

A Spatio-Temporal Drought Analysis for the Midwestern US



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Dr. Shih-Chieh Kao
Oak Ridge National Laboratory, CSE
kao@ornl.gov
<http://www.ornl.gov/~5v1/>

Dr. Rao S. Govindaraju
Purdue University, Civil Engineering
govind@purdue.edu

Dr. Dev Niyogi
Purdue University, Agronomy, EAS
climate@purdue.edu

Drought Overview

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● Challenges

- No clear (scientific) definition
- Phenomenon dependent in
 - Time and space
 - Between various variables (e.g. precipitation, streamflow)

● Classification of droughts

- Meteorological drought
- Hydrologic drought
- Agricultural drought

● Various drought indices

- Palmer Drought Severity Index (PDSI), Crop Moisture Index (CMI), Surface Water Supply Index (SWSI), Vegetation Condition Index (VCI), CPC Soil Moisture, Standardized precipitation index (SPI)

Sept., 2007, George H. Sparks Reservoir
Lithia Springs, Georgia



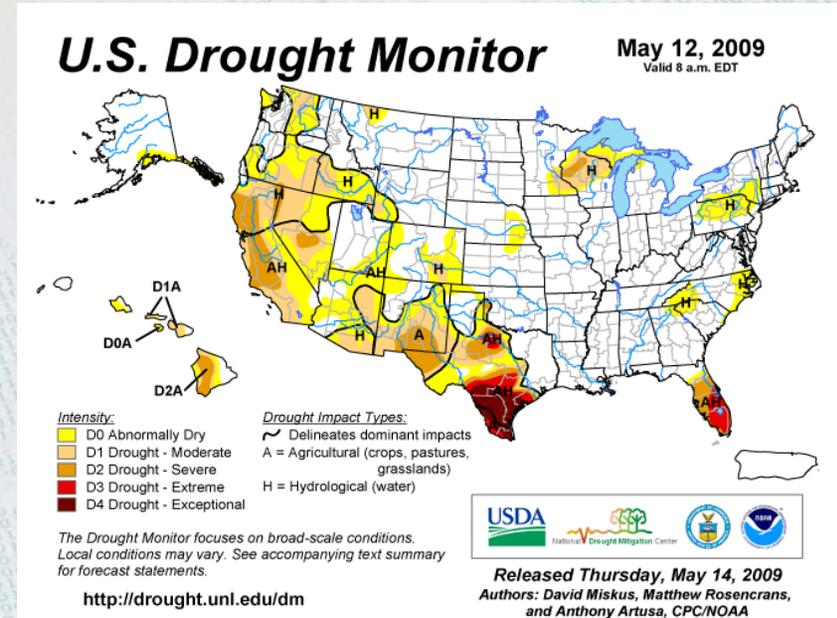
(Barry Gillis, <http://www.drought.unl.edu/gallery/2007/Georgia/Sparks1.htm>)

US Drought Monitor

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- Overall drought status (D0 ~ D4) determined based on various indices together (Svobada *et al.*, 2002)
 - PDSI
 - CPC Soil moisture
 - USGS weekly
 - Percentage of normal
 - SPI
 - VCI



<http://drought.unl.edu/DM/MONITOR.html>

- Linear combination of selected indices (OBDI, objective blend of drought indicator) was adopted as the preliminary overall drought status
- The decision of final drought status relies on subjective judgment

Research Objectives & Study Area

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- **Research Objectives**

- Exploring the dependence structure between various drought indices
- Develop probability-based joint drought index

