all non-negative, and the integrals are all equal to 1 due to the factor involving conductivity:

$$\int_0^{\infty} \kappa_1(\tau) = 1, \quad \kappa_1(\tau) \geq 0, \quad I = 12, 1a, 2a \quad (9)$$

The transmittive weighting functions have a bell-shaped form (Figure 9, top and bottom). They are zero during a first period (around an hour) until the temperature step is felt at the other side. They increase to a maximum at a certain time, and then they slowly decrease to zero for a long time. The absorptive weighting functions decrease monotonously from a very high value (Figure 9, middle).

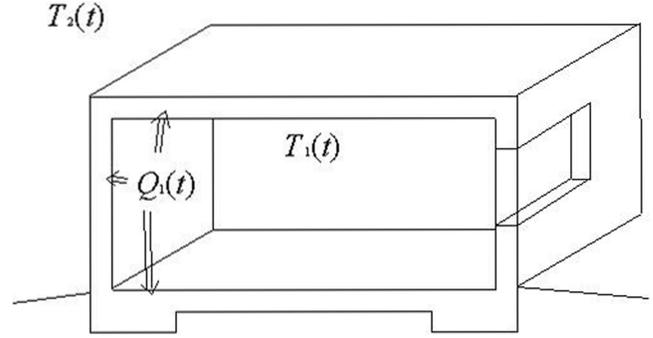


Figure 3 The heat loss from a building.

All equations in this brief outline of the theory of dynamic thermal networks are mathematically exact, or as exact as the numerically calculated step-response functions and their derivatives, the weighting functions. The heat flow problem must be linear so that superposition is applicable.

The heat flow problems considered above have two boundary surfaces with different temperatures. The heat flow problem may in a more general case have N boundary surfaces with different temperatures. Then the dynamic thermal network has N nodes. Each node or boundary surface has an absorptive component. There are transmittive components between all pairs of nodes just as in the corresponding steady-state network (see Claesson 2003). The theory may also be applied for a subsurface of the indoor area. An example is Wentzel (2002), where the heat loss dynamics of different parts of the floor is analyzed. Another application for composite walls is presented in Wentzel and Claesson (2003), where the analyses are extended to the annual heating cost for a variable energy price.

A BUILDING WITH THREE TYPES OF FOUNDATION

We will consider a particular building with heat flow through ceiling, walls with window openings, and floor. Three types of foundation are studied: slab-on-ground, cellar, and split-level. The outdoor temperature, $T_2(t)$, follows a natural climate, while the indoor temperature $T_1(t)$ depends on the heating conditions.

Basic Formulas for the Heat Loss

Our main interest is the heat loss $Q_1(t)$ from the building through ceiling, walls, and floor (see Figure 3). We are not interested in the heat flux through the outdoor surface. We need Equation 1 only:

$$Q_1(t) = K_1 \cdot [T_1(t) - \bar{T}_{1a}(t)] + K_{12} \cdot [\bar{T}_{1i}(t) - \bar{T}_{2i}(t)] \quad (10)$$

The thermal conductance for the whole building is K_{12} (W/K). The factor K_1 (W/K) is the surface thermal conductance at the building's inside. It is equal to the surface area A_1 times the surface heat transfer coefficient α_1 : $K_1 = A_1 \cdot \alpha_1$.

The right-hand terms of the dynamic relations involve thermal conductances multiplied by temperature differences. The temperatures that are used are the present indoor temperature $T_1(t)$ and average temperatures backward in time. The temperature averages are given by Equations 2 and 4. We have:

$$\begin{aligned} \bar{T}_{1i}(t) - \bar{T}_{2i}(t) &= \int_0^{\infty} \kappa_{12}(\tau) \cdot [T_1(t-\tau) - T_2(t-\tau)] d\tau \\ \bar{T}_{1a}(t) &= \int_0^{\infty} \kappa_{1a}(\tau) \cdot T_1(t-\tau) d\tau. \end{aligned} \quad (11)$$

Here, τ assumes values from zero to infinity or sufficiently far back in time. The transmittive weighting function $\kappa_{12}(\tau)$ and the absorptive weighting function $\kappa_{1a}(\tau)$ are discussed below.

Figure 4 shows the dynamic thermal network according to Equation 10 for the building's heat loss. We have an absorptive and a transmittive component.

