Simultaneous Teleportation of the Spectral and Polarization States of a Photon

Travis S. Humble, Ryan S. Bennink, and Warren P. Grice, Oak Ridge National Laboratory, Oak Ridge, TN

Synopsis

Photonic implementations of quantum teleportation transfer the quantum state of one photon onto another. Similar experiments have been implemented to teleport states encoded into either the polarization or field-quadrature degrees of freedom.

An outstanding question is how to simultaneously teleport quantum information encoded in multiple photonic degrees of freedom. The capability to teleport information carried by multiple degrees of freedom could support advanced algorithms for manipulating qubits, e.g., embedded Bell-state analysis, or help overcome experimental inefficiencies, e.g., losses due to spectral filtering.

We report how the spectral and polarization states of a single photon can be simultaneously teleported by using a Bell-state analyzer based on optical sum-frequency generation.

Broad Bandwidth SPDC

A pair of spectrally entangled photons can be generated by spontaneous parametric down conversion. In SPDC, the nonlinear medium induces a high frequency pump photon to decay into a pair of lower frequency photons. Depending on the type of SPDC, the photons may be similarly (type-II) or oppositely (type-0) polarized.

Pump Laser

\[ \text{Emission Cone} \]

The emission cones in type-II SPDC.

The SPDC process conserves energy and momentum and generates a joint spectral amplitude equal to the pump spectrum \( \Psi(\omega) \) times the phase-matching function \( \Psi(0,\omega) \):  

\[ \alpha_0 + \alpha_1 = \alpha_p \]

\[ f(\omega,\omega') = A(\omega_0,\omega) \Psi(\omega_0,\omega) \]

For type-II SPDC, the emission cones of the photons can be overlapped by adjusting the phase-matching angle. The joint spectral amplitude overlap images as intersecting rings.

As positions where the cones overlap, the two spectrally multimode photons enter in a polarization-entangled state.  

\[ |\psi_0\rangle = \frac{1}{\sqrt{2}} \left( |1,0\rangle + |0,1\rangle \right) \]

Depending on experimental conditions, the joint spectral amplitudes \( f(\omega,\omega') \) can be either equal (type-II) or related by permutation of the frequency arguments (type-0).

Spectral Entanglement

If the joint spectral amplitude is factored into the product of single-photon spectral amplitudes \( \alpha_0 \) and \( \alpha_1 \), then the photon pair is spectrally entangled.

\[ f(\omega,\omega') = \alpha_0(\omega) \alpha_1(\omega') \]

Spectral entanglement for a photon pair can be quantified with respect to the Schmidt decomposition of the joint spectral amplitude.

\[ f(\omega,\omega') = \sum_{\alpha} \lambda_\alpha |\alpha\rangle \langle \alpha| \]

The Schmidt coefficient \( \lambda_\alpha \) measures the relative amplitude of the \( \alpha \)-th Schmidt mode, \( \alpha_0 \) and \( \alpha_1 \), while either the Schmidt number \( K \) or the von Neumann entropy \( S \) quantifies the degree of spectral entanglement.

\[ K = \text{Tr} \left( \sum_{\alpha} |\alpha\rangle \langle \alpha| \right) \]

\[ S = \text{Tr} \left( \rho \log_2 \rho \right) \]

In cw-pumped SPDC, the pump spectrum \( \delta(\omega) \) is sharply peaked and the joint spectral amplitude exhibits strong correlation between the frequencies of the down-converted photons modulated by the phase-matching function.

Neglecting the effects of the phase-matching function, and approximating the latter by a constant, all the Schmidt coefficients are unity and the Schmidt modes represent delta distributions.

Quantum Teleportation

Quantum teleportation requires entanglement to establish a quantum communication channel. For teleporting the spectral and polarization states of a photon, quantum teleportation requires both spectral and polarization entanglement.

Kim et al. previously demonstrated quantum teleportation of a polarization Bell state using a complete Bell-state measurement based on a series of type-I and type-II sum-frequency generation (SFG) events.

For Kim et al., photons 1 and 2 pass through a pair of type-II SFG crystals and is measured in the diagonal basis. The polarization entanglement depends on the spectral entanglement via the overlap of the two underlying spectral amplitudes.

\[ C_{ij} = \left| \langle \eta_i | \rho | \eta_j \rangle \right|^2 \]

As measured by the concurrence, the polarization entanglement is less than maximal when the joint spectral amplitudes are distinguishable, although such states can omit certain effects, e.g., the crossed joint spectral amplitudes produced in type-II SPDC optimize Hong-Ou-Mandel interference.

Spectral Polarization Teleportation

Spectral considerations eliminate the SFG phase-matching requirements in Kim et al. but teleportation of the complete polarized states remains a challenge. We modify the complete polarization-Bell-state analyzer by cascading coherently the up-conversion photon. We show that the spectral-polarization states of photon 1 can then be teleported to photon 3.

Consider the initial photons to be in multimode analogs of the conventional polarization states.

\[ |\psi_3\rangle = \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) |\eta_3\rangle \]

Accounting for the four possible SFG events and polarization rotations applied to the up-converted photon prior to detection, we then determine the outcomes for the four possible detection events.

As an example, detection of a horizontal photon in path 3 at a frequency \( \omega_3 \) projects photon 3 into a state that resembles the initial state of photon 1.

\[ |\psi_3\rangle = \left( \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) |\eta_3\rangle \right) = \left( \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) |\eta_3\rangle \right) \]

The joint spectral amplitude serves to transfer the spectral information of photon 1 onto photon 3. For infinite spectral entanglement, the joint spectral amplitudes acts as an identity operator in the frequency domain.

\[ |\psi_3\rangle = \left( \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) |\eta_3\rangle \right) = \left( \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) |\eta_3\rangle \right) \]

Recovering the complete quantum state requires a match in the spectral state. The basis can be achieved using difference frequency generation (DFG), making perfect teleportation possible, in principle.

More generally, the fidelity for the cw-pump limit depends explicitly on the phase-matching function.

\[ F = \left| \langle \eta_1 | \rho | \eta_2 \rangle \right|^2 = \left( \left( \langle \eta_1 | \rho | \eta_2 \rangle \right) \right) \]

In summary, we have shown how the composite spectral-polarization state of a single photon can be teleported using a combination of sum- and difference-frequency generation, an effect that supports the possibility of teleporting the full quantum state of a single photon.

References


Spectral Polarization Teleportation

Schematic of a complete Bell-state measurement based on SFG where photon 3 represents multiple photons, for spectrally and polarization states of photon 1, across photon 3.