- **NOAA time bias corrected divisional dataset (TD-9640)**

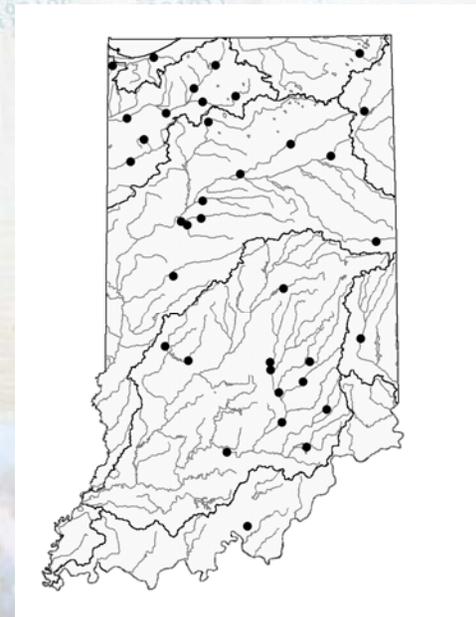
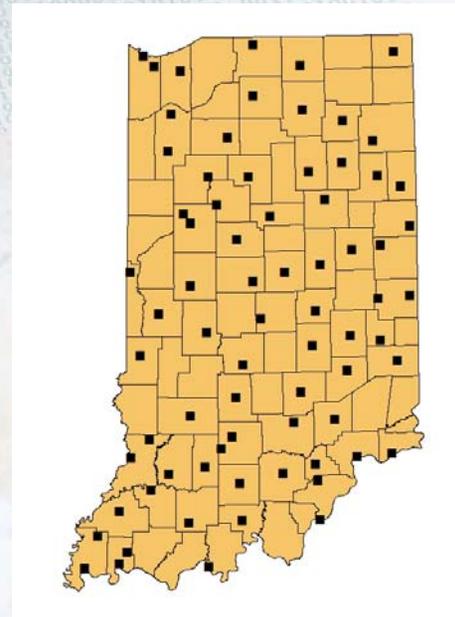
- Monthly precipitation, temperature, SPI, PDSI
- 1895 ~ present

- **NOAA daily precipitation dataset (TD-3200)**

- 73 stations (>80 years)

- **USGS unregulated daily mean flow**

- 36 stations (>50 years)



Standardized Index Method



- **Proposed by McKee *et al.* (1993)**
- **Generalizable to various types of observations**
 - For precipitation: SPI
- **For a given window size, the observed precipitation is transformed to a probability measure using Gamma distribution, then expressed in standard normal variable**

Probabilities of Occurrence (%)	SI Values	Drought Monitor Category	Drought Condition
20 ~ 30	-0.84 ~ -0.52	D0	Abnormally dry
10 ~ 20	-1.28 ~ -0.84	D1	Drought - moderate
5 ~ 10	-1.64 ~ -1.28	D2	Drought - severe
2 ~ 5	-2.05 ~ -1.64	D3	Drought - extreme
< 2	< -2.05	D4	Drought - exceptional

- **Though SIs for different windows are dependent, no representative window can be determined**

Co-occurrence of Droughts



- **Precipitation SIs $\{u_1, u_2, \dots, u_{12}\}$ and streamflow SIs $\{v_1, v_2, \dots, v_{12}\}$ are selected**
 - Annual cycle accounts for the seasonal effect naturally
 - Allow for a month-by-month assessment for future conditions
- **Dependence structure**

