# Probabilistic Analysis of Indoor Surface Hygrothermal Conditions Accounting for Thermal and Hygric Memory of the Building Component

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## ABSTRACT

*A new approach to address the memory effects of the building component is demonstrated in the paper. As an output of the analysis, the probability distribution for the surface temperature and relative humidity is given, which is interesting for the determination of both thermal comfort and the risk of mold formation. Three different wall constructions are used in the example.*

*The paper presents two calculation procedures. The first procedure is the probabilistic approximation based on the wall surface dynamic response (DTN model) to the external and internal climatic load. The probability distributions of moisture and thermal parameters, as well as a reliability analysis of the moisture performance of the walls, are carried out using FORM (first-order reliability method). The results are then compared with the results obtained from the second procedure. In this, a transient one-dimensional HAM model is used for the calculation of the hygrothermal surface conditions, followed by a statistical analysis of the output.*

## INTRODUCTION

Mold growth in the building envelope is an unwanted event for every house owner. To predict the performance failure of a building envelope exposed to certain environmental load is a complex task involving application of reliability methods. Progress in the development of engineering reliability and/or risk analysis has been substantial over the past 20 years. Researchers have gained understanding of uncertainties in the design of structures and systems. In the field of moisture performance, a critical layer for moisture content and its behavior has been studied and identified as the inner layer of a wall structure (Kurkinen and Hagentoft 2002). However, the applications of reliability/risk analysis have been carried out mainly within the area of structural safety (hence, also the terminology is mostly borrowed from this field), while serviceability problems concerning moisture, heat, or ventilation performance, are seldom addressed. As an attempt to fill this gap, a probabilistic model for reliability analysis in building physics design based on limit state approach has been developed (Pietrzyk et al. 2003, 2004) and has been applied to the moisture problem in buildings.

Reliability analysis can be carried out using different types of probabilistic models (random variable models, random process models, random field models) depending on the level of description of uncertainties in a physical model. The first-order reliability method FORM is used as a tool for the reliability analysis. Fundamentals of reliability analysis together with a description of FORM can be found in Ditlevsen and Madsen (1996), Haldar and Mahadevan (2000), and Nowak and Collins (2000). Applying FORM offers savings in computer processing time compared to other techniques, for example, Monte Carlo simulations.

A random variable model is usually used for steady-state problems. The moisture problems in buildings are often described by time-dependent models. Nevertheless, the problem can be reformulated in such a way that the random variable approach can still be applied. In this paper a new procedure called "dynamic thermal network" (DTN) (Claesson 2001, 2003), which addresses memory effects of the

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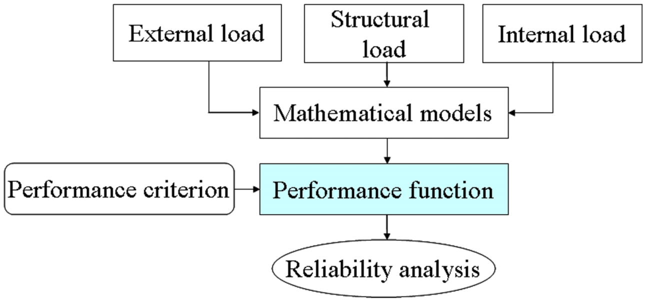


***Figure 1*** *Model for reliability analysis.*

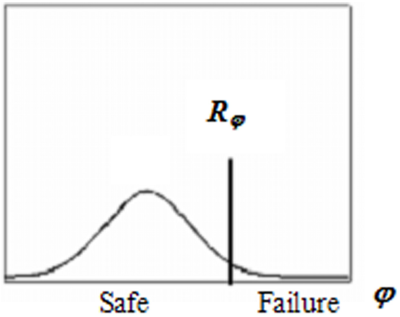


***Figure 2*** *Limit level on the performance variable.*

component, has been applied to hygrothermal calculations. It gives the opportunity to transform a transient moisture model to a steady-state model that can be handled by a random variable probabilistic model. The probability distributions of moisture and thermal parameters are estimated using probabilistic approximation made by FORM. Since the method hasn't been used in this way before, the resulting probability distributions are compared with simulations obtained from the second procedure (1D-HAM). In this, a transient one-dimensional heat, air, and moisture model is used for the calculation of the hygrothermal surface conditions, followed by a statistical analysis of the output. This program is one of many programs in the field of building physics simulation tools (WUFI, DELPHIN, MATCH, etc.). Currently there is a benchmark project funded by the EU for different HAM programs (Hagentoft 2002).

Three different wall constructions are used in the examples to illustrate the comparison between the DTU modeling and the more traditional finite difference modeling, represented here by 1D-HAM. Since this is a first effort in trying to use the DTN model for humidity calculations, simple wall constructions are chosen.

