The Moment Method for Measuring Moisture Diffusivity of Porous Building Materials

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ABSTRACT

A method called the moment method has been developed for measuring the moisture diffusivity of porous building materials. An encased bar of the material is placed on two supports, one of them on a balance and the other on a fixed support. If there is a moisture gradient along the bar, a moisture flow will take place and cause a change of weight over time at the supports and cause a change of the moment of a force around each support. Knowing the moment change due to time we can calculate the diffusivity for the material. The method demands a balance with a high accuracy and very careful preparation of the sample to achieve the wanted moisture gradient and to avoid hysteresis. Measurements have been carried out for various building materials, some results are shown in this paper.

INTRODUCTION

Modern PCs and new calculation programs make it possible to calculate advanced moisture problems, for example moisture transfer in porous material above the hygroscopic area, but the lack of relevant material data and suitable measurement methods is often a hindrance for the calculations. Most of the building materials are porous such as concrete, aerated concrete, timber, bricks etc., and contain an extensive and complicated system of pores where moisture in the shape of ice, water or water vapor may be present. The amount of moisture in the pores and the amount of moisture transfer through porous material is primarily governed both by the structure of the pore system and by the moisture conditions such as relative humidity or suction in the material. A number of methods have been developed to measure the moisture properties for porous materials, often based on studying the moisture profiles during water uptake with e.g. computed tomography or magnetic resonance tomography. These are often expensive methods and give no direct values for the transfer coefficients. There has been a need for methods involving direct measurements of e.g. the moisture diffusivity. The moment method is an example on such a method.

Fundamental and experimental research on moisture properties and the moisture conditions in porous materials have been important issues at Lund University since the 1970s. A research group called the Moisture Research Centre in Lund was established in the 1980s and has been engaged in the development of methods for determining moisture properties for different material, one of these methods is the moment method. The moment method was originally used in soil science and presented by Zaslavsky and Ravina, (1965). Other researchers have later published articles with results from occasional use of the method, as Klute (1972) and Cunningham et al. (1989), but it has not been considered a practical method, due to problems regarding how to carry through measurements with acceptable accuracy. The author and his colleagues at Moisture Research Centre in Lund have tried to improve the measurement technique over many years and have gradually developed a method with good accuracy for measurements of porous building materials. Using this method, moisture diffusivity has been successfully determined for some typical porous building materials. The results have been compared with other methods and have shown good agreement.

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THE MOMENT METHOD

In the moment method, the moisture flow coefficient \( D_w \), called the moisture diffusivity, can be measured at a moisture gradient \( w \) during isothermal conditions in a porous material. The experimental equipment used for the measurements is as follows.

A solid bar of the porous material is enclosed in a moisture and air tight metal tube, placed on two supports, one of them placed on a precision balance and the other on a fixed point outside the balance (see Figure 1).

The bar is conditioned in advance to create a moisture gradient along the bar. Initially there will be the same moisture content \( w(x,0) = w_2 \) over the whole bar, then a certain amount of water \( \Delta w \) is added to one of the halves, which gives it \( w(x,0) = w_1 \). The bar is at once enclosed in the metal tube and placed on the supports. A moisture flow will start in the bar to equalize the moisture difference. The flow will give a change in weight on the balance, which is recorded as the rate of weight change, \( \frac{dm}{dt} \). During the flow period the weight will change linearly with time and the \( \frac{dm}{dt} \) is a constant. The diffusivity \( D_w \) can be calculated as

\[
\tilde{D}_w = \frac{L_o \frac{dm}{dt}}{A \Delta w}
\]

where

- \( D_w \) = diffusivity, m\(^2\)/s
- \( L_o \) = distance between the supports, m
- \( A \) = cross sectional area of the bar, m\(^2\)
- \( w \) = moisture content, kg/m\(^3\)
- \( \frac{dm}{dt} \) = the rate of weight change, kg/s

The value \( \tilde{D} \) is a mean value in the interval \( w_1 \) to \( w_2 \). Because of the large variation of \( D_w \) with \( w \) it is necessary to make the intervals as small as possible. This will of course demand series of measurements to cover as much as possible of the whole range of \( w \).

The final result will be a curve which shows \( D_w \) as a function of \( w \) covering most of both the hygroscopic and the capillary moisture intervals.

THEORY

The bar has the total length \( 2L \) and the cross section \( A \). At time \( t = 0 \), half the bar has the moisture content \( w_1 \) and the other half \( w_2 \). The moisture difference causes a moisture flow \( g \),

\[
g = -D_w \frac{\partial w}{\partial x},
\]

where

- \( g \) = moisture flow, kg/m\(^2\)/s
- \( D_w \) = diffusivity, m\(^2\)/s

The moisture flow \( g \) is the total moisture flow including diffusion and capillary transport.