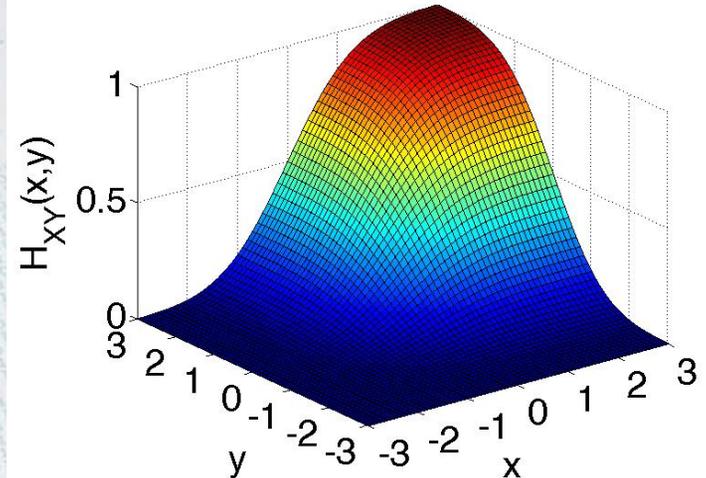
Spearman's r_{ij} between u_i and u_j

		Spearman's r_{ij} between u_i and u_j											
		i	1	2	3	4	5	6	7	8	9	10	11
Spearman's r_{ij} between v_i and v_j	j	1	0.71	0.57	0.48	0.41	0.38	0.37	0.36	0.35	0.33	0.31	0.30
	2	0.89	0.82	0.70	0.61	0.55	0.53	0.51	0.49	0.47	0.44	0.42	
	3	0.80	0.93	0.87	0.76	0.69	0.64	0.61	0.59	0.56	0.54	0.51	
	4	0.73	0.85	0.94	0.90	0.81	0.75	0.70	0.67	0.65	0.62	0.60	
	5	0.67	0.78	0.87	0.95	0.92	0.85	0.79	0.75	0.72	0.69	0.67	
	6	0.63	0.72	0.81	0.89	0.96	0.93	0.87	0.82	0.78	0.75	0.73	
	7	0.59	0.68	0.75	0.83	0.90	0.96	0.94	0.89	0.85	0.81	0.78	
	8	0.57	0.64	0.72	0.79	0.85	0.91	0.97	0.95	0.90	0.86	0.83	
	9	0.55	0.62	0.69	0.75	0.81	0.87	0.93	0.97	0.96	0.91	0.88	
	10	0.53	0.60	0.66	0.72	0.78	0.83	0.89	0.94	0.98	0.96	0.92	
	11	0.51	0.58	0.64	0.70	0.75	0.81	0.85	0.90	0.94	0.98	0.96	
	12	0.50	0.56	0.62	0.68	0.73	0.78	0.83	0.87	0.91	0.95	0.98	

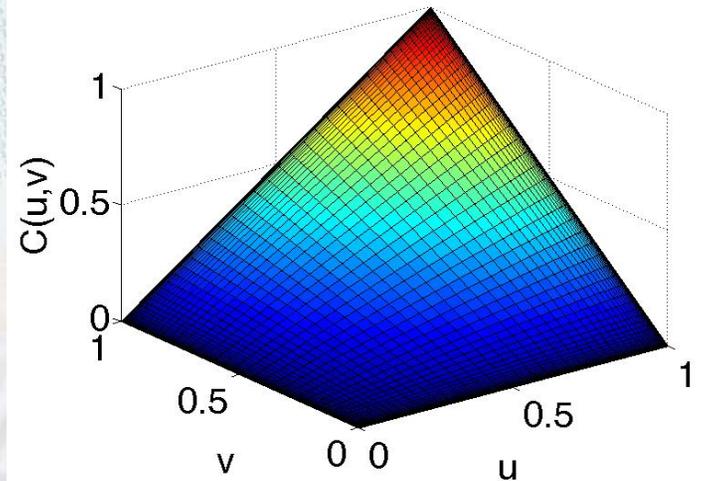


- **Transformation of joint cumulative distribution**
 - $H_{XY}(x,y) = C_{UV}(u,v)$
marginals: $u = F_X(x)$, $v = F_Y(y)$
 - Sklar (1959) proved that the transformation is *unique* for continuous r.v.s
- **Use copulas to construct joint distributions**
 - Marginal distributions => selecting suitable PDFs
 - Dependence structure => selecting suitable copulas
 - Together they form the joint distribution