The weighting functions $\kappa_{1a}(\tau)$ and $\kappa_{12}(\tau)$ in Equation 11 are obtained from a basic step-response solution. We have from Equations 7 and 8,

$$\kappa_{12}(\tau) = \frac{1}{K_{12}} \cdot \frac{dQ_{12}}{d\tau} \quad \kappa_{1a}(\tau) = \frac{-1}{K_1} \cdot \frac{dQ_{1a}}{d\tau} \quad (12)$$

Here, we need the basic step-response heat fluxes for a unit step at the inside surface 1:

$$Q_{11}(\tau) \quad Q_{12}(\tau) \quad Q_{1a}(\tau) = Q_{11}(\tau) - Q_{12}(\tau) \quad (13)$$

See Figure 5. The admittive heat flux $Q_{11}(\tau)$ at the inside boundary and the transmittive flux $Q_{12}(\tau)$ out through the outside boundary have to be determined from the calculation of the step-response problem. The absorptive heat flux, $Q_{1a}(\tau)$, which is used to calculate the absorptive weighting function, is the difference between the admittive and transmittive flux for a unit step at surface 1.

Building and Foundations

A building with three different foundations is studied. The foundations are chosen to represent three common types in Sweden: slab-on-ground, cellar, and a split-level. The building's floor area is 12.5 m × 8 m.

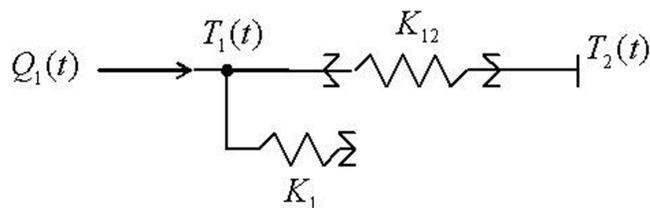


Figure 4 Dynamic thermal network for the heat loss.

The building above the foundation has 0.2-m-thick light expanded clay aggregate (Leca) walls with an external insulation layer of 0.1 m. The roof is made of light concrete with an external insulation layer of 0.1 m. The windows have a total area of 24 m².

The studied **slab-on-ground foundation** consists of a 0.1-m-thick concrete slab on a 0.2-m-thick insulation layer. The edge beams are 0.2 m thick and 0.3 m wide. The exterior walls, which are placed on the slab, are made of 0.2-m-thick light expanded clay aggregate with a 0.1-m-thick exterior insulation layer. (see Figure 6, top).

The studied **cellar foundation** consists of 0.2-m-thick cellar walls of concrete with a 0.1-m-thick exterior insulation layer. The walls are 2.4 m high. The cellar floor consists of a 0.1-m-thick concrete slab on a 0.2-m-thick insulation layer. The edge beams are 0.2 meter thick and are 0.3 m wide. The floor between the cellar and the upper building consist of 0.15 m concrete. The upper exterior walls are the same as for the slab-on-ground foundation. The ground level lies 0.5 m below the cellar ceiling (see Figure 6, bottom, left).

The **split-level foundation** has the same construction as the cellar above. The highest ground level is 0.5 m below the ceiling and the lower ground level is at the same level as the floor (see Figure 6, bottom, right).

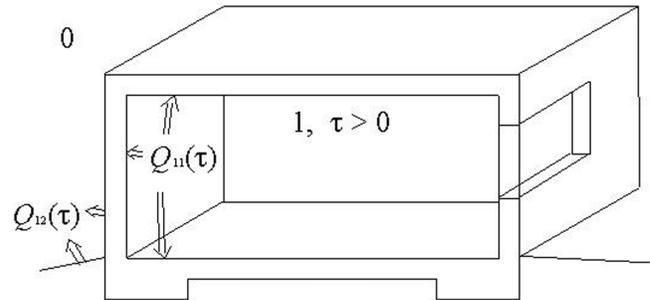


Figure 5 Basic step-response heat fluxes for a unit step at the inside boundary surface.

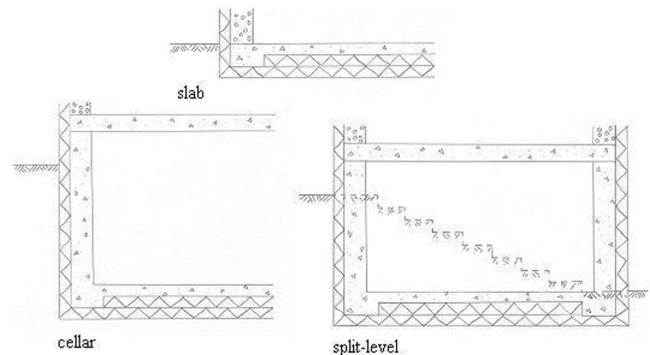


Figure 6 Three types of foundation: slab-on-ground, cellar, and split-level foundation.