## PROBABILISTIC MODEL FOR RELIABILITY ANALYSIS

First, a moisture problem should be identified, for example, with the help of "failure mode and effects analysis" (FMEA) (Nielsen 2002) or "failure tree analysis" (FTA). Then, a probabilistic model for reliability analysis of building components (including quantification of the probability of failure) can be built up as illustrated in Figure 1 (Pietrzyk et al. 2003, 2004).

To be able to predict if a structure fulfills specified serviceability requirements, the performance criterion is set to a value that describes an unwanted event. The performance function describes a chosen performance aspect, in this case, surface relative humidity $\varphi$, in relation to the load parameters and performance criterion $R_\varphi$ stated against failure. The load parameters consist of external parameters such as outdoor temperature and humidity, material properties, and the parameters characterizing internal climate, accounting for the influence of the installations and the living habits of inhabitants, etc. Some of them are treated as random variables. The performance function is built on the basis of the mathematical models describing physical relationships between the variables. It divides the response space into two subsets: *safe* subset and *failure* subset. Probability density function of surface relative humidity (see Figure 2) shows how the results are distributed over the range of $\varphi$. Probability of failure is calculated as $P_f = P(\varphi > R_\varphi)$.

Reliability analysis can be carried out on the basis of calculated probability of failure.

## PROBABILISTIC MODELING OF SURFACE RELATIVE HUMIDITY ON WALLS

In this paper, two mathematical models are used. One that addresses the hygrothermal memory effect of the wall, Dynamic Thermal Networks (DTN) (Claesson 2003; Wentzel 2004), and a more general one to validate this new approach, finite difference model, 1D-HAM (Hagentoft et al. 2003).

The model for reliability analysis works on the basis of the DTN model, which gives the possibility to define the new random variables that include memory effects of a building structure. The performance function describes relative humidity, $\phi$, with the help of these new random variables, and the performance criterion is set to $R_\varphi = 75\%$ of the relative humidity, as this is the criterion for mold growth in organic material (Nevander and Elmarsson 1994).

The load parameters, identified as the outdoor climate with temperature and humidity, the material properties, and the indoor climate described by the internal temperature, moisture production, and air exchange rate, are used as the input for the mathematical models DTN and 1D-HAM.

On the basis of the DTN model, the new parameters are calculated and described by probabilistic characteristics. These parameters form the input random variable to the probabilistic model of relative humidity. The probability of exceeding the critical relative humidity on the surface is calculated using FORM techniques as well from the deterministic simulations done by 1D-HAM. The flow chart in Figure 3 describes the whole process.

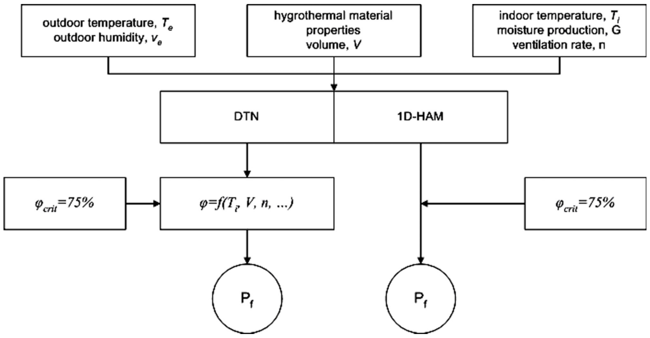


***Figure 3*** *Model for estimating the probability of failure using 1D-HAM and DTN as mathematical models.*

## MATHEMATICAL MODELS

To be able to compare results from the two mathematical models, DTN vs. 1D-HAM, some assumptions have to be made:

- No liquid water flow occurs.
- The sorption isotherms are linearized. Since no condensation occurs in our models, the simplification of the sorption isotherm is done to fit the curve in the range in which we are interested, which is 0-90% relative humidity (see Figure 5).
- Moisture is transferred only by diffusion in vapor phase (the convective part of moisture transfer is not taken into account).

These limitations are done mainly because of the DTN model, which is based on a linear behavior with constant coefficients.

The models are calculated for the hourly averaged mean input data. The indoor temperature is constant 22°C.

### Dynamic Thermal Network

The theory of dynamic thermal networks is based on step response functions (Claesson 2003). By using this theory to calculate the relative humidity on wall surfaces, an analytical expression will be obtained and used in the probabilistic model. The surface temperature is calculated from Equation 1.

$$T_{surf}(t) = T_i - \frac{K_{wall} \cdot (T_i - \overline{T}_{et}(t))}{K_{si}} \qquad (1)$$

where

$T_i$ = indoor temperature (°C),
$K_{wall}$ = walls total thermal conductance including the surface heat transfer coefficient (W/K),
$K_{si}$ = inside surface thermal conductance (W/K),
$\overline{T}_{et}(t)$ = mean values of the outdoor temperatures backward in time (°C).

