Modeling of Air Convection Effects on Hygrothermal Performance of Vented Roofs

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INTRODUCTION

The influence of air flux on the hygrothermal behavior of building envelope systems may play an important role which should not be neglected in many practical building physical problems. Particularly air permeable lightweight constructions may suffer from condensation problems due to air exfiltration in cold climate zones. In warm and humid climate zones the problem might be opposite. Air infiltration can cause deposition of moisture when the temperature in the construction element drops below the dew point of the outdoor air.

Sometimes the hygrothermal simulation of envelope wall constructions can be simplified to two dimensional cases. In many cases (for example vented roofs) the situation is more complicated. The challenge is here the combination of three dimensional air fluxes with heat and moisture transport in porous building materials. There are very few simulation programs available to the public which take these effects into account.

Current research work on multizone building simulation including heat, air, moisture and pollutant transport processes in building envelope systems is being conducted and funded by the U.S. Environmental Protection Agency and the Syracuse Center of Excellence. CHAMPS-BES 1 is a newly designed program available to the public for research and educational purposes which has been built upon DIM1-3 (1987-1999) and DELPHIN4 (2000-2007) being Heat, Air and Moisture (HAM) simulation codes developed at the Institute of Building Climatology (IBK) at the Dresden University of Technology and Delphin 5 (Nicolai et al. 2007) jointly developed by Syracuse University and Dresden University of Technology.

1. Download area for the program, material and climate data, examples and documentation from the web page at http://beesl.syr.edu/champs.htm.

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Technology. Research and developmental works in respect to airflows (and transport of volatile organic compounds - VOC) included implementation of a time-stepwise stationary solution of the air pressure field for calculation of the convective transport in the gas phase.

GOVERNING TRANSPORT EQUATIONS

Since coupled heat and moisture balance equations are being frequently published, there is no need to repeat their notation. The reader may refer to Künzel 1994, Grunewald 2000 or Sasic Kalagasidis 2004 which represent a very small selection of the available literature in this field. Instead, a generalized formulation (Equation 1) is preferred to specify the underlying balance equations and to highlight the necessary extensions to account for air transport.

CHAMPS Balance Equations

A general extensive quantity \( E \) is introduced that stands for all quantities the conservative law is applied to. This can be, for example, any mass component, internal energy, linear momentum or entropy. While the quantities volumetric \( \dot{E} \)-density and volumetric \( \dot{E} \)-production density are related to a Representative Elementary Volume (REV), the \( \dot{E} \)-flux density is related to a Representative Elementary Area (REA). In equations (1) through (6) we make use of Albert Einstein’s summation convention meaning that pairwise \( k \)-indices are summed up over all spatial coordinates.

\[
\frac{\partial \rho_{\text{REV}}}{\partial t} = \sum_{i=1}^{M} \int_{v} \sigma_{\text{REV}} E_{i} \, dV - \sum_{i=1}^{M} \int_{A} j_{k, \text{REA}, i} \, dA + \int_{V} \sigma_{\text{REV}} \, dV
\]

\( E \) = General extensive quantity

e.g. mass, internal energy,
 linear momentum, entropy

\( \rho_{\text{REV}} \) = Volumetric \( \dot{E} \)-density

\( j_{k, \text{REA}, i} \) = \( \dot{E} \)-flux density

\( \sigma_{\text{REV}} \) = Volumetric \( \dot{E} \)-production density (source)

\( x_{k} \) = Spatial coordinates (\( x_{1}=x, x_{2}=y, x_{3}=z \))

In case of CHAMPS: \( E = m_{w+v} \) liquid water and vapor mass

\( m_{a} \) dry air mass

\( m_{w} \) VOC mass

\( U \) internal energy

In case of CHAMPS problems, the masses of liquid water, water vapor, dry air, a pollutant component and the internal energy are the unknowns of the system. Eliminating the phase transmutation term (evaporation or condensation rate) the balances liquid water and water vapor are added to a total moisture mass balance. Assuming local thermodynamic equilibrium between the liquid and the gas phases, the liquid water mass and the water vapor mass can be reduced to one unknown variable: the moisture mass.

