$\hat{x}$: $$x'' - \frac{1}{R}\left(1+\frac{x}{R}\right)\left(-\frac{x'^2}{R+x}\right) + \frac{(\gamma\beta)' x'}{\gamma\beta} =$$

$$= \frac{-q B_0}{p_0 (1+\delta)}\left(1+\frac{x}{R}\right)^2\left(1+\frac{B_{yx}}{B_0}x+\frac{B_{yy}}{B_0}y\right)$$

$1/R$

$\hat{y}$: $$y'' \left(-\frac{x'y'}{R+x}\right) + \frac{(\gamma\beta)' y'}{\gamma\beta} =$$

$$\frac{q B_0}{p_0 (1+\delta)}\left(1+\frac{x}{R}\right)^2\left(\frac{B_{xx}}{B_0}x+\frac{B_{xy}}{B_0}y\right)$$

Ordering: $\frac{x}{R}, \frac{y}{R}, x', y', \delta$ small

$\mathcal{O}(1)$

$$p_0 = \gamma_0 \beta_0 m c$$

$$p = p_0 + \delta p = p_0(1+\delta) \text{ where } \delta = \frac{\delta p}{p_0}$$

$$\frac{q B_0}{p_0} \equiv \frac{1}{R} \qquad \frac{q}{p_0} \equiv \frac{1}{R B_0}$$

$$x'' - \frac{1}{R} - \frac{x}{R^2} + \frac{(\gamma\beta)'}{\gamma\beta}x' = -\frac{1}{R}(1-\delta+\ldots)\left(1+\frac{2x}{R}+\ldots\right) \times \left(1+\frac{B_{yx}x}{B_0}+\frac{B_{yy}y}{B_0}\right)$$

$$y'' + \frac{(\gamma\beta)' y'}{\gamma\beta} = \frac{1}{R}(1-\delta+\ldots)\left(1+\frac{2x}{R}+\ldots\right) \times \left(\frac{B_{xx}x}{B_0}+\frac{B_{xy}y}{B_0}\right)$$

$$x'' - \frac{1}{R} - \frac{x}{R^2} + \frac{(\gamma\beta)' x'}{\gamma\beta} = -\frac{1}{R} + \frac{\delta}{R} - \frac{1}{R}\frac{2x}{R}(1-\delta) - \frac{1}{R}(1-\delta)\left(\frac{B_{yx}}{B_0}x+\frac{B_{yy}y}{B_0}\right)$$

$$y'' + \frac{(\gamma\beta)' y'}{\gamma\beta} = \frac{1}{R}(1-\delta)\left(\frac{B_{xx}}{B_0}x+\frac{B_{xy}}{B_0}y\right)$$

$$\vec{B} = \hat{y}(B_0 + B_{yx}x + B_{yy}y) + \hat{x}(B_{xx}x + B_{xy}y)$$

$$\vec{\nabla}\cdot\vec{B} = 0 = B_{xx} + B_{yy} : \quad B'_s \equiv B_{yy} = -B_{xx}$$

$$(\vec{\nabla}\times\vec{B})_s = 0 = \hat{s}(B_{yx} - B_{xy}) : \quad B'_N \equiv B_{yx} = B_{xy}$$

$$x'' + \frac{(\gamma\beta)' x'}{\gamma\beta} + \frac{x}{R^2} - \frac{2x}{R^2}\delta +$$

$$+ \frac{1}{R}(1-\delta)\left(\frac{B_N'}{B_0}x + \frac{B_S'}{B_0}y\right) = \frac{\delta}{R}$$

$$y'' + \frac{(\gamma\beta)' y'}{\gamma\beta} + \frac{1}{R}(1-\delta)\left(-\frac{B_N' y}{B_0} + \frac{B_S'}{B_0}x\right) = 0$$

$$x'' + \frac{(\gamma\beta)'}{\gamma\beta}x' + \left(\frac{1}{R^2} + \frac{B_N'}{RB_0}\right)x + \left(-\frac{2}{R^2} - \frac{B_N'}{RB_0}\right)\delta x$$

$$+ \frac{B_S'}{RB_0}y - \frac{B_S'}{RB_0}\delta y = \delta/R$$

$$y'' + \frac{(\gamma\beta)' y'}{\gamma\beta} - \frac{B_N'}{RB_0}y + \frac{B_N'}{RB_0}\delta y + \frac{B_S'}{RB_0}x - \frac{B_S'}{RB_0}\delta x = 0$$

$$K_x \equiv \frac{1}{R^2} + \frac{B_N'}{RB_0} \qquad K_{\delta x} \equiv \frac{2}{R^2} + \frac{B_N'}{RB_0}$$

$$S_x \equiv \frac{B_S'}{RB_0} \equiv S_{\delta x}$$

$$K_y \equiv \frac{B_N'}{RB_0} \equiv K_{\delta y}$$

$$S_y \equiv \frac{B_S'}{RB_0} \equiv S_{\delta y}$$

Note $S_x = S_{\delta x} = S_y = S_{\delta y} \equiv S$

$$x'' + \frac{(\gamma\beta)'}{\gamma\beta}x' + K_x x - K_{\delta x}\delta x + S y - S\delta y = \delta/R$$

$$y'' + \frac{(\gamma\beta)'}{\gamma\beta}y' - K_y y + K_{\delta y}\delta y + S x - S\delta x = 0$$

Effects: Acceleration $(\gamma\beta)'$

Focusing K_x, K_y

Dispersion δ/R

Chromaticity $K_{\delta x}, K_{\delta y}$

Coupling S terms.

Title: Oct 5 - 3:18 PM (2 of 2)