Introduction

We study broad bandwidth photon sources for quantum information protocols such as quantum teleportation and entanglement swapping. In contrast to single-mode treatments underlying many proof-of-concept experiments, our multi-mode results quantify the fidelity of experimental protocols by accounting for the joint spectral amplitude inherent to pairs of polarization entangled photons. This work addresses the role of spectral engineering in the development of quantum communication channels and related quantum information technologies, and explores other opportunities for using spectrally engineered entangled photon sources.

Single-mode Analysis

Bennett et al. (1993) showed how quantum teleportation (QT) can transport a quantum state between distant particles. Teleportation of a qubit uses a pair of entangled particles acting as a quantum communication channel, in addition to a classical communication channel.

Experimental realizations of QT using polarization entangled photons (PEP) transfer a qubit encoded in the horizontal and vertical polarizations $h$ and $v$.

Unknown qubit

Polarization entangled photons

$$|\psi_1\rangle = a|h_1\rangle + b|v_1\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} (|h_2,v_1\rangle + |v_2,h_1\rangle)$$

Braunstein and Mann (1995) proposed a linear optical method for conditionally performing a BSM, using a beam splitter and polarizing beam splitters, registered by coincidence detections.

Composite state

Coincidence detection, e.g.,

$$\langle \psi_1 | \psi_2 \rangle \rightarrow a|h_1\rangle + b|v_1\rangle$$

Multi-mode Analysis

A single-mode analysis neglects the spatial and spectra degrees of freedom inherent to a photon. We perform a multi-mode analysis of the QT protocol, focusing on

- spectral effects arising from the generation of polarization entangled photons
- spectral effects inherent to the coincidence detection scheme, i.e. Bell-state measurement
- subsequent effects of the above on teleportation fidelity
- strategies for improving QT in light of spectral effects

Spectral Effects in Entanglement Generation

With a broad bandwidth pump pulse, i.e. ultrashort, polarization entangled photons can be generated "on demand" using spontaneous parametric down-conversion (SPDC)

$$|\psi_{SPDC}\rangle = \frac{1}{\sqrt{2}} (|h_{SPDC}\rangle + |v_{SPDC}\rangle)$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}} (|h_{out}\rangle + |v_{out}\rangle)$$

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}} (|h_{in}\rangle + |v_{in}\rangle)$$

For type-II SPDC

$$|\psi_{SPDC}\rangle = \frac{1}{\sqrt{2}} (|h_{SPDC}\rangle + |v_{SPDC}\rangle)$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}} (|h_{out}\rangle + |v_{out}\rangle)$$

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}} (|h_{in}\rangle + |v_{in}\rangle)$$

Joint spectral probability (calculated) for type-II SPDC in a phase-matched 2 mm BBO crystal pumped by a 35 fs Gaussian pulse centered at 405 nm. Difference frequencies are measured with respect to half the pump energy ($\omega_p$) and darker regions indicate regions of higher probability density.

In type-II SPDC spectra correlate with the polarization

$$f(\omega,\omega') = g(\omega,\omega')$$

$$\langle \psi_{in} | \psi_{out} \rangle = \frac{1}{\sqrt{2}} (|h_{in}\rangle + |v_{in}\rangle)$$

This distinguishing information underlines interference effects (Grice and Walmsley, 1997), and complicates the linear optical BSM (Kim and Grice, 2003).

With post-production spectral engineering (Grice et al., 2001) spectra correlate with path

$$f(\omega,\omega') = g(\omega,\omega')$$

$$\langle \psi_{in} | \psi_{out} \rangle = \frac{1}{\sqrt{2}} (|h_{in}\rangle + |v_{in}\rangle)$$

with a joint spectral state

$$\langle \psi_{out} | \psi_{in} \rangle = \frac{1}{2} (|h_{out}\rangle + |v_{out}\rangle) \otimes |h_{in}\rangle$$

Spectral Entanglement

Approximate joint spectral amplitude

$$f(\omega,\omega') \approx \exp \left[ \frac{(\omega - \omega')^2}{2\sigma^2} \right]$$

$$S = \frac{1}{2} \left[ 1 + \frac{1}{\pi} \ln \left( \frac{1}{\sigma^2} \right) \right]$$

Correlation angle $\theta = \frac{\omega - \omega'}{\sigma}$

Major, minor widths $\sigma_h, \sigma_v$

Marginal widths

$$\sigma_h^2 = \sigma_v^2 = \frac{1}{2} \sigma^2$$

Spectral Entanglement Fidelity

$$F = \frac{1}{1 + \frac{1}{2} \sigma^2}$$

Teleportation Fidelity

$$F = \frac{1}{1 + \frac{1}{2} \sigma^2}$$

Maximal Fidelity

The maximal fidelity for a given correlation angle and ratio of major to minor widths can be significantly less than one, even when the spectra correlate with path. This is an indication of the extent to which spectral interference undermines the BSM.

Joint Spectral Amplitudes

Polarization Correlation

When spectra correlate with path, unit visibility can also be achieved in the typical polarization correlation experiment. But when spectra correlate with polarization the visibility depends on the shape of the joint spectral amplitude.

References cited