Bivariate Gaussian distribution, $\rho = 0.1$



Gaussian Copulas, $\rho = 0.1$



Higher Dimensional Copulas



- **Limited choices because of high mathematical complexity**

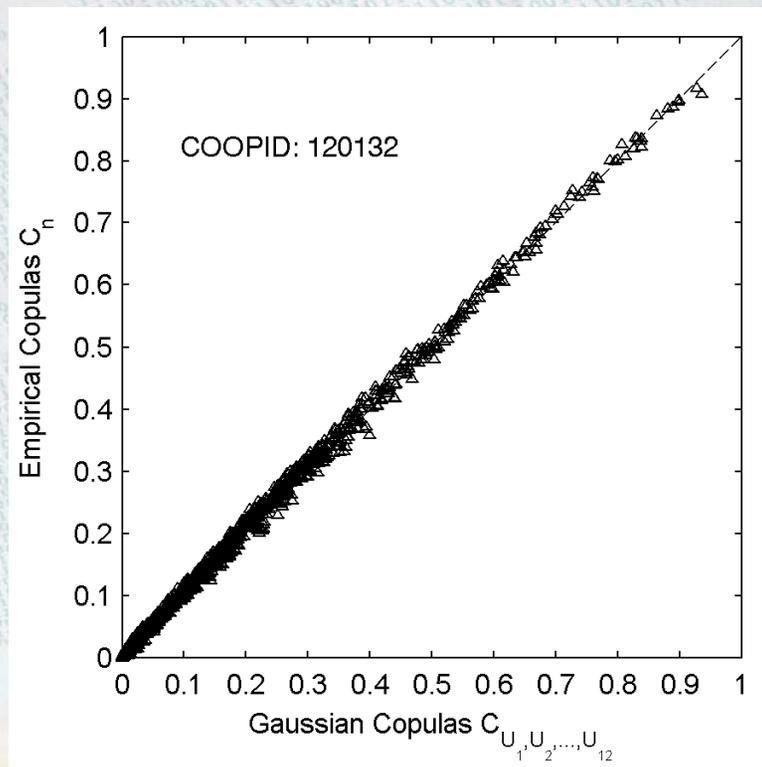
- **Gaussian copulas**

- Derived from the well-known multivariate normal distribution
- Preserving all bivariate marginal dependencies through the correlation matrix Σ

- **Empirical copulas**

- Multi-dimensional rank-based probabilities
- Treated as the observed probabilities when performing model verification

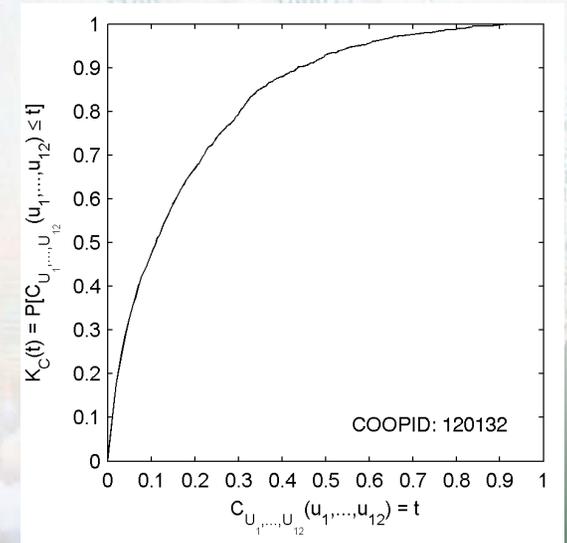
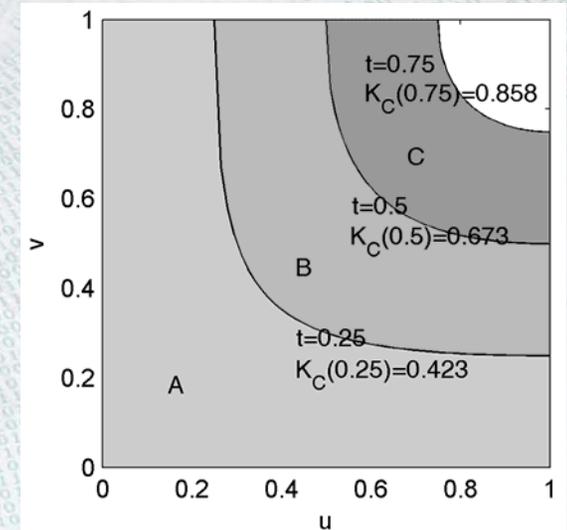
- **Empirical copulas were adopted in this study**



Joint Deficit Index (I)