Numerical Solution

The balances for dry air mass and heat / moisture mass are solved separately by a staggered solution technique. First, the air mass balance is solved under time-stepwise quasi-steady-state conditions. Then the coupled heat and moisture balances are solved (together with a VOC mass balance, if requested). This solution technique (partially decoupled air mass and heat / moisture balances) is advantageous in terms of numerical simulation speed. The convergence behavior of the numerical solution is improved which is of particular importance for two-dimensional simulations under transient conditions.

The balance equations (1) can be numerically solved by using a semi-discretization technique. The solution comprises four main steps. First, balance equations are integrated over an arbitrary volume and the volume integral of the flux divergences in (2) is transformed into a surface integral by using the Gaussian theorem. Then, by discretization of the spatial coordinates (using \( N \) volume elements, each with \( M_{i=1..N} \) sides), the system of integral balance equations (3) is transformed into a system of ordinary differential equations (ODEs) (4) which can be numerically integrated in time. The solution is given by (5) as time-dependent energy and mass densities defined in the representative volume elements (REV) which were used for discretization of the problem. The densities are considered primary state variables of the system while secondary state variable (i.e. temperatures, moisture contents, pressures) can be derived from the primary ones by a decomposition algorithm.

\[
\int_{V} \frac{\partial \rho_{\text{REV}}}{\partial t} \, dV = \sum_{i=1}^{M} \int_{v} \frac{\partial}{\partial x} (j_{k, \text{REA}, i} E_{i}) \, dV + \int_{A} \sigma_{\text{REV}} E_{i} \, dA + \int_{V} \sigma_{\text{REV}} \, dV
\]

\[
\int_{V} \frac{\partial \rho_{\text{REV}, i}}{\partial t} \, dV = \int_{A} j_{k, \text{REA}, i} n_{k} \, dA + \int_{V} \sigma_{\text{REV}, i} \, dV
\]

\[
\frac{\partial \rho_{\text{REV}, i}}{\partial t} \, \Delta V_{i} = \sum_{j=1}^{M} j_{k, \text{REA}, i,j} n_{k} \Delta A_{i,j} + \sigma_{\text{REV}, i} \, \Delta V_{i}
\]

\[
\rho_{\text{REV}, i}(t) = \int_{0}^{t} \left[ \frac{1}{\Delta V_{i}} \sum_{j=1}^{M} j_{k, \text{REA}, i,j} n_{k} \Delta A_{i,j} + \sigma_{\text{REV}, i} \right] \, dt
\]

It is advantageous to integrate (5) by using numerical solver software. DELPHIN4, Delphin 5 and CHAMPS-BES make use of the CVODE package which is a solver for systems of ODEs being part of the SUNDIALS suite developed at Lawrence Livermore National Laboratory at the University of California (Hindmarsh et al. 2005). The CVODE solver allows application of multi-step methods of variable order performing high flexibility at good simulation speed.
Air Mass Balance

Only in exceptional cases, the air mass balance needs to be solved in a fully coupled manner. For building-practical cases, a time step-wise stationary solution might be applicable. This approach can be justified. It can be shown that under application of a jumping boundary condition the time constant for the change in air mass balance is by a factor of $10^5-10^7$ smaller than that for the energy balance. The moisture transport is even slower than the energy. Therefore, the following assumptions are adopted in formulating the air mass balance in the CHAMPS program:

1. The gas phase (moist air) consists of dry air, water vapor and VOC; dry air (which consists of $\text{O}_2, \text{N}_2$, and other minor components) is treated as one component;
2. The gas phase is incompressible and volume fraction of water vapor in the CHAMPS program:
3. Only laminar gas flow is considered; the dry air pressure gradient is taken into account as its driving force; the flow resistance caused by friction forces can be approximated by a transport coefficient, the air permeability, to be introduced by a constitutive equation;
4. The staggered solution implies that the air balance can be solved under heat and moisture conditions of the previous time step; there is no influence of changing hygrothermal conditions on transport of air during the current time step;
5. The transmutation of kinetic energy of the gas phase to internal energy by compression and friction is negligible;
6. The ideal gas equation of dry air can be used for moist air; intrinsic air density means dry air mass divided by volume of the gas phase.