The change of the moisture content \( w \) in every point in the bar will be

\[
\frac{dw}{dt} = \frac{\partial}{\partial x} \left( D_w(w) \frac{\partial w}{\partial x} \right)
\]

The weight at the balance support will change due to the moisture distribution

\[
\frac{dm}{dt} = \frac{d}{dt} \left( m(t) - m(0) \right)
\]

A moment equation for the left support \( x = x_o \) gives

\[
\frac{dm}{dt} = \frac{d}{dt} \left[ \frac{A}{L_o} \int_{-L}^{L} (x-x_o)(w(x,t) - w(x,0)) dx \right]
\]

\[
= \frac{A}{L_o} \int_{-L}^{L} (x-x_o) \frac{\partial w(x,t)}{\partial t} dx + \frac{A}{L_o} \int_{-L}^{L} (x-x_o) \frac{\partial}{\partial x} \left( D_w \frac{\partial w}{\partial x} \right) dx
\]

\[
= \frac{A}{L_o} \left[ \int_{-L}^{L} (x-x_o)(w(x,0)) dx \right] \frac{\partial w}{\partial x} \bigg|_{x=L} - \int_{-L}^{L} \int_{x_o}^{w(x,t)} D_w(w) \frac{\partial w}{\partial x} dx dv
\]

Here we use the fact that the moisture flow is zero at the ends \( x = L \) and \( x = -L \). We have the following exact formula:

\[
\frac{dm}{dt} = \frac{A}{L_o} \int_{w(w_1)}^{w(w_2)} D_w(w) dv = \frac{A}{L_o} (w_1 - w_2) \tilde{D}_w
\]

where \( \tilde{D}_w \) is the mean value for \( D_w(w) \) within the interval \( w_1 \) to \( w_2 \).

As long as the moisture flow is zero at the ends, the values \( w(L, t) \) and \( w(-L, t) \) are equal to the initial values \( w_1 \) and \( w_2 \), respectively, during a first period, until the change in moisture content has reach the ends of the bar. We have during this first period
\[ \dot{D}_w = \frac{L_o \frac{dm}{dt}}{A \Delta w} \]

or

\[ \dot{D}_w = \frac{L \cdot L_o \frac{dm}{dt}}{\Delta W} \]

where \( \Delta W = W_1 - W_2 \), \( W_1 = A \cdot L \cdot w_1 \) and \( W_2 = A \cdot L \cdot w_2 \).

The factor \( \frac{dm}{dt} \) and hence \( \dot{D}_w \), is constant during the first period. This is an important advantage with this method.

**EXPERIMENTS**

To determine the diffusivity \( \dot{D}_w \) within the interval \( w_1 \) to \( w_2 \) we need to measure:

- the length \( L_0 \) and \( L \) (or the area \( A \))
- moisture content \( w_1 \) and \( w_2 \) (or the moisture weights \( W_1 \) and \( W_2 \))
- the rate of weight change \( \frac{dm}{dt} \) at the balance

The change in weight \( m(t) \) from a reasonably typical measurement is shown in Figure 2. The rate of change will have a constant value as long as the moisture content at the ends of the bar can be considered as constant (\( w_1 \) and \( w_2 \), respectively). The slope of the straight line gives the diffusivity.

After the initial period the moisture content at the ends of the bar will be affected by the equalizing moisture flow between the halves. Then \( \frac{dm}{dt} \) will decrease slowly (Figure 2).

**Time Period of Constant Slope**

It is important to determine the length of the time period with constant slope. The analytical solution for the moisture flow equation

![Figure 2](image)

*Figure 2*  The change in weight \( m(t) \) with time \( t \) registered by the balance. The full line shows the first period with constant slope as a straight line.

when \( L = \infty \) and constant \( D_w(w) = \dot{D}_w \) is well known.

\[ w(x, t) = w_o - \frac{\Delta w}{2} \text{erf} \left( \frac{x}{\sqrt{4D_w t_{lin}}} \right) \]

where

\[ w_o = \frac{w_1 + w_2}{2} \]

and \( \text{erf}(\ldots) \) denotes error function.

Let us define \( t_{lin} \) as the time when the changes of \( w \) at the ends is equal to 1% of \( \Delta w/2 \). This gives

\[ \text{erf} \left( \frac{L}{\sqrt{4D_w t_{lin}}} \right) = 0.99 \]

The table for error function gives

\[ \frac{L}{\sqrt{4D_w t_{lin}}} = 1.82 \]

\[ t_{lin} = \frac{1}{1.82^2 \cdot 4 \dot{D}_w} = \frac{1}{13} \frac{L^2}{\dot{D}_w} \]

The corresponding weight change is

\[ \frac{(\Delta m)_{lin}}{\Delta W} = \frac{1}{13} \frac{L}{L_o} \] or \[ \frac{(\Delta m)_{lin}}{\Delta W} = \frac{1}{13} \]

Table 1 gives some values for \( t_{lin} \) for some values of \( \dot{D}_w \) when \( L = 0.1 \) m.