The mean values of the outdoor temperatures $\overline{T}_{et}(t)$ contain the thermal memory effect of a transient problem. This temperature is calculated by weighting the outdoor temperature backward in time with a weighting function (Claesson 2003).

$$\overline{T}_{et}(t) = \int_0^{\infty} \kappa_{ie}(\tau) \cdot T_e(t-\tau) d\tau \qquad (2)$$

where

$\tau$ = time, backward in time,
$\kappa_{ie}(\tau)$ = weighting function for the heat flow between the inside and outside (-),
$T_e(\tau)$ = outdoor temperature (°C).

The weighting function $\kappa_{in}(\tau)$ is the derivative of the wall's transmittive step response function (Claesson 2001). The transmittive step response function is the function that is initiated by a temperature step from zero to one at one side at the wall; the transmittive response function is the heat flow at the wall's other side.

To be able to use the theory of dynamic thermal networks on moisture flow, the heat transfer coefficient has to be transformed to moisture flow coefficients; see Table 1 and Equations 10 and 11 (Wentzel 2004; Hagentoft 2001). The moisture flow coefficients are constant. The surface moisture by volume is calculated from Equation 3.

$$v_{surf}(t) = v_{ia}(t) = \int_0^{\infty} \kappa_{ia}(\tau) \cdot v_i(t-\tau) d\tau \qquad (3)$$

where

$\overline{v}_{ia}(t)$ = mean values of the indoor moisture by volumes backward in time (kg/m$^3$),
$v_i(t)$ = indoor moisture by volume (kg/m$^3$),
$\kappa_{ia}(\tau)$ = weighting function for the moisture flow into the inside surface (-).

The weighting function $\kappa_{ia}(\tau)$ is the derivate of the wall's absorptive step response function. The absorptive step response function is the difference between the transmittive and the addmittive response functions. These response functions are obtained by a moisture step from zero to one. The transmittive response function is the moisture flow out from the wall and the addmittive response function is the moisture flow into the wall. It turns out that it is only the addmittive part that affects the calculations; the transmittive part is very slow (Wentzel 2004).

The relative humidity at the inner surface is given by Equation 4.

$$\varphi(t) = \frac{v_{surf}(t)}{v_s(T_{surf}(t))} \qquad (4)$$

where

$v_s\,(T_{surf}(t))$ = saturated water vapor content in the surface temperature $T_{surf}(t)$ (kg/m$^3$).

By using this method to calculate the relative humidity, a great advantage in time is obtained. Once the structural load is used to calculate the wall weighting function, the external and internal loads may be changed in any way and the new calculations would only take a few minutes. Equation 4 may also be used in the probabilistic approximations made by FORM.

The dynamic formula for the surface relative humidity (Equation 4) may be rewritten in such a way that the different load parameters will be separated from each other (Equation 5). In Equation 5 the indoor moisture by volume is separated into two parts, one for the outdoor moisture by volume and the other one for the moisture supply at the inside. These parts are named $I_1(t)$ and $I_2(t)$ (Equations 6 and 7). The first one, $I_1(t)$, depends on the wall construction and the outdoor moisture by volume. The second one, $I_2(t)$, depends on the wall construction and the moisture supply at the inside. The moisture supply, on the other hand, depends on the family's living habits, the building's volume, and ventilation (Equation 8). The saturated water content at the surface is also divided into two parts, one depending on the constant indoor temperature and the wall construction and the other depending on the outdoor temperature and the wall construction.

$$\varphi(t) = \frac{I_1(t) + I_2(t)}{v_s\left(T_i \cdot \left(1 - \frac{K_{wall}}{K_{si}}\right) + \underbrace{\frac{K_{wall}}{K_{si}} \cdot \int_0^{\infty} \kappa_{ie}(\tau) \cdot T_e(t-\tau)\,d\tau}_{\Delta T(t)}\right)} \tag{5}$$

$$I_1(t) = \int_0^{\infty} \kappa_{ia}(\tau) \cdot v_e(t-\tau)\,d\tau \tag{6}$$

$$I_2(t) = \int_0^{\infty} \kappa_{ia}(\tau) \cdot \Delta v(t-\tau)\,d\tau \tag{7}$$

$$\Delta v = \frac{G}{n \cdot V} \tag{8}$$

By rewriting Equation 4, we hope to find a way to explain how the different load parameters influence the result of the relative humidity on the surface of the wall. Hopefully, this rewriting can be used in future in the probabilistic analysis.

If $\Delta T$ is small, Equation 5 will be written in following way:

$$\varphi(t) = \frac{I_1(t) + I_2(t)}{v_s(\psi)} - C_1 \cdot (\Delta T(t) \cdot I_1(t) + \Delta T(t) \cdot I_2(t)) \tag{9}$$

where

$$\psi = T_i \cdot \left(1 - \frac{K_{wall}}{K_i}\right) \tag{10}$$

$$C_1 = \frac{\partial v_s}{\partial T} \cdot \frac{1}{v_s^2(\psi)} \qquad (T = \psi) \tag{11}$$

### Finite Difference Technique, 1D-HAM

For the second deterministic model a computer program (1D-HAM) has been used.This program solves problems with coupled heat, air, and moisture transport in multi-layered porous materials in one dimension. In 1D-HAM, moisture is transferred by diffusion and convection in vapor phase. No liquid water transport occurs. Heat is transferred by conduction, convection, and latent heat. At the moment the model accounts for one-dimensional air flow only. This is only applicable for some special cases, such as dynamic walls, and other cases where the air flow can be considered approximately one-dimensional.