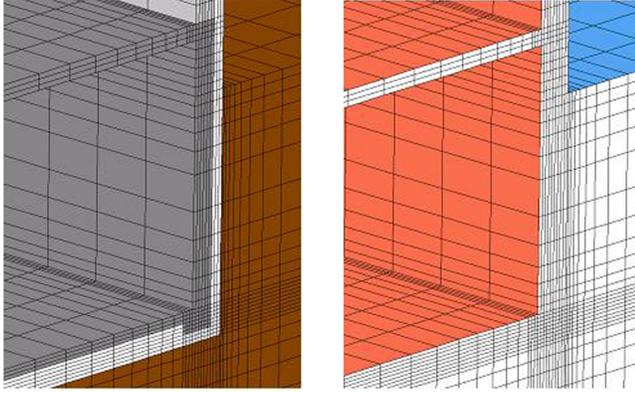


Figure 7 Part of the mesh in Heat 3. Left: The different materials have different shades of gray. Right: The different boundary conditions have different shades of gray.

STEP-RESPONSE AND WEIGHTING FUNCTIONS

To calculate the dynamic heat loss (Equation 10) through the building's foundation, we use the foundation's transmittive and absorptive response functions. These functions give information about how preceding temperature variations influence on the current heat loss.

Numerical Solution

The step-response thermal problem is solved with the three-dimensional numerical code Heat 3 (Blomberg 1998). The mesh involves up to $50 \times 50 \times 50$ (=125,000) nodes. The studied foundations are symmetric, and only one-fourth of them is considered in the numerical program, except for the split-level case where one-half of the foundation is considered. The ground depth is five times the building's length. The European Standards EN ISO 10211-1 (1995) and EN ISO 10211-2 (2001) recommend a ground depth of 2.5 times the building's width, but greater depth gives better precision.

Figure 7 shows parts of the mesh around the cellar foundation. The left-hand figure illustrates the different material layers. The white areas show the insulation outside the concrete. The dark gray area outside the insulation is the soil. The light gray part in the top of the figure is the Leca wall. The right-hand figure shows the boundary conditions. The white part indicates total insulation due to symmetry. The gray area at the cellar's inside (left) has a surface heat transfer coefficient $\alpha_1=7.7$ W/(m²K) and a temperature $T_1=1$ for $\tau > 0$. The other gray area outside the building (right) has a surface heat transfer coefficient $\alpha_2=25$ W/(m²K) and a temperature $T_2=0$.

Analytical Expressions for Longer and Shorter Times

The step-response fluxes for the heat flow through the ground have a very long tail as they approach steady state. It

Table 1. The Foundations' Thermal Conductances and U-Factors

Construction	K_{12} (W/K)	Area (m ²)	U-Factor (W/m ² K)
Walls and ceiling	40.13	184.3	0.218
Slab-on-ground	18.27	100.0	0.183
Cellar	45.07	198.4	0.227
Split-level	48.57	198.4	0.245

takes some time to calculate this part from, say, 5 to 100 years in a numerical model. To handle this problem, we have combined the numerical solution with asymptotic analytical expressions (not yet published). The beginning, the first four to five years, of the response fluxes is calculated numerically. Then analytical expressions for $\tau > \tau_{la}$ (large) are fitted to the numerical results. For the transmittive response and admittive fluxes, we use these expressions:

$$Q_{12}(\tau) = K_{12} - (K_{12} - Q_{12}(\tau_{1a})) \cdot \sqrt{\frac{\tau_{1a}}{\tau}} \cdot \frac{1 - \beta_1 \cdot (\tau_{1a}/\tau) + \beta_2 \cdot (\tau_{1a}/\tau)^2}{1 - \beta_1 + \beta_2} \quad (14)$$

$$Q_{11}(\tau) = K_{12} + (Q_{11}(\tau_{1a}) - K_{12}) \cdot \left(\frac{\tau_{1a}}{\tau}\right)^{1.5} \cdot \frac{1 - \beta \cdot (\tau_{1a}/\tau)}{1 - \beta} \quad \text{for } \tau > \tau_{1a} \quad (15)$$

In the considered cases, t_{la} is five years. The response fluxes $Q_{12}(t_{la})$ and $Q_{11}(t_{la})$ are the last values after five years in the numerical calculation. The coefficients β_1 , β_2 , and β are chosen so that the expressions fit the numerical solutions as well as possible during a year's time before t_{la} . The difference between the above expression and a continued numerical calculation after $\tau = \tau_{la}$ is very small.

The admittive response flux decreases rapidly in the beginning and the numerical solution, in particular the derivative to get the absorptive weighting function, becomes inaccurate. Therefore, an analytical expression is used during the first period. We use the solution for a semi-infinite concrete slab with a surface heat transfer coefficient α_1 with a unit step at the boundary. During a first period, the admittive response flux becomes (Carslaw and Jaeger 1959)

$$Q_{11}(\tau) = A \cdot \alpha_1 \cdot e^{a \cdot \tau/d^2} \cdot \operatorname{erfc}(\sqrt{a \cdot \tau}/d) \quad \text{for } 0 < \tau < \tau_{sm} \quad (16)$$

Here, $\operatorname{erfc}(s)$ is the complementary error function, λ is the thermal conductivity (W/(mK)), ρc the volumetric heat capacity (J/m³), $d = \lambda/\alpha$ an equivalent length of the surface layer, and $\alpha = \lambda/(\rho c)$ the thermal diffusivity (m²/s) of the concrete indoor surface. The time τ_{sm} (small) is chosen as one hour.