Under above considerations, the air mass balance reads as

$$\frac{\partial \rho_{\text{REV}}}{\partial t} + \nabla \cdot \left( \rho_{\text{REV}} \mathbf{v}_{\text{air}} \right) = \sigma_{\text{REV}}$$  (6)

with

- $\rho_{\text{REV}}$: Intrinsic air density, kg/m$^3$
- $\mathbf{v}_{\text{air}}$: Convective air flux, m$^3$/m$^2$s
- $\sigma_{\text{REV}}$: Air sources/sinks in reference volume, kg/m$^3$s

The air mass balance (6) can be applied to air permeable materials and air spaces with friction-dominated transport (with gap width < 2 * thickness of the boundary layer). It probably fails for large air spaces because of the third assumption above.

The buoyancy effect is included by the additional volumetric driving force term (gravity acceleration times intrinsic air density - the gravity acceleration is supposed to be a vector as the air pressure gradient and the convective air flux as well). It builds up air pressure stratification in equilibrium cases. For thermal non-equilibrium (i.e. hot and cold side of an air space), there is an imbalance in volumetric forces to be compensated by the pressure field leading to typical rotating air flux pattern (warm side upward, cold side downward). The air sink/source is a general term for description of a problem-specific time dependent function.

3D-Modeling of Air Flows

From a practical point of view, the contribution of air flux to the hygrothermal behavior of envelope systems should not be neglected. To account for air flux in three dimensions, a rather simplified approach is applied here (the combination of fluid dynamics and heat and mass transport in porous media remains a challenging field of research). In respect to the air mass balance above, two additional airflow contributions are considered:

- air flux in z-direction (assuming discretization of the problem in the x-y plane) and
- exchange between internal air spaces and the indoor / outdoor air.

The contribution of both air fluxes to the heat and moisture balances can be modeled by internal sources and sinks.

$$m_{\text{airchange}} = \rho_{\text{air}} c_{\text{v}} \left( \frac{\partial c_v}{\partial z} \mathbf{v}_{\text{air}} + (c_{v,\text{exterior}} - c_v) \cdot n_{\text{airchange}} \right)$$  (7)

Moisture source in kg/m$^3$s

$$U = \rho_{\text{air}} c_{\text{v}} \left( \frac{\partial T}{\partial z} \mathbf{v}_{\text{air}} + (T_{\text{exterior}} - T) \cdot n_{\text{airchange}} \right)$$  (8)

Energy source in J/m$^3$s

- $\rho_{\text{air}}$: Density of air, kg/m$^3$
- $c_{\text{v}}$: Specific heat capacity of air, J/kgK
- $\mathbf{v}_{\text{air}}$: Volumetric air flux, m$^3$/m$^2$s
- $n_{\text{airchange}}$: Air change rate, 1/s
- $c_v$: Vapor mass concentration, kg/kg
- $T$: Temperature, K

Consideration of air flux in z-direction requires precalculation of the gradients of vapor concentration and temperature in z-direction, as demonstrated in the application case in the next chapter. Preferably, the biggest air flux influ-
ence is simulated first, i.e. the pre-calculation is done in the y-z-plane if the major air flux is in z-direction.

APPLICATION CASE VENTED/NON-VENTED ROOF

Validation of numerical simulation models is a complex process that is preferably done in stepwise manner. The first step should be tests on plausibility, i.e. the software should deliver expected results. Here, the model is applied to a typical roof construction that is sensitive to moisture accumulation. The effect of airflows on heat and moisture balances will be demonstrated by comparison of the results for a vented and a non-vented roof.

