Cascaded ultrabright source of polarization-entangled photons

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An ultrabright source of polarization-entangled photons has been realized using type-II phase matching in the spontaneous parametric down-conversion process in two cascaded crystals. The optical axes of the crystals are aligned in such a way that the extraordinarily (ordinarily) polarized cone from one crystal overlaps with the ordinarily (extraordinarily) polarized cone from the second crystal. This spatial overlapping removes the association between the polarization and the output angle of the photons that exists in a single type-II down-conversion process. Hence, entanglement of photon pairs originating from any conjugate points on the output cones is possible if a suitable optical delay line is used. This delay line is particularly simple and easy to implement.

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I. INTRODUCTION

Entangled photon states generated by spontaneous parametric down-conversion (SPDC) in nonlinear crystals have been extensively used to test basic concepts in quantum mechanics [1]. In addition, entangled photon sources are used in an increasing number of applications, such as quantum teleportation [2], quantum communication [3], quantum cryptography [4], etc. Substantial effort is currently invested in improving the quality of those sources, in terms of both the total number of entangled states produced by the source and the degree of entanglement of those states.

Spontaneous parametric down-conversion can be used to generate entangled photon states using type-I or type-II phase matching. In type-I phase matching, the two paired down-converted photons have parallel polarizations, and they are emitted along concentric cones around the direction of the pump beam. In type-II phase matching, the two paired down-converted photons are orthogonally polarized in the ordinary and extraordinary directions of the nonlinear crystal. Those differently polarized photons are emitted along two different cones with cone axes at opposite sides of the pump beam and where the separation between the cones is determined by the cut angle of the crystal. One of the first realizations of a bright, entangled photon source made use of a type-II SPDC process in a degenerate noncollinear configuration [5] using a $\beta$-BaB$_2$O$_4$ (BBO) crystal. The cut angle of the crystal was chosen to allow some overlapping of the extraordinarily polarized cone with the ordinarily polarized cone. In such a configuration, photon pairs emerging along the overlapping areas can be used to form polarization-entangled states because their polarizations are not known a priori. These states can be simultaneously entangled in polarization, momentum direction, and energy. High pair-production rates along with polarization interference visibilities exceeding 95% were reported. However, the brightness of this type of source is, in general, not optimized, since only a small fraction of the photon pairs produced can be used to form entangled states.

A significant increase in the entangled photon flux was achieved using a different setup, in which two type-I crystals were arranged in cascade with their optical axes perpendicular to each other [6]. By pumping with two orthogonally polarized pump beams (or one beam at 45° with respect to the optical axes of the crystals) one can generate, for example, horizontally polarized photons from the first crystal or vertically polarized photons from the second. With a cw pump, these two down-conversion processes are mutually coherent and biphoton amplitudes from the two crystals can be used to form entangled states. This type of source may also be operated in pulsed mode, although a delay line must be used to allow overlapping in time of photon pairs created in the two different crystals [7]. The advantage of the cascaded crystal source is that entangled pairs can be found at any pair of conjugate points on the output cones. This configuration is also not optimal, however, since the pump is effectively divided between the two crystals.

In this paper, we report on an ultrabright source of entangled photons using type-II SPDC in two crystals arranged in cascade. The source may be operated in either pulsed or cw mode, although pulsed operation results in spectral distinguishability problems similar to those of other type-II sources [8]. The optical axes of the crystals are at angles $\psi$ and $-\psi$ (see Fig. 1) with respect to the pump beam direction. In such an architecture, the extraordinarily (ordinarily) polarized cone of the first crystal and the ordinarily (extraordinarily) polarized cone of the second crystal overlap. This overlapping removes the association between the photons’ polarizations and their direction of propagation. Hence, one can use conjugate points on any part of the output cones as a source for entangled states, providing that a suitable delay line is installed. Moreover, the polarization of the pump is
the same for the two crystals, thus eliminating the need to divide the pump.

