Quantum Spread Spectrum Communication

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Abstract: We demonstrate that spectral teleportation can coherently dilate the spectral probability amplitude of a single photon. In preserving the encoded quantum information, this variant of teleportation subsequently enables a form of quantum spread spectrum communication.

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1. Introduction

For quantum optical communication to mediate the transfer of quantum information between parties, the transmitted field must interface with a quantum physical receiver located downstream. Given the variety of solid-state systems being considered as viable candidates for these interfaces, including trapped ions, atomic vapors, and neutral atoms, physical specifications for the receiver interface can differ widely according to intrinsic physical constraints, e.g., resonances, line widths, lifetimes, etc., as well as device inhomogeneities.

We outline a method for coherently modulating the spectral bandwidth of an optically encoded qubit and, consequently, enabling a quantum transmission to be dynamically tuned to a specified receiver. This method, based on spectral teleportation, serves as a quantum analog of spread spectrum communication. The latter is a well-established classical communication technique that broadens the bandwidth of a signal and, in the classical context, has proven useful to prevent jamming of a transmission and to guarantee a low probability of intercept. These capabilities should also hold true in the context of quantum spread spectrum communication (QSSC), which additionally permits the carrier waveform of the quantum information to be coherently diluted, thereby preserving the encoded quantum state for an optimized coherent interaction with a quantum receiver.

2. Spectral Teleportation

Briefly, the spectral state of a single photon

$$|\psi_1\rangle = \int \alpha(\omega)|\omega\rangle d\omega$$

is expressed in terms of a normalized spectral probability amplitude $\alpha(\omega)$, where the ket $|\omega\rangle$ designates the spectral eigenstate of photon 1 at frequency $\omega$. We assume the spectral amplitude has a mean frequency $\tilde{\omega}$, and a characteristic bandwidth $\sigma_\omega$. Spectral teleportation onto a remote photon 3 is mediated by photon 2, where the latter pair is in the spectrally entangled state

$$|\varphi_{23}\rangle = \int d\omega'| \int d\omega'' f(\omega',\omega'')|\omega'\rangle|\omega''\rangle.$$

States of the form (2) are readily prepared, e.g., using spontaneous parametric down-conversion driven by a pump pulse of carrier frequency $\tilde{\omega}_0$. The entanglement of the state (2) is quantified with respect to the Schmidt decomposition of the joint spectral amplitude $f(\omega',\omega'')$, i.e., the spectral Schmidt number $K$ is given by

$$K = (\sum_n \lambda_n^2) / (\sum_n \lambda_n^2)$$

with $\lambda_n$ the Schmidt coefficient of the $n$th pair of Schmidt modes. Note that $K$ is unity for a normalized, spectrally unentangled joint amplitude, e.g., when the photons are spectrally indistinguishable, whereas $K$ approaches infinity as the spectral correlation between photons increases.

Spectral teleportation is initiated by frequency up-conversion of photons 1 and 2 into photon 4. This entangling transformation is followed by a spectrally resolved measurement of photon 4, e.g., at a frequency $\Omega$, after which the state of photon 3 is given by

$$|\psi_3\rangle = \eta \int d\omega \int d\omega' \alpha(\omega) f(\Omega - \omega, \omega')|\omega'\rangle,$$

where $\eta$ is the normalization factor. It is immediately apparent from Eq. (4) that in the limit that the joint spectrum approaches the Dirac delta distribution, the state of photon 3 is related to the original state of photon 1 by the spectral shift $\Delta = \Omega - \omega_0$. The latter shift can be applied, e.g., using an acoustic optic modulator driven at the frequency $\Delta$. Applying this measurement-dependent shift to the spectral state of photon 3 produces the original spectral state of photon 1, and, consequently, the spectral teleportation fidelity is unity. High fidelity teleportation of the spectral state is to be distinguished from performing an coherent transformation of the spectral amplitude for subsequent interaction with a solid-state device, as the latter may yield a state with poor (teleportation) fidelity.
3. Quantum Spread Spectrum Communication