Response Functions for the Three Foundations

Table 1 gives the thermal conductance, indoor area, and U-factor for the building above ground (walls and ceiling) and

for each foundation. The transmittive response functions, $Q_{12}(\tau)$, for the three foundations and for the building above the foundation are shown in Figure 8 (top). The curves are normalized to one by division with the thermal conductance K_{12} . The left-hand graphs show the first week (168 hours). The building above ground reaches steady state after, say, four days. The response is very different for the foundations. We see that only 20% to 50% of the steady-state value is reached during the first week. In the right-hand graphs, the time axis is shown in log-scale in order to represent both the long tail and the first period properly.

The transmittive response fluxes in Figure 8, top, are virtually zero before one hour ($\tau = 1$). The transmittive response functions have reached 90% of their steady-state values after 18 years, 17 years, and 7 years for slab, cellar, and split-level, respectively.

The absorptive response functions $Q_{1a}(\tau)$ are shown in Figure 8, bottom. All foundations have the same indoor surface (concrete), which means that the normalized response functions are virtually identical during the first hour in accordance with Equation 16.

Figure 9 shows the weighting functions corresponding to the response functions of Figure 8. The maximum of the transmittive weighting functions is the time backward when previous temperature variations have the largest influence on the current heat flow. We see in Figure 9, top, that the maximum values for the different foundations occurs after 5.3 (split-level), 5.7 (cellar), and 22 (slab) hours. The values of the weighting functions are very small after the first week, but the tails of the transmittive weighting functions are very long, and the integrals over later parts involve most of the influence as we have seen above. The bottom curve for the weighting function of the building above ground does not have this long tail. All influence after, say, $\tau = 4$ days is negligible.