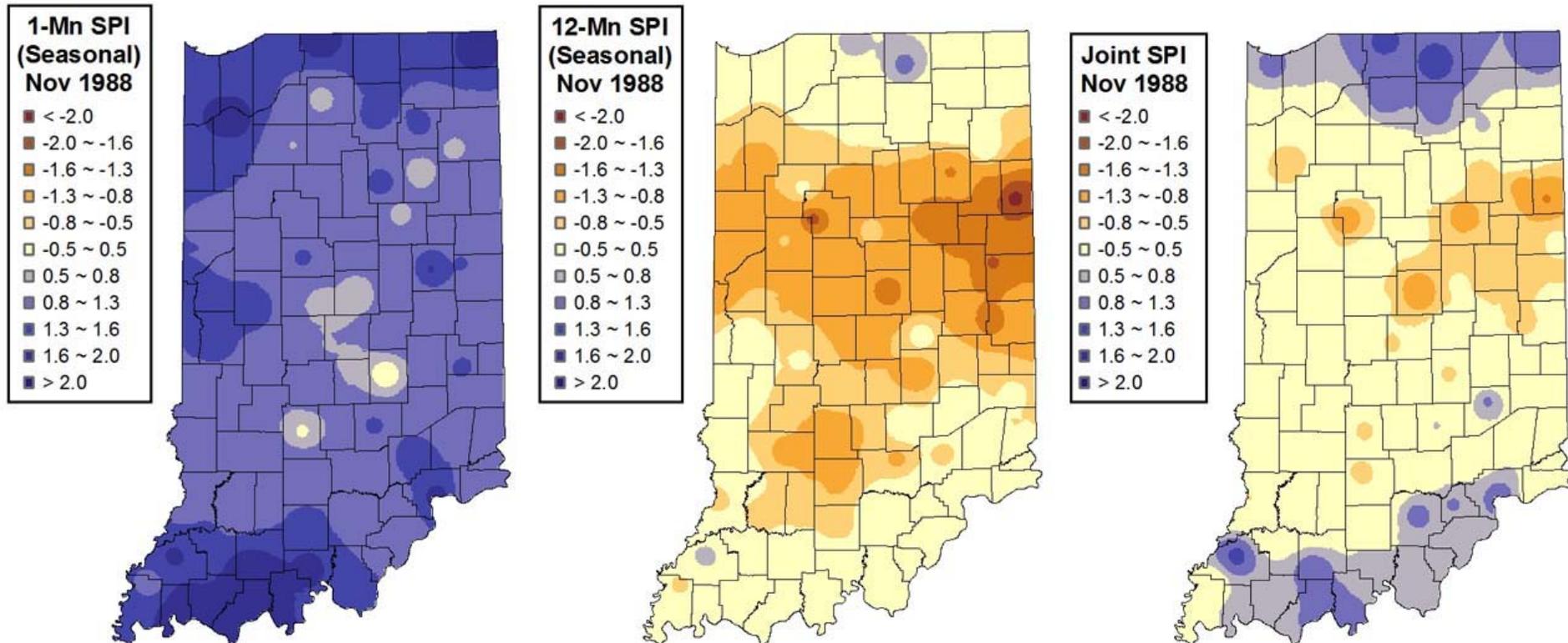
- **Assumption: events with the same value of copulas (joint cumulative probability) have same severity**
 - Copula values are treated as joint deficit status
- **Distribution function of copulas $K_C(t)$**
 - Give probability measure for events with $C(u_1, u_2, \dots, u_{12}) \leq t$
- **Joint deficit index (JDI)**
 - $JDI = \Phi^{-1}(K_C)$
 - Share the same classification with SI



Joint Deficit Index (II)



- **Comparison between 1-mn SPI, 12-mn SPI, and JDI**
 - 12-Mn SPI changes slowly, weak in reflecting emerging drought
 - 1-Mn SPI changes rapidly, weak in reflecting accumulative deficit
 - JDI reflects joint deficit