In order to understand the role of the delay line in the cascaded type-II source, it is instructive to review the single-crystal source. Neglecting, for the time being, the complications due to pulsed pumping, the two-photon state for the single-crystal configuration may be written as

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |H_A, \tau_p + \tau_o/2 \rangle \langle V_B, \tau_p + \tau_e/2 | + |V_A, \tau_p + \tau_e/2 \rangle \langle H_B, \tau_p + \tau_o/2 | \right). \] (1)

This expression describes a state in which two photons are emitted into beams A and B (corresponding to the regions where the \(o\)- and \(e\)-polarized cones overlap, as described above). Either polarization (H or V) may be found in each of the beams, although the two photons will always be orthogonally polarized. Also included in this expression are subscripts describing the average times of emission relative to the time that the center of the pump pulse enters the crystal. The delay \(\tau_p\) is the time required for the pump to travel the full length of the crystal, while \(\tau_o\) and \(\tau_e\) are the propagation times for the \(o\)- and \(e\)-polarized photons, respectively. Since the average times of emission correspond to down-conversion events occurring at the crystal center, the ordinarily polarized photon, for example, will have an average emission time equal to \(\frac{1}{2} (\tau_p + \tau_o)\). This is the time required for the pump to travel to the crystal center and for the \(o\)-polarized photon to subsequently propagate to the exit face.

For a cw pump, the arrival time of the pump pulse is undefined and so it is the relative times of emission for the two photons that are relevant. Pairs of photons created near the output surface of the crystal have identical emission times while pairs created near the entrance face of the crystal have the largest difference in emission times. On average, the faster photon precedes the slower by \(\frac{1}{2} |\tau_o - \tau_e|\). Thus, information about the creation of a particular photon pair is carried in the relative emission times. If polarization entanglement is to be observed with this type of source, this timing information must be eliminated. One way that this may be accomplished is to allow the photons to pass through birefringent elements in order to delay the faster photon in each beam with respect to the slower by \(\frac{1}{2} |\tau_o - \tau_e|\). Including these delays, the source may then be described as emitting one photon into each beam, with an average arrival time in each beam that is independent of polarization.

In the cascaded configuration, photons of either polarization are found in all portions of the output cones, with entangled pairs found in conjugate beams. If, for example, beam A corresponds to one portion of the \(e\)-polarized cone from the first crystal (and, therefore, the \(o\)-polarized cone from the second), then the conjugate photons are found on the opposite side of the pump in beam B, corresponding to a

FIG. 1. Emission cones from a cascaded type-II SPDC for the degenerate case. The optical axes of the two crystals \(OA_1\) and \(OA_2\) are at angles \(\psi\) and \(-\psi\) with respect to the pump beam.

FIG. 2. (a) The calculated average emission times as functions of the angle \(\phi\), the angle between the \(x\) axis and the projection of the photon \(k\) vector onto the \(x-y\) plane. The emission times of the photons depend on their polarizations and on the crystal in which they were down-converted. The legends used in this figure are as follows: 1 \(e\) (1 \(o\)) is an extraordinarily (ordinarily) polarized photon originating in the first crystal; 2 \(e\) (2 \(o\)) is an \(e\) (\(o\)) polarized photon originating in the second crystal. The inset shows the values of the angles for different points on the output cones. The two shaded circles indicate the spatial directions used in this experiment. (b) Optimal delay as a function of \(\phi\).
portion of the $o$-polarized cone from the first crystal (and, therefore, the $e$-polarized cone from the second). For any two such beams [see the inset of Fig. 2(a) for an example], there are two means by which photons of different polarizations may be emitted, just as in the single-crystal case described above. In this case, however, the two processes correspond to down-conversion events in different crystals. Because the two events occur in different regions of the source, a particular photon's average emission time will depend not only on its polarization, but also on whether it originated in crystal 1 or crystal 2. Once again neglecting the complications associated with pulsed pumping, the two-photon state in this case may be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[ |H_{A,\tau_p/2+3\tau_o/2}V_{B,\tau_o+\tau_e/2+\tau_e'}\rangle + |V_{A,3\tau_p/2+\tau_e/2}H_{B,3\tau_p/2+\tau_e/2}\rangle \right],$$

where the delays are referenced to the time when the center of the pump pulse enters the first crystal. Upon comparison to Eq. (1), it is seen that additional delays are included: since the pump pulse must pass through the first crystal before reaching the second, all down-conversion events in the second crystal are delayed by $\tau_p$; and $o$- and $e$-polarized photons produced in the first crystal are delayed by $\tau_o$ and $\tau_e$, respectively, as they propagate through the second crystal. Here, $\tau_p$, $\tau_o$, $\tau_e$, and $\tau_e'$ are defined as the propagation times through one of the cascaded crystals. The prime is included because the $e$-polarized photon generated in the first crystal has a different group velocity in the second.