The governing equations for temperature used in this program can be described as

$$-\frac{\partial}{\partial x}\left(-\lambda \frac{\partial T}{\partial x} + q_a \rho c_a \cdot T\right) + r\frac{\partial w}{\partial t} = \rho c \frac{\partial T}{\partial t}\,. \tag{12}$$

Here the thermal conductivity for the wall material is denoted by $\lambda$ (W/m·K). The density is $\rho$ (kg/m$^3$) and the heat capacity is $c$ (J/kg·K). The ambient air volumetric heat capacity is denoted by $\rho c_a$ (J/(m$^3$·K)). The latent heat of evaporation is described by $r$ (J/kg).

The governing equation for the humidity by volume is

$$-\frac{\partial}{\partial x}\left(-\delta_v \frac{\partial v}{\partial x} + q_a \cdot v\right) = \frac{\partial w}{\partial t}\,. \tag{13}$$

Where, $w$ (kg/m$^3$) denotes the water content of the material. The vapor diffusion coefficient is denoted by $\delta_v$ (m$^2$/s).

The constant flow resistance $R_p$ (Pa/(m$^3_{\text{air}}$/(m$^2$·s))) together with the pressure difference over the structure $\Delta P$ determine the air flow rate.

$$q_a = \frac{\Delta P}{R_p} \tag{14}$$

In our calculations the pressure difference has been set to zero over the wall structures. The convective part has been neglected and set to zero to be able to compare the results from the 1D-HAM simulations with the simulations done with the DTN method.

The numerical model for this program is based on the finite difference technique with explicit forward differences in time. The coupling between neighboring computation cells is based on analytical solutions for the coupled processes of air flow through two layers of different materials in contact with each other.

The transfer of moisture to and from the cells is governed by humidity by volume in the cells and the humidity at the boundary. The increase of the moisture content $\Delta w_i$ (kg/m$^3$) of cell $i$ with the width $\Delta x_i$ due to the net flow rate of moisture $\Delta g_i$ kg/(m$^2$·s) is given by

$$\Delta w_i = \Delta t \cdot \frac{\Delta g_i}{\Delta x_i}. \tag{15}$$

Here $\Delta t$ (s) is the time step considered. The program determines this time step automatically. It is updated at each time step.

The program handles the sorption isotherm as three straight lines. The look of this chart should be chosen in order to fit the true sorption isotherm. In our case the sorption isotherm is a straight line in order to be able to compare the results from 1D-HAM with the results from DTN simulations.

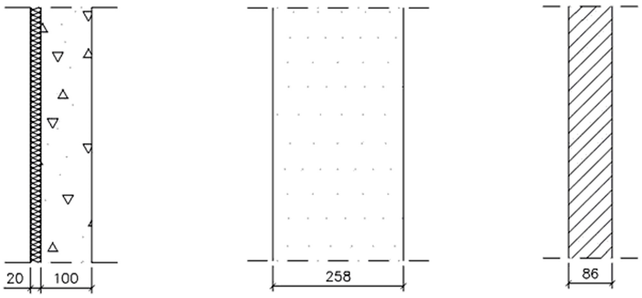


***Figure 4*** *The three different walls used in the case study. From left, a concrete wall with insulation, a lightweight concrete wall, and a wooden wall.*

To be able to simulate a specific wall structure, the program needs climatic data. This data file, i.e., boundary conditions, specifies the temperature and the humidity both outside and inside the building. When the simulation is done, temperatures, humidity by volume, and relative humidity can be plotted for each layer of interest. In this case the closest inner layer is 0.5 mm from the inner climate.

## CASE STUDY

### Three Different Walls

To be able to compare the two mathematical models for us in a probabilistic (statistical) method, three different wall structures have been simulated (Figure 4). The criterion was that the U-factor for these walls be the same 1.048 W/m$^2$K. This means that the walls are badly insulated for a Nordic climate. The surface resistances are set to $R_{se}$= 0.04 m$^2$K/W for the external side and $R_{si}$ = 0.30 W/m$^2$K for the internal side. This higher indoor surface resistance is used to symbolize an unwanted case, for example, a wall with a painting hanging on it or a bookcase standing in front of the inner surface.

Though one can think that this would not be a problem, the most common thing is that when durability problems occur, they certainly do not occur were one can notice it!

The first wall is a 100-mm-thick concrete wall with 20 mm of insulation on the exterior side. The second wall is lightweight concrete with a thickness of 258 mm, and the third wall is wooden with a thickness of 86 mm. Material properties for these wall structures can be found in Table 1.