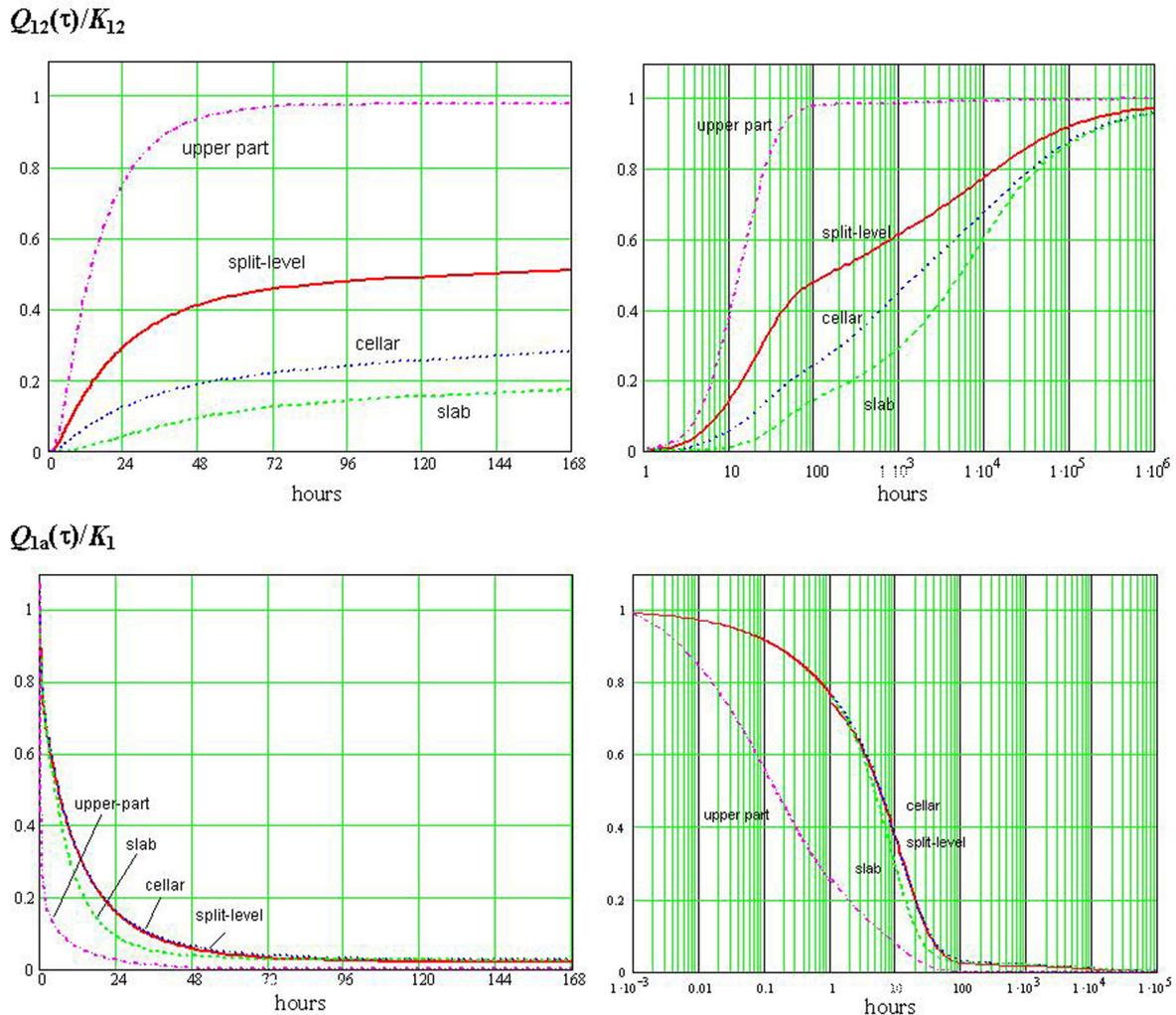


Figure 8 Transmittive (top) and absorptive (bottom) step-response functions for the building above ground and for the slab, cellar, and split-level foundations. Left: The first week. Right: Logarithmic time from 1 hour to 10^6 hours or 114 years.

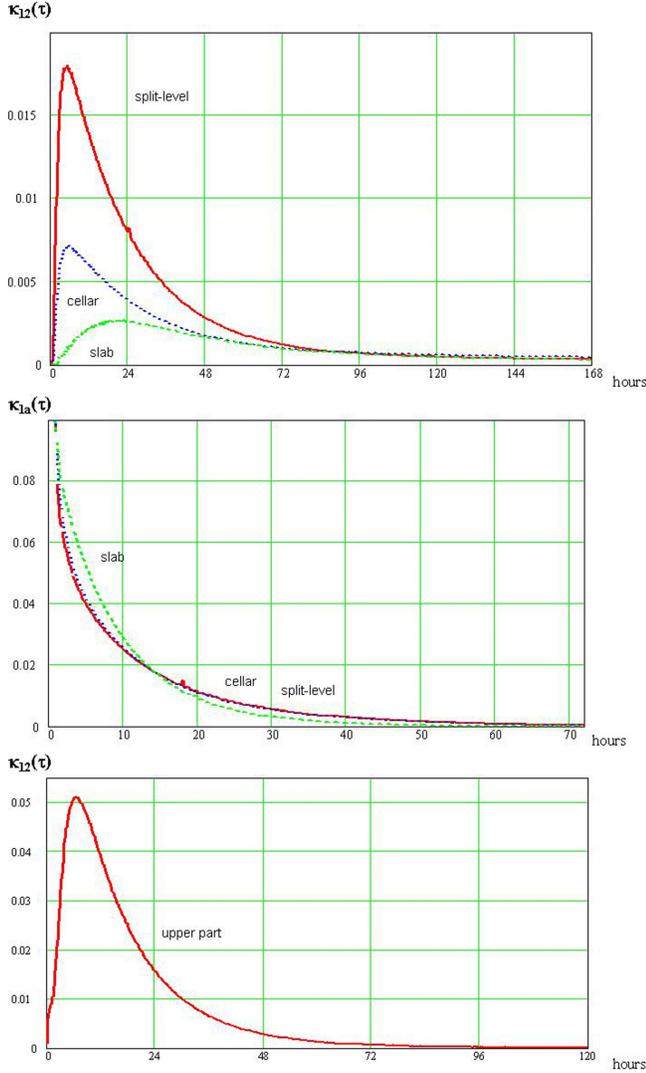


Figure 9 Transmittive (top) and absorptive (middle) weighting functions for the slab, cellar, and split-level foundations. Bottom: Transmittive weighting function for the building above ground.

RELATIVE INFLUENCE OF BACKWARD TIME INTERVALS

The weighting functions $\kappa(\tau)$ give the relative influence of boundary temperatures $T(t-\tau)$ backward in time from the considered time t , ($\tau > 0$). A wall, or a corner in two or three dimensions, will have a response time scale of up to a few days. After this time, the weighting factor tends to zero exponentially. The integrals (Equations 3 and 4) in τ may be terminated after a few days or a week. The basic reason for this limited time scale is that the heat flow region has limited extensions.

The heat flow region in the ground below and around the building has very large extensions downward and outward. This leads to much longer time scales. The tail of the weighting must, in a precise description, be accounted for during several years.