**Examples.** Consider a bar with the length \( L = 200 \) mm made of aerated concrete or brick.

Aerated concrete: \( w = 50 \) kg/m\(^3\) gives \( D_w = 4 \cdot 10^{-7} \) m\(^2\)/h

The moisture content at the ends can be considered a constant during the time

**Table 1.**  Time \( t_{lin} \) for the Constant Slope at Some Values of \( \dot{D}_w \)

<table>
<thead>
<tr>
<th>( \dot{D}_w ) (m(^2)/h)</th>
<th>10(^{-2})</th>
<th>10(^{-3})</th>
<th>10(^{-4})</th>
<th>10(^{-5})</th>
<th>10(^{-6})</th>
<th>10(^{-7})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{lin} ) (min)</td>
<td>5</td>
<td>48</td>
<td>7 hours</td>
<td>3 days</td>
<td>32 days</td>
<td>320 days</td>
</tr>
</tbody>
</table>
Brick: \( w = 200 \text{ kg/m}^3 \) gives \( D_w = 8 \cdot 10^{-3} \text{ m}^2/\text{h} \)

\[
t_{\text{fin}} = \frac{1}{13} \frac{L^2}{D} = 1923 \text{ h} \approx 80 \text{ days}
\]

\[
t_{\text{fin}} = \frac{1}{13} \frac{L^2}{D} = 5.77 \text{ min}
\]

**Moisture Conditioning of the Bars**

One of the most difficult parts in the development of the Moment method has been how to precondition the bar to achieve the desired moisture content in each half. In the very first experiments we used a bar that was divided in two halves which were conditioned to \( w_1 \) and \( w_2 \). To avoid problems due to the contact resistance between the two halves, we abandoned this use of two halves and used an undivided bar. The full bar is conditioned to the initial moisture content \( w_2 \), which must be evenly distributed across and along the full bar. However, it is difficult to achieve this due to confined air in the pores and hysteresis. To minimize these problems we have used a sample with a square cross section that was sliced longitudinally into four plates (see Figure 3). Every plate was placed with air spaces between two other plates, which were saturated with moisture (see Figure 4). A vapor flow will start over the air space. This arrangement allows us to get a slow and well-distributed sorption in the sample. Then we spray water to one half of each plate in order to get the moisture content \( w_1 \). If we want to study samples under desorption, we use a rapid absorbing material like a dish sponge, wound round the half of the plates, to suck water from it.

When we start a new test it is necessary that the moisture in the sample is fully equalized before we add water to one of the halves.

**The Measurement Device**

The measurement device has been improved step by step to achieve more accurate measurements. In one of the first set-ups we used an edge at one of the supports and a roll at the other as shown in Figure 1. This caused however problems with friction and horizontal movements which resulted in incorrect values on the balance. To eliminate these problems we developed new set-ups, the best of them has been to use an air cushion as one of the supports instead of the roll and a sharp edge as support on the balance. This gives more accurate measurements without problems with friction and horizontal movements.

**RESULTS**

We have performed tests primarily for aerated concrete, brick and gypsum. Figure 5 shows \( D_w(w) \) for aerated concrete both in the absorption and desorption phases. Figure 6 shows \( D_y(w) \) for gypsum and also the suction curve \( w(P_s) \) for gypsum measured by a suction apparatus. In Figure 6 we can see large hysteresis between the \( D_y(w) \)-curves in the absorption and desorption phases in the interval \( w = 200 \) to \( 400 \text{ kg/m}^3 \). For \( w = 300 \text{ kg/m}^3 \), \( D_w \) is around six times higher than in the desorption phase. This is primarily due to the large differences in moisture capacity between the phases. By determining the moisture capacity, i.e. the slope of the suction curve \( dw/dP_s \), we can calculate the moisture flow coefficient \( D_y = dw/dP_s \cdot D_w \). Where \( D_y \) gives the moisture flow when using a difference in capillary suction as the driving force. By calculating \( D_y \) both for the absorption and desorption phases at the same moisture content, we can see that the hysteresis is small.

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**Figure 3** The sample with a square cross section and sliced longitudinally into four plates.

**Figure 4** Test material plates placed with air spaces between plates saturated by moisture.
For example at $w = 300 \text{ kg/m}^3$, $D_s$ in the desorption phase is not more than around 20% higher than in the absorption phase.

**CONCLUSIONS**

Determination of moisture transport using the moment method is suitable for determination of the moisture diffusivity $D_w$ for use in calculation programs and for studies of the nature of moisture phenomena.

The method can be used within the whole moisture interval but it is most suitable in the capillary interval. The materials tested were brick, aerated concrete, gypsum and porous glass. In another laboratory wood has been tested, see Cunningham (1995). It will however be more difficult to use the method for materials subject to geometrical changes during a moisture change e.g. wood with its swelling/shrinkage, which could affect the moment.

The moment method will, as shown above, give direct values for moisture diffusivity $D_w$. The vapor permeability $D_v$, equal to $D_w$ multiplied by the derivative $dw/dv$, can also be determined if we know the sorption isotherm.

The following requirements must be fulfilled:

- very accurate balance
- negligible friction forces between the balance and sample support
- very careful conditioning of the sample
- no residual moisture flow from previous tests
- choice length of the sample to get reasonable times and magnitudes for the moisture flows
- dimensionally stable materials.

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