**Table 1. Material Properties for the Wall Structures**

| **Material** | **$\lambda$ (W/mK)** | **$C$ (J/m$^3$K)** | **$\delta_v$ (m$^2$/s)** | **$w_{100}$ (kg/m$^3$)** | **$a_v$ (m$^2$/s)** | **$\rho c_v$ (-)** |
|---|---|---|---|---|---|---|
| Concrete | 1.7 | $2.07 \cdot 10^6$ | $0.5 \cdot 10^{-6}$ | 88 | $7.29 \cdot 10^{-11}$ | 6859 |
| Mineral wool | 0.036 | $0.012 \cdot 10^6$ | $20 \cdot 10^{-6}$ | 0.4 | $4.70 \cdot 10^{-7}$ | 42.5 |
| L.weight concrete | 0.42 | $0.5 \cdot 10^6$ | $3 \cdot 10^{-6}$ | 44 | $8.75 \cdot 10^{-10}$ | 3430 |
| Wood/spruce | 0.14 | $0.63 \cdot 10^6$ | $1 \cdot 10^{-6}$ | 112 | $1.15 \cdot 10^{-10}$ | 8730 |

where

$\lambda$ = thermal conductivity, W/mK

$C$ = volumetric heat capacity, J/m$^3$K

$\delta_v$ = vapor permeability, m$^2$/s

$w_{100}$ = moisture content mass by volume, kg/m$^3$

$a_v$ = vapor diffusivity, m$^2$/s, defined as

$$a_v = \delta_v \cdot \frac{v_s(15°\mathrm{C})}{w_{100}} \tag{16}$$

and $\rho c_v$ is the moisture capacity

$$\rho c_v = \frac{\delta_v}{a_v} \tag{17}$$

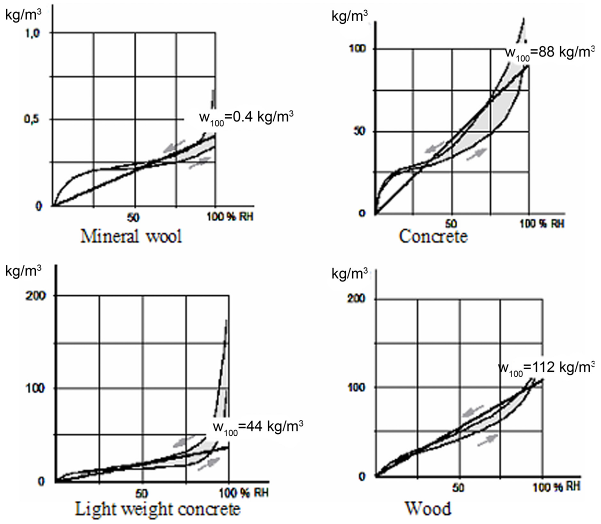