In order to analyze this problem, we divide the backward time $0 < \tau < \infty$ in J intervals:

$$0 = \tau_0 < \tau_1 < \dots < \tau_j < \dots < \tau_J = \infty \quad (17)$$

The integral of the weighting function over interval j , $\bar{\kappa}_{12,j}$, gives the relative importance of the temperatures $T(t-\tau)$ in that interval $\tau_{j-1} < \tau < \tau_j$ on the transmittive mean temperature. Using Equation 7, we have

$$\bar{\kappa}_{12,j} = \int_{\tau_{j-1}}^{\tau_j} \kappa_{12}(\tau) d\tau = \frac{Q_{12}(\tau_j) - Q_{12}(\tau_{j-1})}{K_{12}} \quad j = 1, \dots, J. \quad (18)$$

This means that the *increase* of $Q_{12}(\tau)/K_{12}$ over any time interval represents the relative importance of that particular interval.

The mean temperature in interval j (with the weighting function $\kappa_{12}(\tau)/\bar{\kappa}_{12,j}$) is

$$\bar{T}_{1t,j} = \frac{1}{\bar{\kappa}_{12,j}} \cdot \int_{\tau_{j-1}}^{\tau_j} \kappa_{12}(\tau) \cdot T_1(t-\tau) d\tau \quad j = 1, \dots, J. \quad (19)$$

The transmittive mean temperature is then

$$\bar{T}_{1t}(t) = \sum_{j=1}^J \int_{\tau_{j-1}}^{\tau_j} \kappa_{12}(\tau) \cdot T_1(t-\tau) d\tau = \sum_{j=1}^J \frac{Q_{12}(\tau_j) - Q_{12}(\tau_{j-1})}{K_{12}} \cdot \bar{T}_{1t,j}. \quad (20)$$

This mathematically exact formula shows very clearly the influence from any time interval. There is a corresponding formula for $\bar{T}_{2t}(t)$. The formulas for the absorptive mean temperature are of the same type as the ones above except for the minus sign in Equation 8. We have

$$\bar{T}_{1a}(t) = \sum_{j=1}^J \frac{Q_{1a}(\tau_{j-1}) - Q_{1a}(\tau_j)}{K_1} \cdot \bar{T}_{1a,j}. \quad (21)$$

This means again that the *decrease* of $Q_{1a}(\tau)/K_1$ over any time interval represents the relative importance of that interval.

We may choose the times τ_j so that the J intervals all have the same relative importance, i.e., $1/J$. We get a set of intervals for the transmittive and for the absorptive mean temperatures:

$$\frac{Q_{12}(\tau_{j,t})}{K_{12}} = \frac{j}{J}, \quad \frac{Q_{1a}(\tau_{j,a})}{K_1} = 1 - \frac{j}{J}, \quad j = 0, 1, \dots, J \quad (22)$$

Then we have

$$\bar{T}_{1t}(t) = \frac{1}{J} \cdot \sum_{j=1}^J \bar{T}_{1t,j}, \quad \bar{T}_{1a}(t) = \frac{1}{J} \cdot \sum_{j=1}^J \bar{T}_{1a,j}. \quad (23)$$

Figure 10 shows these time intervals for $J=10$ for the slab-on-ground foundation. We see that the first 10% of the transmittive heat loss is due to outdoor temperatures from 0 (rather than 3) to 52 hours. The last 10% is due to temperatures more than 163,000 hours = 17 years backward. The limits for all ten intervals are

0, 52 h, 12 d, 44 d, 108 d, 217 d, 1.1 y, 2.1 y, 4.8 y, 17 y, ∞

We see that the transmittive heat loss involves preceding temperatures during more than a decade.

The behavior for the absorptive heat flux is very different. The first 10% of the absorptive heat loss is due to the preceding indoor temperatures from 0 to 0.15 hours (9 minutes). The last 10% is due to temperatures more than 23 hours backward. The limits for all ten intervals are

0, 0.15 h, 0.74 h, 1.8 h, 3.2 h, 4.9 h, 7.0 h, 9.9 h, 14 h, 23 h, ∞

We see that the absorptive heat loss involves preceding temperatures during a few days only.

CALCULATION OF THE HEAT LOSS

In a numerical solution, we must use a discrete approximation of the backward temperatures and the integrals (Equation 11). Our approach is reported in Claesson (2003). We use a time step h , and an indoor and an outdoor temperature at each time step. We assume a linear variation of the temperatures during each time step.

The heat flux is obtained for each time step from the corresponding discrete form of Equation 10. The integrals (Equation 11) are replaced by a sum of backward temperatures. The transmittive and absorptive weighting functions are replaced by weighting factors, which are obtained directly from integrals of the response functions.