Consider spectral teleportation for the case of a joint amplitude having marginal bandwidths $\sigma_2$ and $\sigma_1$ for photons 2 and 3, respectively. In the limit of infinite spectral entanglement, let the joint amplitude approach the form of a skewed Dirac delta distribution over the relevant bandwidth, i.e.,

$$f(\omega, \omega') \rightarrow \delta(\omega_0 - \omega - \kappa \omega')$$

(5)

with $\kappa = \sigma_2 / \sigma_1$ being the bandwidth ratio, cf. experimental preparation below. Substituting (5) into (4) yields

$$|\tilde{\psi}_1\rangle = \kappa^{1/2} \int d\omega \alpha(\kappa \omega)|\omega - \Delta\rangle_1,$$

(6)

which represents the spectral state of photon 1 dilated by the factor $\kappa$ and shifted by the amount $\Delta' = (\Omega - \omega_0)/\kappa$. By using an appropriate choice of $\kappa$, and applying a measurement-dependent spectral shift to photon 3, the dilated spectral amplitude can be made resonant with the spectral profile of desired quantum receiver. Moreover, as broadening the spectral amplitude narrows the temporal waveform, spectral dilation may better resolution in the time of arrival. Hence, QSSC should prove useful as an optical front-end in narrow bandwidth, solid-state quantum transceivers that require shaping of the transmitted/received waveforms.

As a result of spectral teleportation, the mean frequency of the original spectral amplitude is shifted by the amount $\Delta$, which depends on both the detected frequency $\Omega$ and the bandwidth ratio $\kappa$. This randomized shifting of the spectral mean represents a form of classical frequency hopping spread spectrum techniques, albeit using a truly random noise code to frustrate interception. Of course, by replacing photon 1 with a spectrally entangled biphoton state, it should also be possible to use spectral teleportation to relay and distribute these randomized frequency correlations between remote parties, and thereby generate a suitable pseudo-noise code for subsequent (classical) transmissions, a la QKD.

Dilation of the amplitude $\alpha(\omega)$ is controlled by tuning the form of the joint amplitude. Of particular interest is the joint spectrum produced from type-II SPDC, which is sensitive to experimental control. For example, matching the crystal birefringence and pulse bandwidth yields a strongly correlated joint spectrum having finite but disparate marginal bandwidths, cf. Eq. (5). However, finite spectral entanglement should be anticipated. The performance of QSSC under more realistic experimental conditions, e.g., finite bandwidth and entanglement, is presented in Fig.1. By invoking a Gaussian approximation for the joint amplitude to dilate a Gaussian amplitude $\alpha(\omega)$ having linear frequency chirp $\phi$, we calculate the QSSC fidelity as the overlap between the dilated initial state and the resulting teleported state. These results are plotted with respect to spectral entanglement, as quantified in terms of the spectral cross correlation of the joint Gaussian state.

![Quantum Spread Spectrum Communication Fidelity](image)

Fig. 1. QSSC fidelity with respect to spectral entanglement for the cases (a) $\sigma_1 \rightarrow 2\sigma_1$ with $\kappa = \frac{1}{2}$ and $\phi = 0$, (b) $\sigma_1 \rightarrow 2\sigma_1$ with $\kappa = \frac{1}{2}$ and $\phi = \frac{1}{2}\sigma_1$, and (c) $\sigma_1 \rightarrow 2\sigma_1$ with $\kappa = 1$ and $\phi = \frac{2}{\sigma_1}$. The inset plots the initial, unchirped and the dilated, chirped waveforms. While cases (a) and (b) reach unit fidelity in the limit of maximal entanglement, case (c) uses a suboptimal value of $\kappa$ for the spectral dilation $\sigma_1 \rightarrow 2\sigma_1$.

4. Conclusions

By using spectral teleportation to coherently dilate the spectral amplitude, we have demonstrated that the spectral state of an optical carrier can be tuned, e.g., to interact with a downstream quantum receiver. As with classical spread spectrum techniques, quantum spread spectrum communication may also be useful to prevent jamming in transit, establish a low probability for intercept, temporally compress transmission duration, or frequency hopping.

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