***Figure 5*** *Simplified sorption isotherms for the materials.*

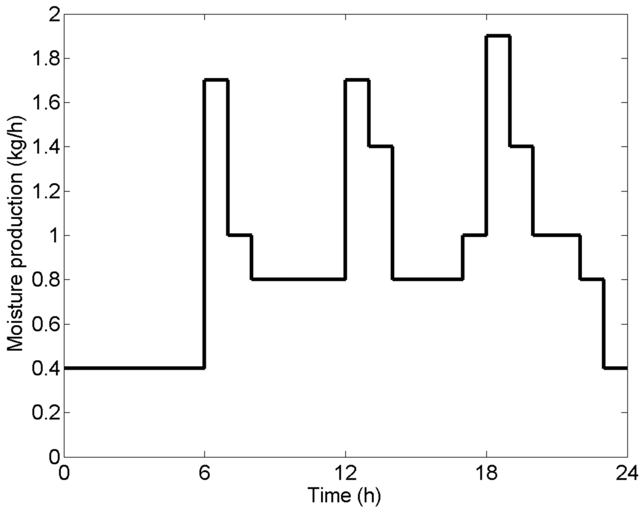


***Figure 6*** *Daily moisture production (kg/h) for the family.*

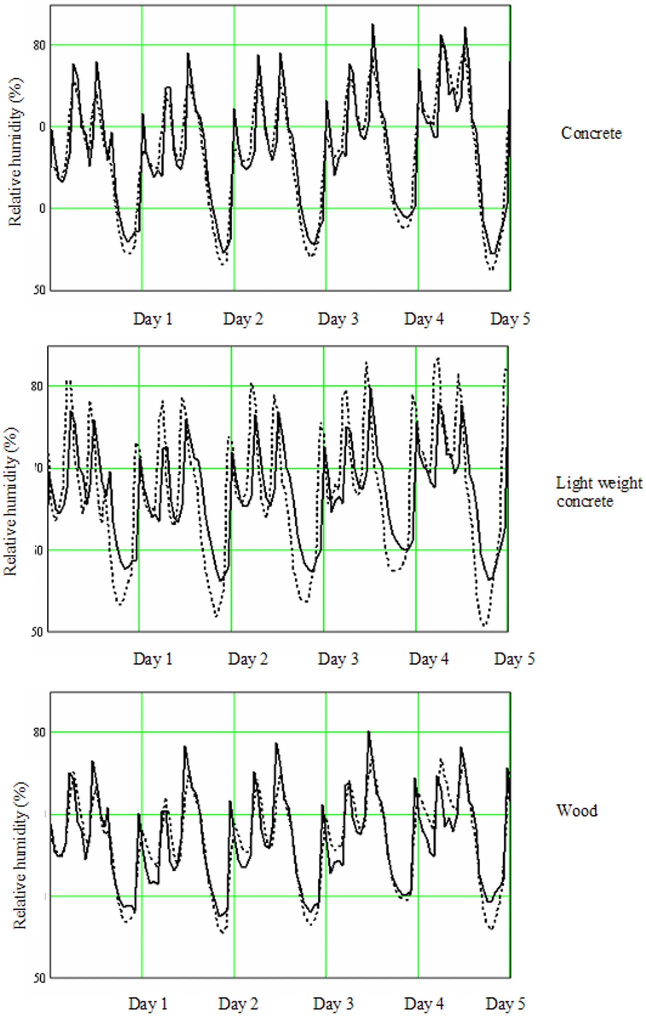


***Figure 7*** *Comparison between results from DTN simulations and simulations done with 1D-HAM. Solid lines show calculations with the DTN method, and dashed lines show calculations with 1D-HAM.*

The sorption isotherm is simplified in order to be able to compare results from the DTN simulations with the simulations made in 1D-HAM. The look of the simplified isotherm is a straight line (Figure 5).

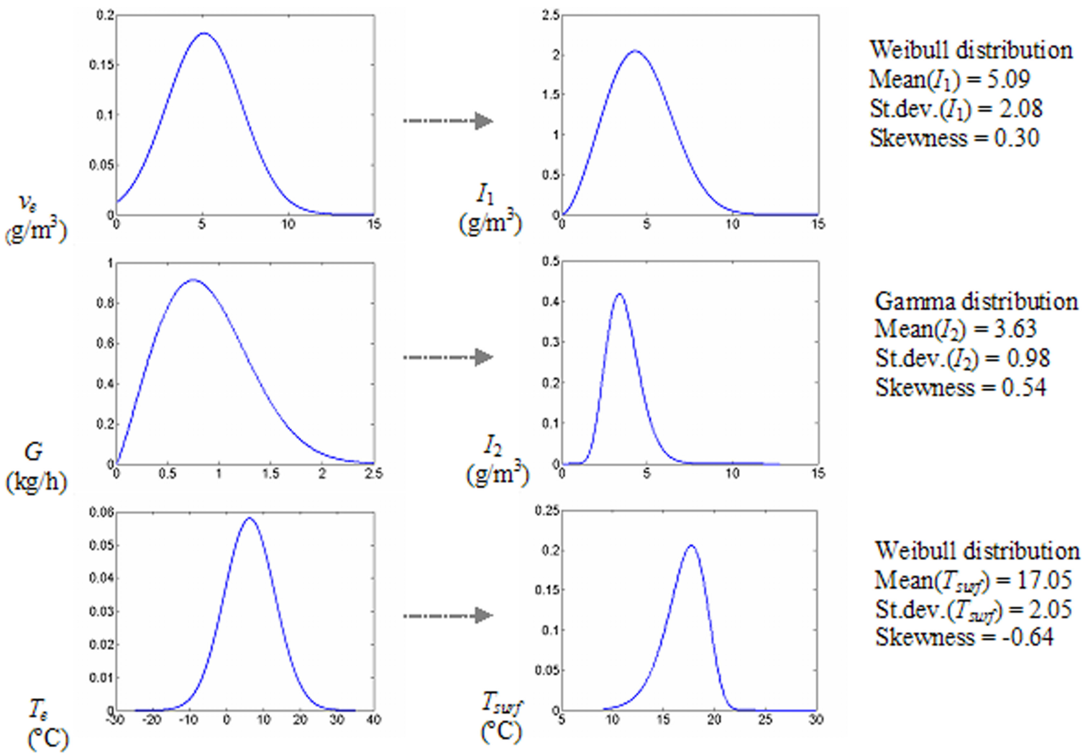
The climatic file is the governing input data. These external climatic data are taken from an airfield Säve close to Gothenburg in Sweden. This file contains hourly climatic data with both indoor and outdoor temperatures and indoor and outdoor humidity by volume.

The indoor moisture production is based on a family of five, with a daily moisture production as illustrated in Figure 6.