The roof depicted in Figure 1 used here for simulative analysis is a roof construction typically built in mixed to warm climate zones. The construction elements are:

- Tiles: red concrete or clay, thickness = 15 mm, external surface temperature in summer up to 80°C
- Battens: 30x30 mm, distance between battens: 32 cm
- Counter-battens: 50x30 mm
- Ventilation layer between the underlay and the tiles
  — Average thickness: 40 mm
  — Openings between the tiles: 30 cm/m
  — No openings at the eaves and ridge
- Bituminous underlay: sd-value = 30 m
- Ventilation layer between the underlay and the insulation
  — Thickness: approx. 75 mm
  — Openings at the eaves: 100 cm/m
  — Openings at the ridge: 50 cm/m
- Mineral wool: thickness = 80 mm
- No vapor control layer
- Gypsum board: thickness = 12 mm

The material properties of the construction elements specified above were used directly. Additional properties required for the simulation were obtained according to Table 1 using values from the DELPHIN4 material database.

Discretization of the Problem

For input to numerical simulation programs one typically selects a characteristic subsection of the construction (discretized area as shown in Figure 1) that is located next to a symmetry plane. This allows cutting off the half of the construction which reduces the size of the output data files and results in significantly higher simulation speed.

The construction has to be slightly geometrically simplified since the DELPHIN4 program uses rectangular volume elements for discretization. Curved construction parts like tiles are modeled as rectangular and no sagging of the bituminous underlay has been taken into account. The discretized roof section is presented in Figure 2. The air and the bituminous underlay are treated as building materials. In case of air a temperature depended sorption isotherm is automatically generated by DELPHIN4 from the ideal gas equation for water vapor. The bituminous underlay is discretized as well (alternatively it can be modeled just as resistance between the two air spaces with no thickness and without storage capacity).

Table 1 reports the basic material parameters used in the simulation.

Initial and Boundary Conditions

The boundary conditions of the problem are given by the climate components of Syracuse (annual course of temperature, relative humidity and radiation, see Figure 3) assigned to the top side of the roof. In addition, constant indoor climate of temperature T = 20°C and relative humidity RH = 50% is assigned to the bottom side. There are no boundary conditions at the left and right sides which are interpreted, by default, as zero flux conditions.

Beside the air temperature, the short wave radiation and long wave radiation are two climate components having major influence on the thermal behavior of the building envelope and the building as whole. The mechanisms of transformation of direct radiation into diffuse radiation are adsorption and dispersion (extinction) in the atmosphere (by ozone, carbon dioxide, water vapor, dust, pollution). Using a direct sun radiation model, the radiation flux density normal to the roof surface is calculated from the direct and diffuse sun radiation components on a horizontal surface shown in Figure 3 taking the roof parameters orientation, inclination and latitude into account. The short wave radiation is a net influx (gain) of energy of the roof.

The long wave radiation is expressed as a balance, a sum of absorption and emission. Both, absorption and emission are calculated by Boltzmann’s law that relates the radiation flux to the temperature of the radiating body. Radiating bodies are here the roof (long wave emission) and the sky (atmospheric counter radiation). The atmospheric counter radiation depends on the cloudiness and the humidity in the air.

The climatic components are reported in Figure 3 for one year starting with the 1st of January and ending with the 31st of December. The minimum temperature in February and December is below T = -20°C which might be critical to the roof since no vapor control layer is used. Vapor can enter the roof construction and condensation can accumulate if the outdoor temperature is low.

The initial conditions of temperature T = 19°C and relative humidity RH = 80% are not very decisive in this case since the total simulation time is one year and the results are evaluated after the roof had enough time to reach a quasi-stationary (recurring every year) state.

Discussion of Simulation Results for the Non-Vented Roof

The results of the simulation comprise fields of temperature, relative humidity and moisture content as well as single values as function of time like integral moisture masses or temperatures, relative humidities and moisture contents at specific critical locations. One of the critical locations is the
 resume: It is clearly shown that the roof is not well adapted to the climate. The critical moisture situation might be controlled by installation of a vapor retarder beneath the gypsum board but this is not intended here. The research interest of this paper is to test the software and the implemented air flow model under critical circumstances.