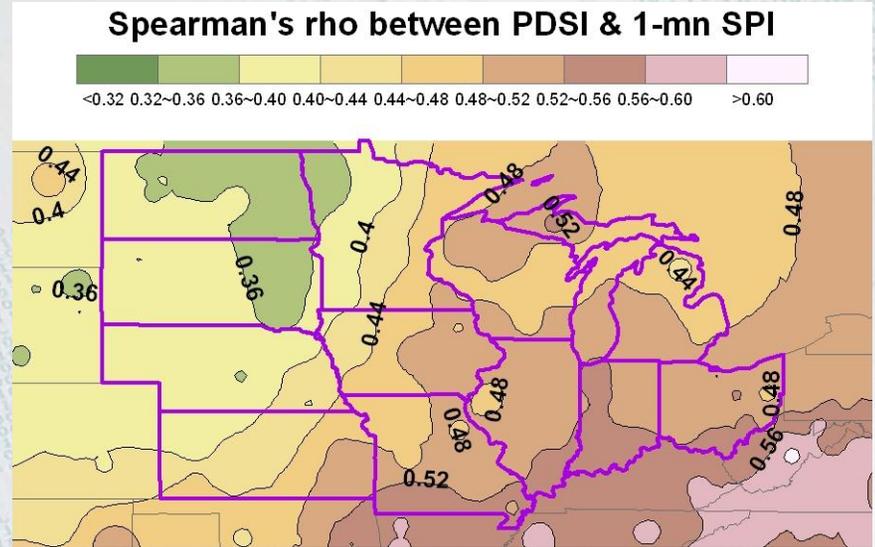
In Comparison with SPI & PDSI



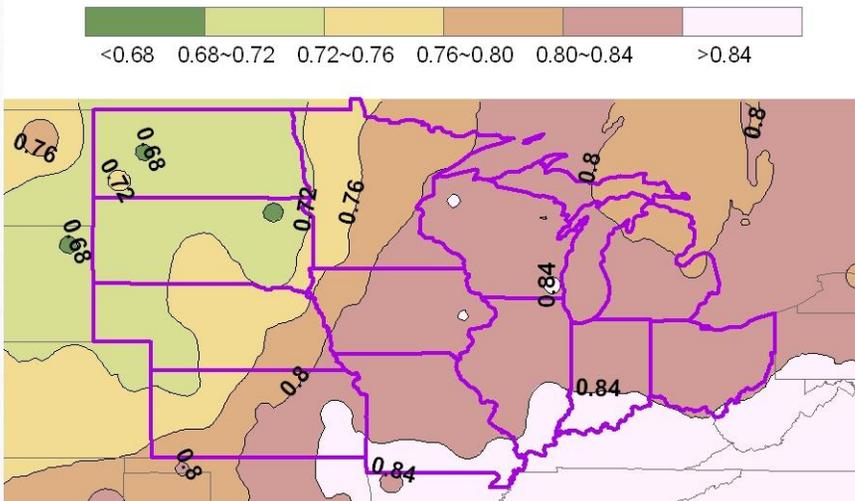
PDSI

- Based on both precipitation and temperature
- Highly correlated to JDI

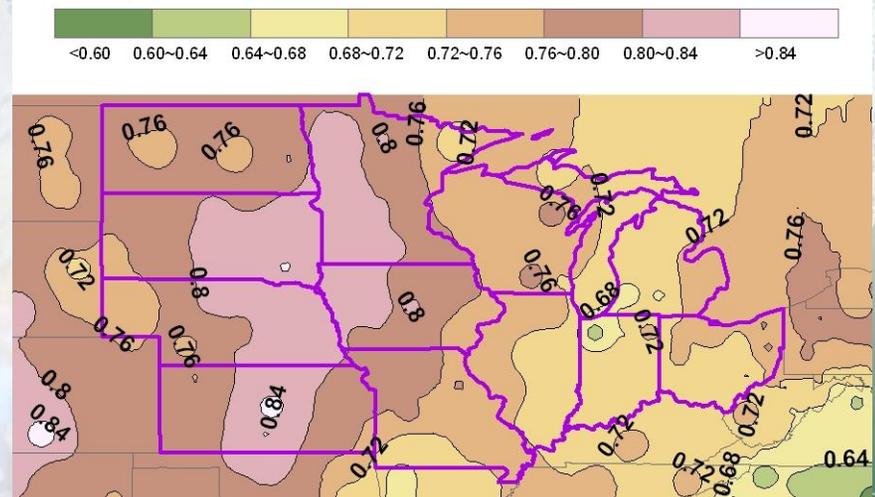
	SPI ₁	SPI ₃	SPI ₆	SPI ₉	SPI ₁₂	JDI
JDI	0.72	0.80	0.79	0.76	0.70	
PDSI	0.45	0.66	0.75	0.77	0.76	0.79