As with the single-crystal source, it is necessary to eliminate the correspondence between average emission time and polarization. Inspection of the time subscripts in Eq. (2) shows that the timing information contained in the relative times of emission can be eliminated if the $e$-polarized photons in beams A and B are delayed with respect to the $o$-polarized photons by

$$\tau_A = \frac{1}{2} (3\tau_o - \tau_e - 2\tau_p),$$
$$\tau_B = \frac{1}{2} (\tau_o - \tau_e - 2\tau_e' + 2\tau_p).$$

In contrast to the single-crystal case, these two delays are not identical.

In general, the average times of emission are angle dependent. This is due not only to the angle-dependent index of refraction for the $e$-polarized photons, but also to the additional crystal material encountered in the beams making larger angles with the pump beam. This can be seen in Fig. 2, in which the calculated average arrival times are plotted for different parts of the output cones. The parameters used for the calculation (crystal material and thickness, center wavelength, etc.) are the same as in the experimental setup described below. As can be seen in Fig. 2(a), the first photon to exit the crystal (on average) is $e1$, followed by $e2$, $o1$, and then $o2$. [For the two-photon state above, of course, only two photons are actually emitted: either $o1$ and $e1$, or $o2$ and $e2$. Each of the birefringent delays given in Eq. (3), then, corresponds to a delay between two potential photons, only one of which is actually present in a given beam.] Since the emission times depend on angle, the optimal delays also depend on angle in the general case. When the calculation is carried out, however, a simple constant delay is found to be satisfactory for all output angles. This can be seen in Fig. 2(b), which shows the optimal delays for various portions of the output cones. Note that the optimal delay is relatively insensitive to angle, suggesting that a single pair of birefringent delays (one for each cone) should be sufficient for all portions of the output cones.

II. EXPERIMENT

A sketch of the cascaded type-II source operated in pulsed mode is shown in Fig. 3. An argon-ion laser is used to pump a femtosecond mode-locked Ti:sapphire laser, producing a 76 MHz train of optical pulses centered near 790 nm. A second-harmonic generation (SHG) process in a 6-mm-thick LiB$_3$O$_5$ (LBO) crystal is used to convert the beam from 790 to 395 nm. The measured spectral bandwidth of the pulse in the uv is 1 nm and the average power in the second harmonic was typically 200 mW. An aperture and a pair of fused silica prisms separated by 10 cm are used to remove the fundamental before the beam is passed through two consecutive BBO crystals (1.07 mm each) in which the optical axes of the two crystals are at angles $\psi = 43.65^\circ$ and $-\psi$ with respect to the direction of the pump beam, as described above.

The alignment of the source was verified via a scan of the output pattern from the crystal pair. The output pattern consisted of two intersecting circles, just as would be expected from an identically oriented single type-II SPDC (see the inset in Fig. 2). This indicates that the output cones from the two crystals completely overlap. The output pattern was also scanned with a polarizer in front of the scanning head. Regardless of the polarizer angle, the shape of the output pattern was the same and the intensity was half the intensity.
recorded without the polarizer. As expected, both vertically and horizontally polarized photons reside on both the upper and the lower cones.

In order to demonstrate the unique character of the cascaded source, we chose to study the space-time and polarization interference of photon pairs emerging from regions where the upper and lower cones do not overlap. Two variable circular irises located 20 cm after the crystals were used to select the two specific spatial directions marked A and B in the inset of Fig. 2. Quartz plates and wedges placed after the irises served as continuously adjustable birefringent delay lines. Polarizing beam splitters preceded by half-wave plates were used as polarization analyzers, and broadband spectral filters with cutoff wavelength at 715 nm were used to reduce the background noise. The coincidence rate at the two single-photon detectors (EGdG model SPCM-AQR-14) was measured for various orientations of the half-wave plates and for various settings of the birefringent delays \( \tau_A \) and \( \tau_B \).