A particular problem is the long time scale for the heat loss through the foundation. We have seen that we should account for boundary temperatures more than ten years back in time for the transmittive heat loss. With a time step of one hour, this means that we must consider more than 10^5 temperatures at preceding time steps. But the variation of the weighting factors become smaller and smaller as the backward time increases. This means that we only need average backward temperature values for increasing time intervals. This is clearly illustrated in Equation 20. But these averages should be as correct as possible.

In order to minimize the storage of preceding temperatures, we proceed in the following way. We use each preceding time step until the weighting factor has decreased well below half of its maximum value. Then we use the mean value of two consecutive temperatures. When the weighting factor is well below one-fourth of its maximum, we use the average of

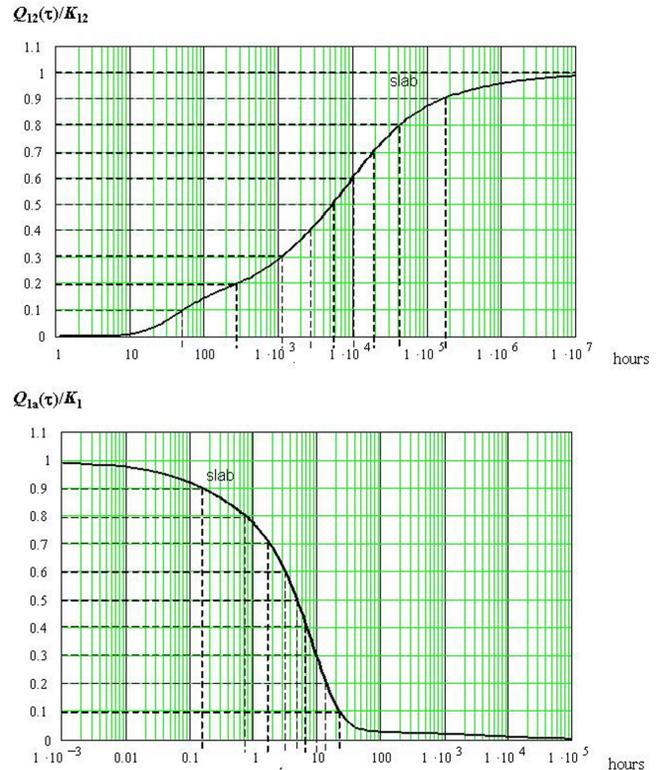


Figure 10 Time intervals of equal influence (10%) on the heat fluxes for the slab-on-grade foundation. Top: Transmittive heat flux. Bottom: Absorptive heat flux.

$2^2 = 4$ consecutive temperatures. We double the intervals in this systematic way. The number of preceding temperatures that we have to store and use decreases drastically; in a typical case from 10^5 - 10^6 to 100. The calculation of the heat loss during a year may involve 10^4 time steps, where each time step requires a few hundred additions and multiplications. The computer time requirement is then a few seconds only. It should be noted that we retain then full three-dimensional accuracy via the step-response fluxes. The method requires that the two step-response fluxes be calculated separately and given as an input.

The original sequence of preceding temperatures is shifted one step backward for each time step. In the condensed representation, with mean values of 2, 4, 8, etc., original temperatures, this backward shift must be done in a particular way in order to avoid what we call time-shift dispersion. This problem and our solution will be reported in another publication.

HEAT BALANCE MODELS

The above theory of dynamic thermal networks is particularly designed to incorporate dynamic heat flow in solid components in heat balance models. We will illustrate this with a simple example.

We consider a building with an indoor temperature $T_1(t)$ and a given outdoor temperature $T_2(t)$. There is a given heat input $Q_h(t)$ (W) and a given heat gain from solar radiation $Q_r(t)$. The heat input due to ventilation is determined from a prescribed time-dependent ventilation conductance $K_v(t)$ (W/K). The sum of these heat inputs is equal to the heat flux $Q_1(t)$ at the inside boundary of the building. Using Equation 10, we have the heat balance

$$\begin{aligned} Q_1(t) &= K_1 \cdot [T_1(t) - \bar{T}_{1a}(t)] + K_{12} \cdot [\bar{T}_{1r}(t) - \bar{T}_{2r}(t)] \\ &= Q_h(t) + Q_r(t) + K_v(t) \cdot [T_2(t) - T_1(t)]. \end{aligned} \quad (24)$$

The corresponding dynamic network is shown in Figure 11.