**Consideration of Air Flows**

According to the air mass balance (6) and the sink and source model (7) and (8) introduced above, the air flows of the roof are decomposed into three main components (see Figure 5):
Figure 3  Climate data from syracuse, NY: temperature (upper left), relative humidity (upper right), direct sun radiation (lower left) and diffuse sun radiation (lower right).

Figure 4  Simulation results of the non-vented roof: fields of temperature (left) and relative humidity (right) after 90 days (in the beginning of December).
• Exfiltration air flux through leakage in construction parts (gypsum board, bituminous underlay, roof tiles) and air permeable materials (insulation)

• Air exchange in the air space underneath the roof tiles with the outdoor air

• Ventilation air flux through openings at eaves and ridge.

From the solution of the air mass balance, the air pressure field is calculated in the discretized x-y-domain. The air pressures in combination with the permeabilities yield the air fluxes. Additional air exchange and ventilation air flux have no direct influence on the air pressure field, i.e. superposition of the three effects above will affect the heat and moisture balances only. No 3D-effects will be contained in the air pressure field.

Exfiltration and air exchange can be directly simulated with the input information provided by the 2D-problem (x-y-plane) above. For consideration of air flux in the third dimension, the gradients of temperature and vapor mass concentration need to be known. Since DELPHIN4 does not allow a full 3D-solution of the air flow field, the air flux in z-direction and the temperature and vapor mass concentration gradients are pre-calculated by a simulation in y-z-direction.

Pre-Calculation of Temperature and Vapor Mass Concentration Gradients

A pre-calculation of the mean temperature and vapor concentration gradients over the roof length has been carried out for the configuration as shown in Figure 6 using air flow ventilation in z-direction. Air exfiltration and air change through roof tiles has not been taken into account in this simulation. One half of the roof (5 m from the eaves to the ridge) is discretized and subjected to outdoor and indoor climate conditions at three constant air flux rates of 0 m³/hm, 1.5 m³/hm and 3 m³/hm (volumetric air flux in m³/h, given per m roof length in x-direction). The volumetric air flux density (air velocity) depends on the openings area (100 cm²/m at the eaves, 50 cm²/m at the ridge and 1000 cm²/m in the air space). Corresponding values are 0 m³/m²/h, 150 m³/m²/h and 300 m³/m²/h at the eaves or 0 m³/m²/h, 15 m³/m²/h and 30 m³/m²/h in the air space, respectively.

For the three airflow rates the temporal course of the temperatures and the vapor mass concentrations are reported as spatial mean values over the marked areas shown in the construction view in Figure 6. In Figure 7 the results (snapshot of temperature field after 12 days and the mean temperatures as function of time) are shown for the case \( V_{air} = 3 \text{ m}^3/\text{hm} \).

The values in the middle of the roof are reported to control if the temperature gradient is uniform over the roof height. It turns out that a higher air flux has a positive effect. At zero air flux there is no remarkable gradient from middle to ridge but from eaves to middle. The differences between values at eaves-middle and middle-ridge become smaller with higher air flux and are almost zero at \( V_{air} = 3 \text{ m}^3/\text{hm} \).

From the difference eaves-ridge the mean gradients can calculated according to (9) and (10). For an air flux of 3 m³/hm the mean gradients of temperature and vapor mass concentration are shown in Figure 8.

\[
\frac{\partial T}{\partial z}(t) = \frac{T_{eaves} - T_{ridge}}{h_{roof}}
\]

Mean temperature gradient over roof height, K/m

\[
\frac{\partial c_v}{\partial z}(t) = \frac{c_{v,eaves} - c_{v,ridge}}{h_{roof}}
\]

Mean vapor concentration gradient over roof height, kg/kgm

Most of the time in the year there is a cooling and drying potential (for cooler and drier outdoor air in comparison to the indoor air). There is only a short period in summertime where the temperature and the vapor concentration is higher outside. The data shown in Figure 8 are applied to the roof construction depicted in Figure 2 and the simulation is repeated under consideration of the air mass balance and additional sources and sinks as described under modeling of 3D air flows above.