The house has a volume of 486 m$^3$ and a constant air exchange rate of 0.5 air changes per hour. Since the interest was to gain knowledge of the relative humidity at the surface, the input parameters are both temperatures (inside and outside) $T_i$ and $T_e$, the vapor content by volume outside $v_e$, the volume of the building $V$, the moisture production inside the house $G$, and the air exchange rate $n$ in the house (Figure 3).

## Validation of the Results from DTN and 1D-HAM Simulations

In order to go on with the probabilistic approach, the two deterministic methods were compared. The simulations were done for a period of four years, where values only for the last year were compared. In Figure 7 the results are shown for a period of five days during a summer month in Säve. This week was chosen to illustrate peaks of relative humidity over 75%,



Weibull distribution
Mean($I_1$) = 5.09
St.dev.($I_1$) = 2.08
Skewness = 0.30

Gamma distribution
Mean($I_2$) = 3.63
St.dev.($I_2$) = 0.98
Skewness = 0.54

Weibull distribution
Mean($T_{surf}$) = 17.05
St.dev.($T_{surf}$) = 2.05
Skewness = -0.64

***Figure 8*** *Probability density functions of the climatic load (left) and probability density functions and statistical description of the input variables to FORM approximations of the surface relative humidity for the concrete wall (right).*

which is the performance criterion, and also because it shows for the whole period typical differences between the results obtained by the two models. The results show that this agreement is quite good for the concrete wall and for the timber wall. The lightweight concrete wall has somewhat different values. These must be studied more. This fault might derive from differences in discretization or the effect of latent heat in 1D-HAM.

The dashed lines in these charts represent the simulations done with 1D-HAM and the solid line represents the simulations done with DTN. In the Y-axis we have the relative humidity, which is the variable of interest in this paper.

The results for the cases with the concrete wall show best agreement. In general the curves tend to give higher values with the DTN method than with the simulations done with the 1D-HAM program. The small differences can be explained in that the 1D-HAM simulations account for the latent heat and the DTN simulations do not. The surface relative humidity in the 1D-HAM simulations represents the situation at the depth of 0.5 mm, i.e., the center of the surface computational cell, while the DTN method is based on analytical expression and calculates the relative humidity directly on the surface.

## PROBABILISTIC APPROXIMATION

The probabilistic model for reliability analysis illustrated in Figure 1 has been applied. Probability density function of surface relative humidity is evaluated using FORM sensitivity analysis (Pietrzyk 2000) for the performance function given by Equation 18 (see also Equations 1, 6, and 7). It depends on statistical parameters of the joint distribution of three random variables: inner surface moisture by volume $I_1$ resulting from the outside humidity, inner surface moisture by volume $I_2$ representing a contribution of the moisture production, and the surface temperature $T_{surf}$. Those three variables reflect the dynamic response of the wall surface to the external and internal climatic load characterized by external moisture by volume $v_e$, moisture production G, and external temperature $T_e$. The probability density functions of those variables corresponding to the case with the concrete wall and the air change rate of 0.5 changes per hour are shown in Figure 8.

$$\varphi(t) = \frac{I_1(t) + I_2(t)}{v_s(T_{surf}(t))} \tag{18}$$

where, after Nevander

$$v_s(T_{surf}) = 1000\frac{288.68(1.098 + T_{surf}/100)^{8.02}}{461.4(T_{surf} + 273.2)} \qquad (T_{surf} > 0)\,. \tag{19}$$

The probability density functions of climatic parameters are presented at the left side of Figure 8. They could be the input data for the probabilistic random variable model describ-

**Table 2. The Results of Probabilistic Approximations of the Surface Relative Humidity for the Concrete Wall**

| Case | Mean Value | St. Dev. | Distribution |
|---|---|---|---|
| Uninhabited house | 33.8 | 10.47 | Truncated Weibull |
| ACH = 0.5 | 58.0 | 10.77 | Normal |
| ACH = 0.4 | 65.3 | 11.80 | Weibull |

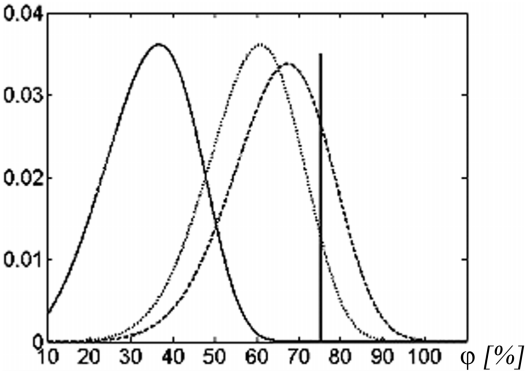


***Figure 9*** *Probability density functions of the surface relative humidity approximated for the case of uninhabited house (solid line), the house with ach=0.5 (dotted line), and the house with ach=0.4 (dashed line).*

**Table 3. The Results of Reliability Analysis**

| Case/Model | $P_f$ (validation) | $P_f$ from FORM |
|---|---|---|
| Uninhabited house | 0.000 | 0.000 |
| ACH = 0.5 | 0.051 | 0.057 |
| ACH = 0.4 | 0.219 | 0.194 |