The indoor temperature $T_1(t)$ is readily solved from the heat balance (Equation 24). We have

$$\begin{aligned} T_1(t) &= \frac{Q_h(t) + Q_r(t) + K_v(t) \cdot T_2(t) + Q_A}{K_1 + K_v(t)} \\ Q_A &= K_1 \cdot \bar{T}_{1a}(t) + K_{12} \cdot [\bar{T}_{2r}(t) - \bar{T}_{1r}(t)]. \end{aligned} \quad (25)$$

The indoor temperature is equal to the heat influx to the indoor node of Figure 11 from heating, solar radiation, ventilation with the outdoor temperature, absorptive influx with the absorptive backward mean $\bar{T}_{1a}(t)$, and a transmissive influx with the temperature difference $\bar{T}_{2r}(t) - \bar{T}_{1r}(t)$, all divided by the sum of conductances $K_1 + K_v(t)$ to the node.

The above simple case with prescribed heating may readily be extended to more complex relations for $Q_h(t)$ and to more complex networks. The dynamic thermal networks for heat flow in solid regions may in the above way be added and incorporated within the conceptual framework of steady-state networks. The level of complexity is higher than that of the corresponding steady-state network. We have to add an absorptive component at each node and store and use preceding boundary temperatures for the absorptive and transmissive components.

CONCLUSION

The calculation methodology “Dynamic Thermal Network” is described and used to illustrate the time-dependent or dynamic heat loss for three different foundations, slab, cellar, and split-level. The dynamic heat loss consists of two parts: absorptive and transmissive. The theory of dynamic thermal networks is based on step-response functions and their derivatives (weighting functions).

The response functions for the three-dimensional foundations are calculated by a combination of a numerical solution and analytical expressions for short and long times. This combination saves computer time.

The response functions show the different thermal behavior of the foundations. The absorptive heat loss is similar for all foundations because they have the same inside material, concrete (Figure 8, bottom). The absorptive component has a time scale of days only. The transmissive response functions show that the foundations have reached 90% of their steady-

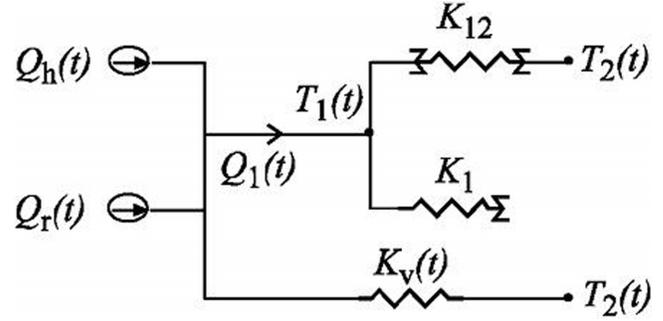


Figure 11 Dynamic thermal network for the indoor temperature with heating, solar radiation, and ventilation.

state value after 18, 17, and 7 years for the slab, cellar, and split-level foundations, respectively. This means that the last 10% of the heat flux is determined by outdoor temperatures more than a decade ago.

The weighting functions $\kappa(\tau)$ give the relative influence of boundary temperatures $T(t-\tau)$ backward in time from the considered time t , ($\tau > 0$). The heat flow region in the ground below and around the building has very large extensions downward and outward. This leads to the long time scales. The tail of the weighting must, in a precise description, be accounted for during several years. A way to illustrate this is to divide the backward time $0 < \tau < \infty$ in, say, 10 intervals. Each interval accounts for 10% of the backward influence. For the slab foundation, the transmissive heat loss involves preceding temperatures during more than a decade. The behavior for the *absorptive heat flux* is very different. The first 10% of the absorptive heat loss is due to preceding indoor temperatures from 0 to 0.15 hours (9 minutes). The last 10% is due to temperatures more than 23 hours backward.

NOMENCLATURE

A	=	area (m^2)
α	=	surface heat transfer coefficient (W/m^2K)
K_{12}	=	thermal conductance (W/K) between boundary surfaces 1 and 2
K_1	=	thermal surface conductance (W/K) of surfaces 1 ($=A_1\alpha_1$)
κ/τ	=	weighting function ($1/s$)
$\kappa_{12}(\tau)$	=	transmissive weighting function ($1/s$) between 1 and 2
$\kappa_{1a}(\tau)$	=	absorptive weighting function ($1/s$) for surface 1
$Q(t)$	=	heat loss (W)
$Q_{11}(\tau)$	=	admittive step-response function (W/K)
$Q_{12}(\tau)$	=	transmissive step-response function between 1 and 2 (W/K)
$Q_{1a}(\tau)$	=	absorptive step-response function for surface 1 (W/K)

$T(t)$	=	temperature (°C)
$T_{1t}(\tau)$	=	transmittive mean boundary temperature for surface 1 (°C)
$T_{1a}(\tau)$	=	absorptive mean boundary temperature for surface 1 (°C)
t	=	time (s)
τ	=	preceding time (s)

Indices

1	=	surface 1
2	=	surface 2
12	=	between surface 1 and 2 (<i>transmittive</i>)
11	=	step at surface 1, flux at surface 1 (<i>admittive</i>)
t	=	transmittive
a	=	absorptive

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