Discussion of Simulation Results: Non-Vented Versus Vented Roof

As first step in validation of the air flow model rather simple conditions should be applied. Therefore, constant air flow rates have been selected for comparison of the non-vented roof with the vented roof. This is of course not a realistic scenario since the air flow depends on climatic conditions as wind speed, wind direction and temperature. In case of the non-vented roof no air flow has been taken into account. For the vented roof three air flow conditions are considered.

• Exfiltration from indoor to outdoor: 1 m³/hm (given per m roof in z-direction)

• Exchange through roof tiles: 4 m³/hm (given per m roof in z-direction)

• Ventilation from eaves to ridge: 3 m³/hm (given per m roof in x-direction)

The snapshot of relative humidity fields in Figure 9 shows a clear difference between both variants: the vented roof is much drier than the non-vented one. The time point after 270 days simulation time (at the end of May) is selected since the outside temperature has already increased after the winter time. While in the non-vented roof there is still a high moisture content which slowly starts to decrease the vented roof is already dry.

For comparison of the results the most critical point of the construction (the upper left edge of the spruce rafter) has been selected. The relative humidity at this point is called the critical relative humidity which is shown in Figure 10 for both variants over one year simulation time. Even if the vented roof reaches a high relative humidity level as well (which indicates that this roof type is not well adapted to cold humid climate)

\[
\frac{\partial T}{\partial z}(t) = \frac{T_{eaves} - T_{ridge}}{h_{roof}}
\]
Figure 5  Consideration of air flows by superposition of exfiltration (left), exchange (middle) and ventilation (right).

Figure 6  Pre-calculation of the gradients of temperature and vapor mass concentration in z-direction and details of modeling of the y-z-section of the roof in DELPHIN4.

Figure 7  Results of pre-calculation: snapshot of temperature field after 12 days (at the 12th of October) and mean temperatures at eaves, middle and ridge as function of time.
Figure 8  Results of pre-calculation: average gradients of temperature and vapor mass concentration in z-direction as function of time for air flux of $3 \text{ m}^3/\text{hm}$ (note here: $t=0$ is the 1st of January and $t=365$ is the 31st of December).

Figure 9  Simulation results under consideration of the pre-calculated gradients of temperature and vapor mass concentration after 270 days (at the end of May): field of relative humidity of the non-vented roof (left) and of the vented roof (right).

Figure 10  Effect of air flows: Comparison of the critical relative humidity (at the edge of the rafter) of the vented and non-vented roof (total simulation time was 1 year, started at the 1st of September).
there is a remarkable influence of ventilation. The vented roof takes longer to reach the maximum relative humidity of almost 95% and it dries faster out in spring time. There is a difference of about 40% at the end of May.

The results indicate that the outdoor air will cause drying of the roof which is due to the simplified conditions under which this roof has been simulated. In real situations this can be different. Due to long wave emission the temperature in the ventilation cavity can drop below outdoor air dew-point temperature during clear nights. There may be situations where a roof could be cooled by night radiation such that outdoor ventilation actually causes deposition of moisture in the roof.

**SUMMARY AND CONCLUSIONS**

For many practical building physical problems the influence of air flux on the hygrothermal behavior of building envelope systems may play an important role which should not be neglected. The paper introduced a model for coupled heat and moisture simulation and a simplified approach to account for air flux in three dimensions. The air mass balance equation is solved prior to the heat and moisture balances under stepwise constant conditions. The constitutive air flux equation was derived by replacing the friction term in the momentum balance by a transport coefficient (application of the air permeability approach).

As a first step towards validation of the model a typical vented roof construction was simulated under simplified air flow conditions. The contribution of the air ventilation (air flux in z-direction from eaves to ridge) to the hygrothermal behavior of the roof was pre-calculated and applied to a second simulation in the x-y-plane taking into account the exfiltration and air exchange flux. The results appear reasonable and encouraging. The limitations of the model need to be clarified by further testing.

**REFERENCES**