The normalized coincidence rate at the detectors is

\[
R(\theta_A, \theta_B; \tau_A, \tau_B) = \int \int |\hat{a}_A(t)\hat{a}_B(t')|^2 dt \, dt',
\]

where the annihilation operators correspond to the output modes of the polarizers and where the integrals are to be evaluated over the duration of the coincidence window \( T \). For polarization analyzers with settings \( \theta_A \) and \( \theta_B \), the operators \( \hat{a}_A(t) \) and \( \hat{a}_B(t) \) are related to the input operators by

\[
\hat{a}_A(t) = \cos \theta_A \hat{a}_{A,0}(t) + \sin \theta_A \hat{a}_{A,\phi}(t),
\]

\[
\hat{a}_B(t) = \cos \theta_B \hat{a}_{B,0}(t) + \sin \theta_B \hat{a}_{B,\phi}(t).
\]

The subscripts on the operators denote both the beam (A or B) and the polarization (0 or \( \phi \)). The two-photon state incident on the polarization analyzers is

\[
|\psi\rangle = \frac{1}{\sqrt{2}} [|\psi_1\rangle + e^{i\phi_0}|\psi_2\rangle],
\]

where \( |\psi_1\rangle \) and \( |\psi_2\rangle \) are the two-photon states from the first and second crystals, respectively. The relative phase \( \phi_0 \) between \( |\psi_1\rangle \) and \( |\psi_2\rangle \) is related to the phases picked up by the various fields as they propagate through the crystals. Including the birefringent delays \( \tau_A \) and \( \tau_B \), the two-photon states for the two crystals are

\[
|\psi_1\rangle = N \int dt_0 dt_p s(t_0 - \tau_A, t_p - \tau_B) \times \hat{a}_{A,0}^\dagger(t_0)\hat{a}_{B,0}^\dagger(t_p)|\text{vac}\rangle,
\]

\[
|\psi_2\rangle = N \int dt_0 dt_p s(t_0 - \tau_B, t_p - \tau_A) \times \hat{a}_{A,\phi}^\dagger(t_0)\hat{a}_{B,\phi}^\dagger(t_p)|\text{vac}\rangle,
\]

where \( N \) is a normalization constant. Note that with respect to the output from a single crystal the photons generated in the first crystal are delayed by \( \tau_A \) and \( \tau_A' \), the photons generated in the second crystal are delayed by \( \tau_B \), and the \( \phi \)-polarized photons are delayed by \( \tau_A \) and \( \tau_B \). For a pump field proportional to

\[
\exp\left[ -\left( \frac{\omega - 2\omega_t}{\sigma} \right)^2 \right],
\]

the probability amplitude is given by

\[
g(t_0, t_p) = \exp\left[ -\frac{\sigma^2}{4}(\tau_p - \tau_0)^2 + (\tau_p - \tau_0)^2 \right] 
\]

\[
\times \Pi(t_e - t_0, \tau_p - \tau_0, 0) e^{-i\omega(t_e + t_p)},
\]

where

\[
\Pi(t\; a, b) = \begin{cases} 1 & \text{for } a < t < b, \\ 0 & \text{otherwise}. \end{cases}
\]

Although this expression includes the effects of group velocity walk-off, higher-order dispersion is neglected [8]. It is also assumed that the pump pulse is transform limited. While this is not the case in the experiment described here, it has been shown elsewhere that reduced visibility in experiments of this type can be attributed to spectral distinguishability [9]. Thus, it is the coherence time of the pump that is important, rather than the pulse duration.