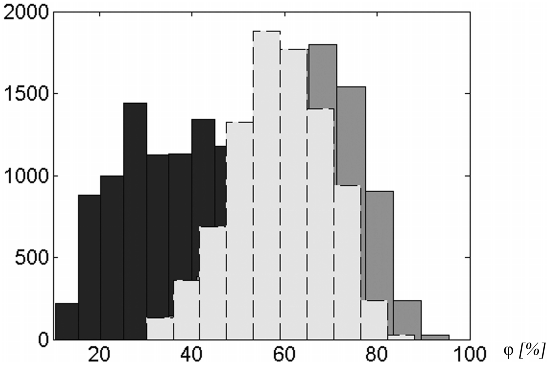


***Figure 10*** *Histograms of the surface relative humidity for the case of uninhabited house (solid line to the left), the house with ach=0.5 (dashed line), and the house with ach=0.4 (solid line to the right).*

ing steady-state moisture conditions. In the case of dynamic moisture conditions, those parameters are recalculated with the help of DTN simulations according to Equations 6, 7, and 1 to the form of $I_1$, $I_2$, and $T_{surf}$. Their probability density functions and the correlations between them are the input data to the probabilistic approximation of φ carried out by FORM. The correlation between the inner surface temperature $T_{surf}$ and the inner surface moisture by volume resulting from the outside humidity $I_1$ is assumed equal to the correlation between the external moisture by volume $v_e$ and the external temperature $T_e$ and it gets a value of 0.93. Correlations between $I_1$ and $I_2$ as well as correlation between $I_2$ and $T_{surf}$ are taken equal to 0.

Probabilistic approximation of the surface relative humidity is carried out for the concrete wall. Three cases characterized by different moisture supply are considered:

- the uninhabited house without moisture production,
- the inhabited house with moisture production $G$ (see Figure 6) and ventilation rate of 0.5 air changes per hour
- the inhabited house with moisture production $G$ (see Figure 6) and ventilation rate of 0.4 air changes per hour

Probability density function of the surface temperature shown in Figure 8 reflects the influence of thermal memory of the concrete wall. It is skewed to the left (negative value of skewness). It means that a mode of the density function is moved toward higher values, which reflects the influence of the cumulated heat on the surface temperature.

The results of the probabilistic approximations of relative humidity obtained by FORM are shown in Table 2 and in Figure 9. They include the mean value, the standard deviation, and the type of probability distribution giving the best fit according to the Kolmogorov-Smirnov test to the results obtained by FORM.

The probability of performance failure $P_f = P(\varphi > 75\%)$ is given in Table 3 for the case of probabilistic approximation and the validation case. The probability of failure corresponds to the area under the probability density function for the relative humidity exceeding 75% as shown in Figure 9.

The validation results are calculated from the deterministic simulations carried out by the 1D-HAM program and presented in Figure 10 in the form of histograms. The results are grouped in adjoining intervals of length $\Delta\varphi_i$. For $\Delta\varphi_i \rightarrow d\varphi$ and a suitably increasing number of results within the intervals, the relative frequency of number of observations per interval approaches the probability density. Probability of failure has been obtained by counting up the results of the deterministic calculations exceeding the prescribed critical value.

The $P_f$ estimated from the probabilistic model are very close to the $P_f$ calculated from the validating model. It means that the probability density function of the relative humidity, and especially their right tail, has been correctly approximated by FORM.

## SUMMARY AND DISCUSSION

A new approach to address the memory effects of the building component has been demonstrated. As an output of the analysis, the probability distributions for the surface temperature and relative humidity are given, which can be used for the reliability analysis of thermal comfort or the risk analysis of mold formation.

Probabilistic approximations are carried out on the results of a new technique for hygrothermal simulations based on the dynamic thermal network method (DTN). The results of deterministic calculations carried out by that method vs. the 1D-HAM transient model show qualitative agreement, although more research must be done to gain more knowledge of how to apply the sorption isotherm to the dynamic thermal network model. The advantage of the calculations carried out with DTN is the computational time. With the program 1D-HAM and with a fairly new computer, the average time for simulations with 50 cells took about five hours. The DTN calculations with a known weighting function for the same wall structure took one-and-a-half minutes. In future work a comparison between different calculation programs and the DTN model should be done.

Application of the reliability methods requires more accurate modeling of the peaks of the high values of relative humidity than is obtained from the simulations carried out for the examples of timber or light concrete walls. The results shown for the concrete wall are considered satisfactory, and they have been used for the reliability analysis. The changes of the probability of failure caused by different moisture supply have been demonstrated in three examples calculated for the concrete wall. Analyzing the distributions of $I_1$, $I_2$, and $T_{surf}$ for different constructions and different climatic conditions, one maight draw conclusions about their typical patterns, which can help in preparing the input data to probabilistic approximation of the surface relative humidity.

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