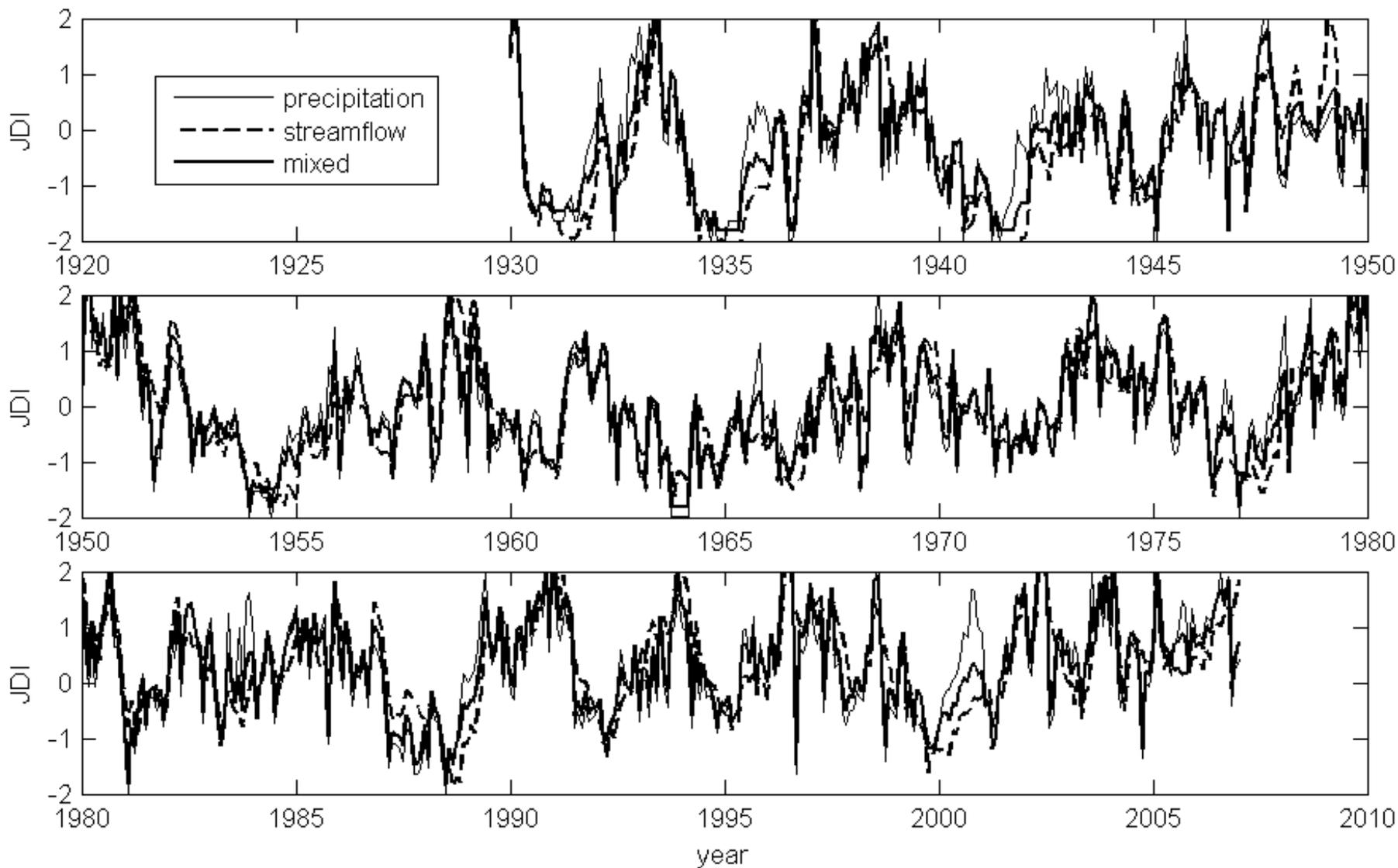
Spearman's rho between PDSI & JDI



Spearman's rho between PDSI & 12-mn SPI



Precipitation vs. Streamflow

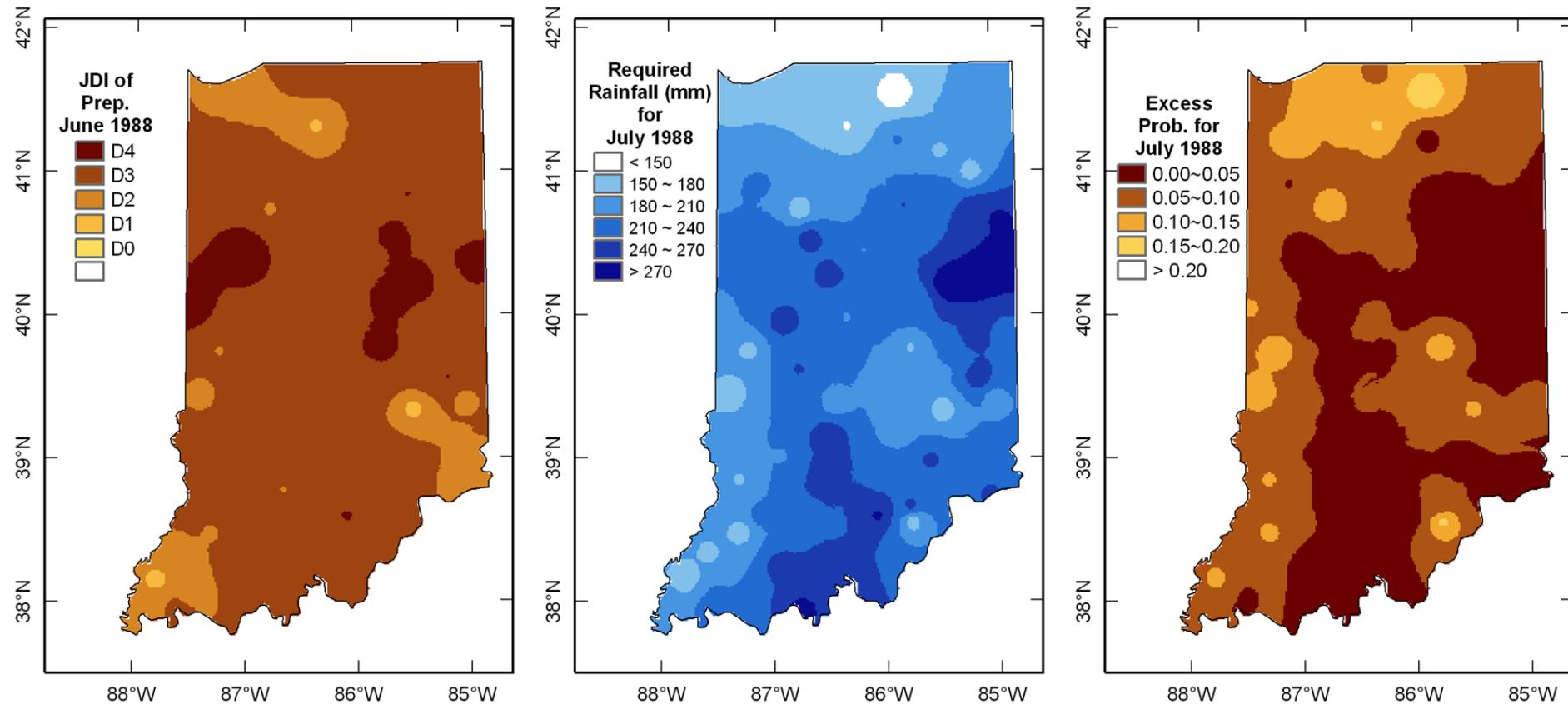


Potential of Future Droughts



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- Required precipitation for reaching joint normal status ($K_C = 0.5$) in the future
- Probability of drought recovery



Conclusion



- **JDI can offer an objective and probability-based overall drought description. It is capable of capturing both emerging and prolonged droughts in a timely manner**
- **High correlation was founded between PDSI and JDI, suggesting the applicability of JDI.**
- **JDI has potential to be applied on different types of hydrologic variables, and can be used to derive an inter-variable drought index**
- **Potential of future droughts can be assessed by using JDI, where the required precipitation and its exceedance probability can be determined**



Thank you
Questions?

Dr. Shih-Chieh Kao
kao@ornl.gov; <http://www.ornl.gov/~5v1/>