The duration of the coincidence window is several nanoseconds, much longer than the photon wave packets, and so the limits of integration in Eq. (4) may be extended to \( \pm\infty \). With the expressions given in the subsequent equations, the coincidence rate is then calculated to be

\[
R_c(\theta_A, \theta_B; \tau_A, \tau_B, \sigma) = \frac{1}{2} \left[ \cos(\theta_A)\sin(\theta_B) \right]^2 + \left[ \cos(\theta_B)\sin(\theta_A) \right]^2 
\]

\[
+ 2 \cos(\theta_A)\sin(\theta_A)\cos(\theta_B)\sin(\theta_B) 
\]

\[
\times V(\tau_A, \tau_B, \sigma) \cos(\phi_0 + \omega(\tau_A - \tau_B)).
\]

As can be seen from this equation, the coincidence rate depends both on the angles of the polarizers and on the delay times, as well as on the pump bandwidth. Hence, both polarization interference and space-time interference will be observed. Note that the birefringent delays have the largest effect when the factor \( \cos(\theta_A)\sin(\theta_B)\cos(\theta_B)\sin(\theta_B) \) is largest in magnitude. This occurs when \( \theta_A \) and \( \theta_B \) are \( \pm\pi/4 \).

The space-time interference is composed of a fast oscillating component given by \( \cos(\phi_0 + \omega(\tau_A - \tau_B)) \) and a slowly varying envelope given by

\[
\exp\left[ -\left( \frac{\omega - 2\omega_t}{\sigma} \right)^2 \right].
\]
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\[
V(\tau_A, \tau_B, \sigma) = \frac{\sqrt{2\pi}}{(2\tau_p - \tau_o - \tau_e)\sigma} \Pi(\tau_A + \tau_B; \tau_o, \tau_e; 3\tau_o - \tau_e - \tau_e') \\
\times \left\{ \text{erf} \left[ \frac{\sigma}{4\sqrt{2}} \left( \tau_A - \tau_B + 4\tau_p - 2\tau_o - \tau_e - \tau_e' + 2\tau_p - \tau_o - \tau_e \right) \right] \\
- \text{erf} \left[ \frac{\sigma}{4\sqrt{2}} \left( \tau_A - \tau_B + \tau_e - \tau_e' + 2\tau_p - \tau_o - \tau_e \right) \right] \right\}.
\]

(12)

For given settings of the polarization analyzers, the visibility of the space-time interference is largest when \(V(\tau_A, \tau_B, \sigma)\) is maximized. As expected, this occurs when \(\tau_A = \frac{1}{2}(3\tau_o - \tau_e - 2\tau_p)\) and \(\tau_B = \frac{1}{2}(\tau_o - \tau_e - 2\tau_e' + 2\tau_p)\). The calculated coincidence rate is shown in Fig. 4 as a function of the delay time in path \(B\). For this plot, \(\theta_A = \theta_B = \pi/4\), the delay in path \(A\) is constant and equal to \(\tau_A = \frac{1}{2}(3\tau_o - \tau_e - 2\tau_p)\), and \(\sigma\) was chosen to match experimental conditions. The oscillatory behavior and slowly varying envelope are evident in this plot. The inset shows the interference pattern on a smaller scale for \(\tau_B = \frac{1}{2}(\tau_o - \tau_e - 2\tau_e' + 2\tau_p)\). The maximum visibility of the interference pattern in Fig. 4 is 86%. The reason that it does not reach 100% is related to the pulsed nature of the pump and can be attributed to residual timing information that cannot be eliminated with delay lines [8]. The maximum visibility attains its highest value in the limit \(\sigma \rightarrow 0\), i.e., in the limit of a monochromatic pump.

With \(\theta_A = \theta_B = \pi/4\), coincidence counts were measured for various values of \(\tau_A\) and \(\tau_B\). The general procedure involved setting \(\tau_A\) and \(\tau_B\) and then measuring the coincidences as small changes were made to \(\tau_B\). Enough data points were collected to map out several fringes. This measurement was repeated many times for different starting values of \(\tau_A\) and \(\tau_B\). Figure 5(a) shows data for \(\tau_A = 31 \text{ fs} (1\text{ mm of quartz})\) and \(\tau_B = 440 \text{ fs} (14.2 \text{ mm of quartz})\), which represent the delays yielding the highest space-time visibility, as determined experimentally. The data in Fig. 5(a) have a visibility of 74%, which is somewhat below the theoretically predicted maximum of 86%. The observed fringe spacing in all our measurements was found to be 2.7 fs, which is in good agreement with the expected spacing of 2.63 fs. The small deviation of 2.5% from the expected result can be attributed to small misalignments of the crystalline quartz.

**FIG. 4.** The normalized coincidence rate as a function of the delay time in path \(B\), as computed from Eq. (11). The delay time in the path \(A\) is kept constant and equal to \(\tau_o\). The angles of the polarizers in front of the detectors are \(\pi/4\). The inset in the figure is a smaller scale of the interference pattern around \(\tau_A = \tau_B\).

**FIG. 5.** The coincidence counts as functions of the delay time in path \(B\). The delay time in path \(A\) is 31 fs and the delay time in path \(B\) was varied around 440 fs. The angles of the polarization analyzers in (a) are \(\pi/4\) and the observed visibility is 74%. The angles of the polarization analyzers in (b) are 0 and \(\pi/2\) and the observed visibility is practically zero.
wedges. The difference in the optimal delay time for path $B$ between the calculated value (410 fs) and the experimentally observed value (440 fs) can be attributed to small differences in the actual cut angle and thickness of the crystals.

The space-time interference pattern depends also on the angles of the polarizers in front of the detectors. The coincidence counts were measured for different angles of the polarizers and the results are in accordance with Eq. (11). Consider Fig. 5(b), where the coincidence rate as a function of the delay time in path $B$ is shown. The angles of the polarizers are $0$ and $\pi/2$. The delay lines in paths $A$ and $B$ were set near their optimal values, namely, 1 and 14 mm of quartz, respectively. As expected from Eq. (11) the observed visibility is nearly zero.

The visibility of the space-time interference was extracted from the coincidence data and is plotted in Fig. 6 as a function of the delay time in path $B$. The delay time in path $A$ was kept constant and equal to 31 fs. The solid line is the visibility calculated from Eq. (12).

In addition to space-time interference, the cascaded source also exhibits polarization interference, which is manifested as a variation in the coincidence rate as a function of the relative angles between the polarization analyzers. This is typically observed in the following way: One of the polarization analyzers is set to a particular angle and the coincidence rate is measured as the other analyzer is rotated. For a maximally entangled state, the visibility will be unity, independent of the setting of the first analyzer. This would be the case for the cascaded source when pumped by a monochromatic pump, as long as $\tau_A$ and $\tau_B$ were optimized. For a pulsed pump, however, the additional timing information reduces the visibility. (Of course, if one of the analyzers is set to 0 or $\pi/2$, the visibility will be 100%, regardless of the delays and of the properties of the pump. However, this is not a property of the entanglement but of the experimental setup.)

The visibility is smallest (representing the most exciting test of polarization entanglement) when the fixed analyzer is set to $\pi/4$. The visibility of the polarization interference in this case is predicted to be the same as the visibility of the space-time interference. This is evidenced in the polarization interference patterns shown in Fig. 7. For these measurements, the delay lines were kept constant around their optimal values ($\tau_A \approx 31$ fs and $\tau_B \approx 440$ fs), the first polarization analyzer was then held fixed, and coincidences were recorded as $\theta_B$ was varied. As expected, the observed polarization interference visibility is close to the space-time interference visibility observed in Fig. 5(a).

III. DISCUSSION

Practical implementations of entangled-photon sources require robustness, ease of implementation, high visibility, and brightness. The type-II cascade source is an advance in this direction, especially the increased brightness with respect to single type-II sources and the ease of implementing the required delay lines. It was shown that simple delay lines can be used to temporally overlap photons generated in the different crystals over the entire area of the output cones. Of course, one delay for each cone is difficult to realize in the region where the cones overlap. For these special points, entangled pairs may be obtained from either crystal in the usual manner [5]. This unique property of the cascaded type-II structure greatly simplifies the actual construction of ultrabright entangled-photon sources. The cascaded source...
may also be useful in experiments involving two or more pairs of photons. Unlike single-crystal configurations [10], multiple pairs of polarization-entangled photons may be generated in distinct paths with only a single pass of the pump beam.

The maximum expected theoretical visibility of the cascaded source for our experimental conditions is limited to 86%. This limitation is typical of SPDC’s pumped by a short pulse and is more pronounced in type-II SPDC’s than in type-I SPDC’s. To overcome this limitation one can choose crystals and wavelengths to minimize this effect [11] or use a specific experimental technique developed to overcome this limitation [12].

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